

SUGGESTIONS FOR TEACHING
on the implementation of practical classes
by discipline
DSP Technology and Application

11.03.02 Infocommunication Technologies and Communication Systems
"Intelligent Data Processing in Infocommunication Systems and Networks"

Practical lesson No 1 INVESTIGATION OF A NON-RECURSIVE DIGITAL LOW-PASS FILTER

1. Purpose of the work

One of the most important features of non-recursive digital filters is that, with the appropriate choice of parameters, they can have a strictly linear phase-frequency response (PFC). This property is, for example, decisive if it is necessary to ensure undistorted signal conversion. Non-recursive digital filters are also used in cases where the requirements cannot be met with Butterworth and Chebyshev filters, for example, to perform differentiation and integration of signals. The aim of the work is to study the features of non-recursive digital LPFs with linear PFC, to master the method of calculating its coefficients by the weighting method and to study the effect of order N and the sampling period T on the frequency characteristics of the filter.

Basic concepts and calculation formulas 1.1. Suggestions for Teaching In this work, a harmonic analysis of a periodic signal formed from pulses of a given shape is carried out (see Appendix 1). Fig. 1.2 shows a graph of a signal derived from triangular pulses.

Fig. 1.2. Example of a periodic signal

When calculating the coefficients of the Fourier series, it is necessary to specify the initial t_0 of the integration period. As a rule, the value of t_0 is chosen from the computation simplification condition. We recommend taking $t_0=0$

. The lower and upper limits of integration will be equal to 0 and T_c , respectively. Note that the standard function in the MatchCAD programming environment defines the main values limited by limits. Therefore, if the formula recommended in the literature is used to determine the harmonic phase, you can get incorrect results. In order to get the correct results, the signs of the coefficients should be taken into account and the necessary corrections should be made to the calculation formulas. When performing the work, we suggest using the function available in the MatchCAD system.

Programme of work Main Task

1. Get analytical expressions for Fourier series expansion coefficients of the

periodic sequence of square pulses. Calculate the values of harmonics for arbitrary $T\tau$.

2. To form a mathematical model in the MathCAD environment for a given signal. Plot the signal. *Note: The shape of the signal under study and the values of the signal parameters are set by the teacher in accordance with Appendix 1 and Table 1.1.*

Table 1.1

Parameters	Variant Numbers							
	1	2	3	4	5	6	7	8
A, B	1	2	5	10	20	25	40	50
τ , ms	0.3	0.5	0.4	0.5	0.8	1.2	0.8	1.2
$T_c T_c$, ms	0.4	0.7	0.6	0.8	1.0	1.5	1.0	1.5
T, ms	0.5	0.8	1.0	1.2	1.5	2.0	2.4	3.0

1. Create a program to calculate the coefficients. Calculate the values of these coefficients and enter them in the table. Build amplitude and phase spectral diagrams based on the data obtained.

2. To form an approximating signal by summing a constant component and a given number of first harmonics. Build graphs of the original and approximating signals on one figure. Draw conclusions.

3. Get error signals for different values of N. Plot their graphs.

4. Calculate the values of the standard mean errors for different N. Draw conclusions about the impact of the number of counted harmonics on the magnitude of the mean quadratic error.

5. Calculate the average power of the signal under study and the average power of the periodic signal described by the truncated Fourier series. Draw conclusions about the impact of the number of harmonics on the value of the average signal power.

Control questions and tasks

1. Describe the basic properties of periodic signals.
2. Plot the first few basic functions of the Fourier series.
3. Explain the features of the decomposition of odd and even functions into a Fourier series.

4. Write down the formulas that relate the coefficients of the trigonometric and complex Fourier series.
5. How will the change in the countdown position affect the Fourier series values?
6. How does the periodic spectrum change if the scale is changed along the time axis?
7. How will the spectrum of the sequence of square pulses change if the duration and the period are reduced by two times?
8. Why can't discontinuous functions have an exact approximation of Fourier series?
9. What is the Gibbs effect?

Practical Lesson No 2

HARMONIC ANALYSIS OF NON-PERIODIC SIGNALS

Purpose of the work Most of the signals that are processed are non-periodic in nature. A feature of the harmonic analysis of non-periodic signals is that the relationship between the time function and its image in the frequency region is determined by integral relations that make up a pair of Fourier transforms. The purpose of the work is to study the direct and inverse Fourier transformations and to acquire practical skills in their use to calculate the spectral characteristic of a signal and restoration of the function from the spectral characteristic.

Basic concepts and calculation formulas Let the nonperiodic signal be described by the time function defined on the interval. For the function, the condition of absolute integrability is performed.

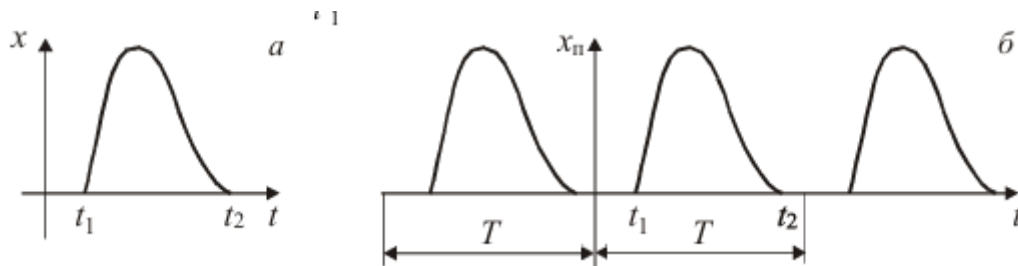


Fig. 2.1. Formation of an auxiliary periodic function: a – nonperiodic function; b – periodic function

By repeating the function with a period greater than the interval length, we form an auxiliary periodic function. A fragment of the function is shown in Fig. 2.1, b. It is obvious that the non-periodic function is the limit of the auxiliary periodic function as the period approaches infinity. The spectral characteristic allows finding a one-to-one correspondence between the time domain and frequency domain representations. Using Euler's formula, the spectral characteristic can be split into real and imaginary parts (even and odd functions of frequency respectively).

Suggestions for Teaching The programme of work provides for the determination of the spectral characteristics of the given signal using the direct Fourier transform formulas and the restoration of the signal using the inverse Fourier transform formula. The signal proposed for research is a single pulse given at the time interval (see Appendix 1). Its spectral characteristic is calculated using integration limits corresponding to the pulse duration. To achieve the necessary accuracy, at least 50 samples of the given signal are taken.

Programme of work Main Task

1. Generate a mathematical model of a signal, which is a pulse of a given shape, in the MathCAD environment.
2. Create a program for calculating the spectral characteristic of a given signal according to the formulas of the direct Fourier transformation. Construct the amplitude and phase spectral characteristics.
3. Draw up a program for restoring the signal based on the received spectral characteristic using the inverse Fourier transformation formula. Construct graphs of the initial and restored signals. Draw conclusions.
4. Change the integration interval and repeat the recovery procedure. Compare with the previous results.
5. For a given signal graph, obtain an analytical expression for the spectral characteristic and write analytical expressions for its real and imaginary parts.
6. Compile a program for calculating the spectral characteristics of the studied signal.

Control questions and tasks

1. Explain the difference between the concepts of the series and the Fourier integral.
2. Under what conditions can the Forward Fourier transform formula be used?
3. Isлисть the principle of superposition valid for the Fourier transformation?
4. Find the spectral characteristic for a given delta-pulse function.
5. Explain the difference between unilateral and bilateral Fourier transforms.
6. Name the features of the spectral characteristics of the signals described by odd and even functions.
7. What is the relationship between the spectrum of a single pulse and the spectrum of a periodic signal formed from such pulses?
8. How do the amplitude and phase spectral characteristics of the signal change when it is delayed?
9. What happens to the spectral response during compression (stretching) of the signal?
10. How to calculate the signal energy using the Fourier transform?

Practical lesson No 3

SIGNAL DECOMPOSITION ACCORDING TO THE LAGUERRE FUNCTION SYSTEM

Purpose of the work Among all the variety of orthogonal function systems used, the Laguerre function system occupies a prominent place. They have found application in the problems of forming mathematical models of industrial control objects on the basis of time characteristics obtained experimentally. The aim of the work is to study the system of Laguerre functions, to approximate a given signal using Laguerre functions, and to study the effect of the number of series terms on the quality of approximation.

Basic concepts and calculation formulas Laguerre functions are formed with the help of orthogonal polynomials. The first five polynomials and their modified forms (Laguerre functions) are used to form an orthonormal system.

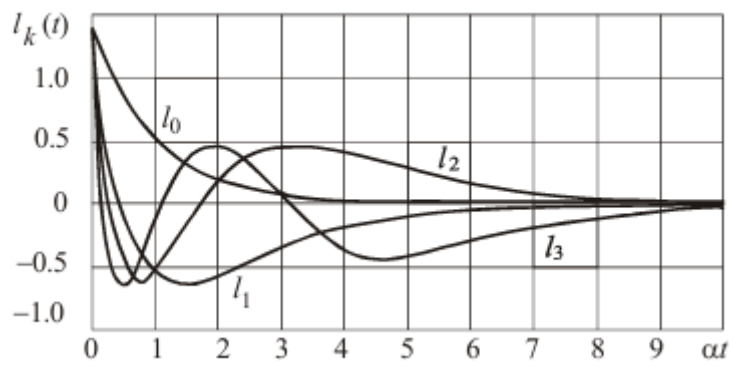


Fig. 3.1. Graphs of the first four Laguerre functions

The decomposition of the signal according to the system of Laguerre functions is written as a series. In practical problems, the signal is set on a limited interval and approximated by a truncated series. Theoretical studies show that the necessary accuracy of approximation of weakly oscillatory signals by a truncated series is achieved at $N = 4-7$.

Suggestions for Teaching Laguerre functions are often used to approximate the temporal characteristics of dynamical systems, in particular, the impulse transient function (IPF) which represents the response of the system to a delta-function input. In this work, the object of research is the IPF of a second-order system (with real or complex-conjugate poles). An important aspect of using Laguerre functions is the choice of the scale coefficient α , which should be chosen so that the durations of the signal under study and the Laguerre functions are approximately equal.

Programme of work Main Task

1. Create a program for modeling a given signal (IPF) and plot its graph. Select the value of α .
2. Create a simulation program for the first five Laguerre functions and plot them for the found value of α
3. Calculate integral values for several arbitrary values of indices and explain the result.
4. Create a program for calculating the coefficients of the series and build a spectral diagram.
5. Create a program for calculating the approximating function for different values of N . Compare the graphs with the given IPF and draw conclusions.

6. Plot approximation errors for different N and analyze their characteristic features.
7. Calculate the part of the energy that falls into the first N terms of the decomposition.
8. Calculate Mean Square Error values and draw conclusions.
9. Investigate the effect of the scale factor α on the value of approximation errors.

Control questions and tasks

1. Why are orthogonal functions used in signal decomposition?
2. Explain the main features of functions that can be represented as a Laguerre function decomposition.
3. Which approximation criterion leads to the formula for calculating the coefficients of the series?
4. What is the purpose of introducing a normalizing multiplier in Laguerre functions?
5. What is the role of the scale parameter α ?
6. Explain the relationships used to determine the value of α .
7. What is the spectral range of signal decomposition according to the system of Laguerre functions?
8. Record the Laplace image of the Laguerre function.
9. Indicate the main differences between Laguerre and Walsh orthogonal functions.

Practical lesson No 4

SIGNAL DECOMPOSITION BY WALSH FUNCTION SYSTEM

Purpose of the work With the development of computer technology, piecewise constant functions have become more often used as basic functions. The main among these functions are the Walsh and Haar functions. Walsh functions are used in speech signal processing, biology, medicine, digital image processing, and digital holography. The aim of the work is to study the system of orthogonal Walsh functions, to decompose a signal of a given shape, and to investigate the effect of the number of terms of the series on the

approximation error.

Basic concepts and calculation formulas Walsh functions were developed as a development of the Rademacher functions. They form a complete and orthogonal system, taking values of $+1$ or -1 .

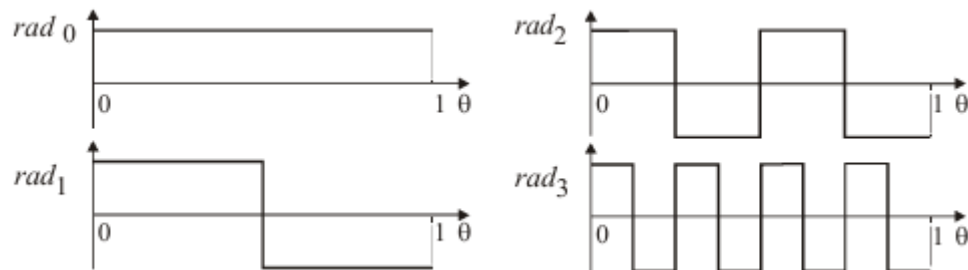


Fig. 4.1. Rademacher functions / Fig. 4.2.

Walsh system Walsh functions possess the property of multiplication, meaning the product of two Walsh functions gives a new Walsh function from the same system. This significantly reduces computational costs during spectral analysis.

Suggestions for Teaching To construct a program suitable for analyzing functions at intervals of different durations, Walsh functions are constructed first on the interval $[0, 1)$ of relative time with at least 1000 samples. Rademacher functions can be formed using conditional operators (if).

Programme of work Main Task

1. Create a program for modeling Walsh functions for $N = 16$ and observe their graphs.
2. Verify that Walsh functions satisfy the orthogonality condition by computing the values of integrals.
3. Form a given signal function in MathCAD and plot its graph.
4. Calculate the values of the expansion coefficients in the Walsh function basis.
5. Create a program for calculating the truncated Walsh series and construct a spectral diagram.
6. Plot the approximation error and analyze its features.
7. Form an approximating signal using the first 8 terms and calculate the mean square error for $N = 8$ and $N = 16$.

Control questions and tasks

1. What are the advantages of using Walsh functions over others?
2. Write a specific Walsh function using Rademacher functions.
3. Show that Walsh functions are orthogonal.
4. Show that the product of any two Walsh functions is also a Walsh function (multiplicative property).
5. Explain the existing ways of ordering Walsh functions.
6. Point out the feature of Walsh functions that simplifies coefficient calculation.

Practical lesson No 5

DECOMPOSITION OF SIGNALS ACCORDING TO THE HAAR FUNCTION SYSTEM

Purpose of the work Modern compression algorithms for image processing are based on wavelet analysis methods, among which the classical orthogonal Haar basis occupies a prominent place. Unlike Walsh functions, Haar functions are localized on separate parts of the interval under study, which allows estimating local properties (Haar wavelets).

Basic concepts and calculation formulas The system of Haar functions consists of piecewise constant functions defined on $[0, 1]$. They can be built using group principles and single or double numbering.

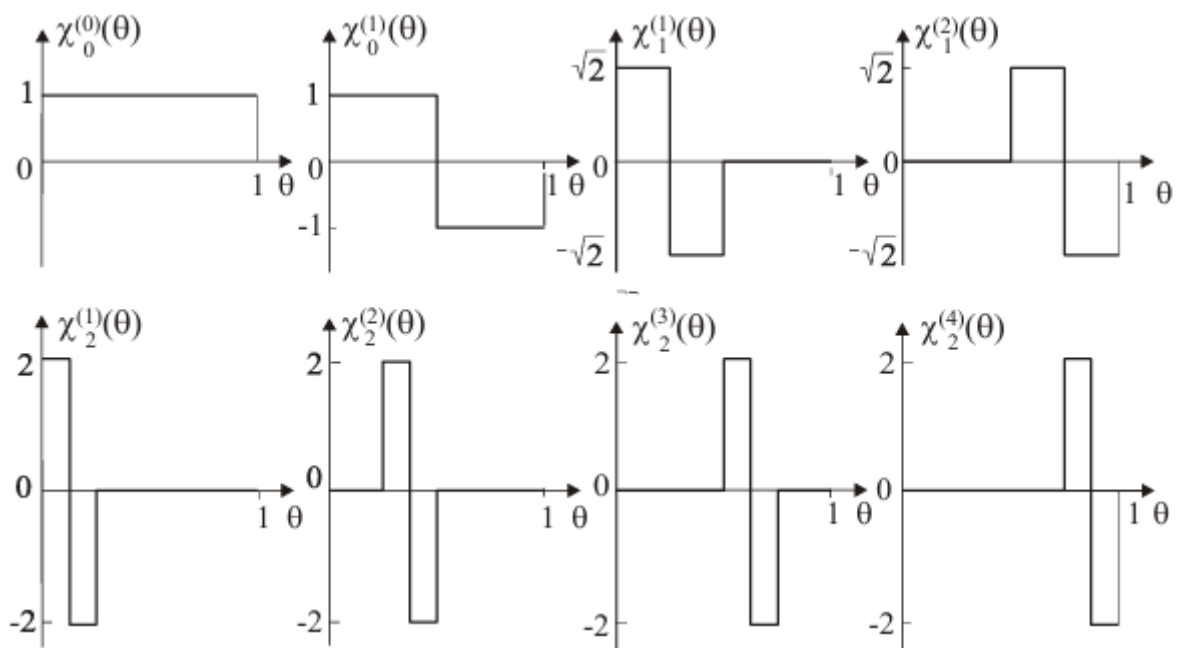


Fig. 5.1. Graphs of the first eight Haar functions

Haar functions are orthogonal and normalized, making them effective for analyzing

signals with strongly pronounced local features (short-term bursts and oscillations).

Suggestions for Teaching To analyze functions of different durations, Haar functions are formed on the relative time interval $[0, 1)$ using a "mother wavelet" defined via conditional operators.

Programme of work Main Task

1. Create a program for modeling Haar functions for $N = 16$.
2. Check the orthogonality condition by calculating integrals.
3. Form a given signal function and plot its graph.
4. Calculate expansion coefficients in the Haar basis and construct a spectral diagram.
5. Calculate the approximating function and plot the approximation error for $N = 8$ and $N = 16$.
6. Calculate mean square errors and draw conclusions.

Control questions and tasks

1. Define Haar functions.
2. Plot a combination of specific Haar functions.
3. How many functions make up the Haar system for a given number of groups?
4. Show that Haar functions are orthogonal and find the energy of a specific Haar function.

Practical lesson No 6

STUDY OF ANALOG LOW-PASS AND HIGH-PASS FILTERS

Purpose of the work The solution of the problem of approximating ideal filter characteristics (LPF and HPF) using stable and physically realizable transfer functions led to standard filter families, most notably Butterworth and Chebyshev filters.

Basic concepts and calculation formulas Normalized transfer functions of Butterworth and Chebyshev filters can be represented as ratios of polynomials.

- Butterworth filters have the flattest frequency response in the passband (maximally flat magnitude).
- Chebyshev filters of the first kind have equal ripples in the passband and a

steeper transition band. Operations of denormalization (frequency scaling) and transformation allow obtaining filters with required cutoff frequencies and types (LPF to HPF conversion).

Suggestions for Teaching Frequency responses are calculated directly from the transfer function frequency response. Pole locations are calculated using MathCAD functions (polyroots), and impulse transient functions are found via inverse Laplace transforms.

Programme of work Main Task

1. Calculate the coefficients of transfer functions for LPF and HPF of 2nd and 3rd orders with given cutoff frequencies.
2. Calculate and plot the frequency responses (amplitude and phase) and compare filters of different orders.
3. Calculate and plot poles on the complex plane.
4. Calculate and plot impulse transient functions and compare them.

Control questions and tasks

1. Define amplitude-frequency response.
2. Explain the concepts of passband and stopband.
3. Explain the meaning of denormalization and transformation operations.
4. How do Butterworth and Chebyshev filter responses differ in the passband?

Practical lesson No 7

REPRESENTATION OF SIGNALS USING THE KOTELNIKOV SERIES

Purpose of the work The first stage of digital processing is discretization (sampling), replacing a continuous signal with samples taken at equal time intervals. Kotelnikov's theorem establishes the conditions for exact signal reconstruction from its samples.

Basic concepts and calculation formulas A band-limited signal can be perfectly reconstructed using the Kotelnikov series (sampling theorem formula) utilizing sinc-like reference functions.

Suggestions for Teaching A single pulse signal is used to study discretization, signal reconstruction, and recovery errors. The sampling period is determined based on the

spectral characteristic boundary frequency.

Programme of work Main Task

1. Find the spectral characteristic of the reference function and plot it.
2. Form a mathematical model of a given signal and plot its graph.
3. Determine the cutoff frequency and the Kotelnikov sampling interval.
4. Create a program to reconstruct the signal from its discrete samples and plot the error signal.
5. Repeat with a reduced sampling period and compare results.

Control questions and tasks

1. How is a continuous signal sampled over time?
2. Show that reference functions are orthogonal.
3. What is meant by the number of degrees of freedom of a signal?
4. What methods, apart from the Kotelnikov series, are used for signal reconstruction?

Practical lesson No 8

DISCRETE INTEGRATION AND DIFFERENTIATION

Purpose of the work Study algorithms for digital integration and differentiation of signals presented as finite discrete sequences, realized in real time (rectangles, trapezoids, parabolic approximation, simple differences, recursive/non-recursive differentiators).

Basic concepts and calculation formulas Discretized versions of integration and differentiation are represented via difference equations and evaluated using z-transforms (transfer functions $H(z)H(z)$).

Programme of work Main Task

1. Calculate and plot frequency responses (amplitude and phase) for discrete integrators (rectangles, trapezoids) and compare with the ideal integrator.
2. Calculate output sequences for test harmonic signals and compare with ideal responses.
3. Study discrete differentiators (simple difference, non-recursive, and recursive filters) and their frequency/time responses.

Control questions and tasks

1. Given a harmonic signal, write its discrete representation.
2. Find the output of a discrete integrator for a given input (e.g., delta function or step).
3. Draw structural diagrams of discrete integrators.

Practical lesson No 9

DISCRETE SEQUENCE SMOOTHING

Purpose of the work Study algorithms for smoothing experimental data affected by additive noise (sliding averaging, smoothing with fourth differences, exponential smoothing).

Basic concepts and calculation formulas

- Sliding averaging calculates arithmetic means over a sliding window of length L .
- Smoothing with fourth differences uses a least-squares parabolic approximation over 5 points.
- Exponential smoothing uses a recurrence relation with a smoothing constant α .

Programme of work Main Task

1. Form a test sequence consisting of a useful harmonic component and centered random noise.
2. Implement sliding averaging smoothing for various L values and evaluate the quality criterion.
3. Implement smoothing with fourth differences and exponential smoothing for various α values.
4. Compare the frequency characteristics (amplitude and phase responses) of all smoothing filters.

Control questions and tasks

1. When is data smoothing recommended?
2. Why does a large window size L in sliding averaging distort the useful component?
3. What is the least-squares approximation?

4. Explain the role of the smoothing constant

α

in exponential smoothing.

Practical lesson No 10 / 11

INVESTIGATION OF DIGITAL AND NON-RECURSIVE FILTERS

Purpose of the work Investigation of second-order digital filters (derived via bilinear transformation from analog prototypes) and non-recursive digital low-pass filters with linear phase-frequency characteristics designed using the weighting (window) method.

Basic concepts and calculation formulas

- Non-recursive filters are described by finite impulse responses (FIR) and have linear phase if coefficients are symmetric.
- Window functions (Bartlett, Hann, Hamming, Blackman) are used to suppress the Gibbs phenomenon during truncation of the ideal impulse response.

Programme of work Main Task

1. Calculate transfer function coefficients for digital LPF/HPF based on variant tables.
2. Calculate and plot frequency characteristics and test them with input sequences containing two sinusoidal components.
3. Investigate the effect of filter order N and window types on the impulse and frequency responses of non-recursive filters.

Control questions and tasks

1. Define a digital filter and distinguish between recursive and non-recursive types.
2. Explain the difference between causal and non-causal filters.
3. Why are non-recursive filters always stable?
4. Explain the role and characteristics of window functions.