

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
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NORTH CAUCASIAN FEDERAL UNIVERSITY

GUIDELINES
for completing laboratory work
in the discipline " **Engineering Mathematics / Инженерная математика** "
for students in the field of study
02.04.01 " Mathematics and Computer Science "
direction (profile)
"Training Program for Electronic Information"

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Laboratory work No. 1

Topic: Introduction to Elementary Error Theory

Test questions and tasks:

1. Give a definition of the absolute error of an approximate number.
2. Give a definition of the relative error of an approximate number.
3. Specify the relationship between absolute and relative errors.
4. What does the entry mean: $a=2.52(\pm 0.01)$?
5. Is the entry correct : $a = 3.47((\pm 0.033))$?
6. Indicate the correct signs in the broad sense of the approximate number if $b=1.345(\pm 0.052)$.
7. Indicate the correct signs in the narrow sense of the approximate number if $b=2.3345(\pm 0.0052)$.
8. Find the absolute error of the sum of numbers a^* and b^* , if $\Delta(a^*)=0.01$, $\Delta(b^*)=0.02$.
9. Find the absolute error of the product of numbers a^* and b^* , if $a^*=3.14$, $b^*=2.72$, $\Delta(a^*)=0.01$, $\Delta(b^*)=0.02$.
10. Find the absolute error of the partial numbers a^* and b^* , if $a^*=3.14$, $b^*=2.72$, $\Delta(a^*)=0.01$, $\Delta(b^*)=0.02$.
11. Find the relative error of the product of numbers a^* and b^* , if $a^*=3.14$, $b^*=2.72$, $\Delta(a^*)=0.01$, $\Delta(b^*)=0.02$.
12. Find the relative error of the partial numbers a^* and b^* , if $a^*=3.14$, $b^*=2.72$, $\Delta(a^*)=0.01$, $\Delta(b^*)=0.02$.
13. Find the absolute error in calculating the value of the function $y = \sin x$ if $x^*=0.547$ (all signs are correct in the broad sense).
14. Find the absolute error in calculating the value of the function $z = \sin(x \cdot y)$ if $x^*=0.547$, $y^*=0.825$ (all signs are correct in the broad sense).

I. Exercise.

- 1) Determine which equality is more accurate.
- 2) Round off the questionable digits of the number, leaving the correct signs:
 - a) in a narrow sense;
 - b) in a broad sense.Determine the absolute error of the result.
- 3) Find the maximum absolute and relative errors of numbers if they have only true digits: a) in the narrow sense; b) in the broad sense.

Option No. 1	Option #16
1) $\sqrt{44} = 6,63$; $19/41 = 0.463$,	1) $5/3 = 1.667$; $\sqrt{38} = 6,16$
2) a) $22,553 (\pm 0.016)$; b) 2.8546 ; $\delta = 0.3\%$	2) a) 3.7542 ; $\delta = 0,32\%$ b) $0.98351 (\pm 0,00042)$
3) a) 0.2387 ; b) 42.884 .	3) a) 62.74 ; b) 0.389 .

Option #2 1) $7/15 = 0.467$; $\sqrt{30} = 5.48$ 2) a) 17.2834; $\delta = 0.3\%$ b) 6.4257 (± 0.0024) 3) a) 3.751; b) 0.537.	Option #17 1) $49/13 = 3.77$; $\sqrt{14} = 3,74$. 2) a) 83,736 ; $\delta = 0,085\%$; b) 5.6483 ($\pm 0,0017$). 3) a) 5.6432; b) 0.00858.
Option No. 3 1) $\sqrt{10.5} = 3.24$; $4/17 = 0.235$. 2) a) 34,834; $\delta = 0.1\%$ b) 0.5748 (± 0.0034) 3) a) 11,445; b) 2,043	Option #18 1) $13/7 = 1.857$; $\sqrt{7} = 2,64$ 2) a) 2.8867; $\delta = 0,43\%$ b) 32.7486 ($\pm 0,0012$) 3) a) 0.0384; b) 63.745.
Option No. 4 1) $15/7 = 2.14$; $\sqrt{10} = 3.16$ 2) a) 2.3485 (± 0.0042) b) 0.34484; $\delta = 0.4\%$ 3) a) 2.3445; b) 0.745.	Option #19 1) $19/12 = 1.58$; $\sqrt{12} = 3,46$ 2) a) 4.88445 ($\pm 0,00052$) b) 0.096835; $\delta = 0,32\%$ 3) a) 12,688; b) 4,636.
Option #5 1) $6/7 = 0.857$; $\sqrt{4.8} = 2.19$ 2) a) 5,435 (± 0.0028) b) 10.8441; $\delta = 0.05\%$ 3) a) 8.345; b) 0.288	Option #20 1) $51/11 = 4.64$; $\sqrt{7} = 5,91$ 2) a) 38.4258 ($\pm 0,0014$) b) 0.66385 ; $\delta = 0,34\%$. 3) a) 6.743; b) 0.543
Option No. 6 1) $12/11 = 1.091$; $\sqrt{6,8} = 2,61$ 2) a) 8.24163; $\delta = 0,2\%$ b) 0.12356 ($\pm 0,00036$) 3) a) 12.45; b) 3.4453.	Option #21 1) $18/7 = 2.57$; $\sqrt{22} = 4,69$ 2) a) 0.39642 ($\pm 0,00022$) b) 46,453; $\delta = 0,15\%$ 3) a) 15,644; b) 6,125.
Option No. 7 1) $2/21 = 0.095$; $\sqrt{22} = 4,69$ 2) a) 2.4543 ($\pm 0,0032$); b) 24.5643; $\delta = 0,1\%$ 3) a) 0.374; b) 4.348.	Option #22 1) $17/9 = 2.11$; $\sqrt{17} = 4,12$. 2) a) 5.8425 ; $\delta = 0,23\%$; b) 0.66385 ($\pm 0,00042$). 3) a) 0.3825; b) 24.6.
Option No. 8 1) $23/15 = 1.53$; $\sqrt{9,8} = 3,13$. 2) a) 23,574; $\delta = 0,2\%$ b) 8.3445 ($\pm 0,0022$). 3) a) 20.43; b) 0.576.	Option #23 1) a) $16/7 = 2.28$; $\sqrt{11} = 3,32$. 2) a) 24.3872 ; $\delta = 0,34\%$; b) 0.75244 ($\pm 0,00013$). 3) a) 16,383; b) 5,734.
Option No. 9 1) $6/11 = 0.545$; $\sqrt{83} = 9,11$ 2) a) 21.68563 ; $\delta = 0,3\%$; b) 3.7834 ($\pm 0,0041$). 3) a) 41.73; b) 0.678.	Option #24 1) $21/13 = 1.54$; $\sqrt{63} = 7,94$. 2) a) 2.3684 ($\pm 0,0017$); b) 45.7832 ; $\delta = 0,18\%$. 3) a) 0.573; b) 3.6761.
Option #10	Option #25

1) $17/19 = 0.889$; $\sqrt{52} = 7,21$; 2) a) $13,537$ ($\pm 0,0026$); b) $7,521$; $\delta = 0,12\%$. 3) a) 5.634 ; b) 0.0748	1) $12/7 = 1.71$; $\sqrt{47} = 6,86$; 2) a) $72,354$; $\delta = 0,24\%$; b) 0.38725 ($\pm 0,00112$). 3) a) 18.275 ; b) 0.00644 .
Option #11 1) $21/29 = 0.723$; $\sqrt{44} = 6,63$ 2) a) 0.3567 ; $\delta = 0,042\%$ b) 13.6253 ($\pm 0,0021$). 3) a) 18.357 b) 2.16 .	Option #26 1) $6/7 = 0.857$; $\sqrt{41} = 6,40$ 2) a) 0.36127 ($\pm 0,00034$); b) 46.7843 ; $\delta = 0,32\%$. 3) a) 3.425 ; b) 7.38 .
Option #12 1) $50/19 = 2.63$; $\sqrt{27} = 5,19$ 2) a) 1.784 ($\pm 0,0063$); b) 0.85637 ; $\delta = 0,21\%$ 3) a) 0.5746 ; b) 236.58 .	Option #27 1) $23/9 = 2.56$; $\sqrt{87} = 9,33$ 2) a) 23.7564 ; $\delta = 0,44\%$; b) 4.57633 ($\pm 0,00042$). 3) a) 3.75 ; b) 6.8343 .
Option #13 1) $13/17 = 0.764$; $\sqrt{31} = 5,56$. 2) a) 3.6878 ($\pm 0,0013$); b) $15,873$; $\delta = 0,42\%$. 3) a) 14.8624 ; b) 8.73	Option #28 1) $27/31 = 0.872$; $\sqrt{42} = 6,48$ 2) a) 15.8372 ($\pm 0,0026$) b) 0.088748 ; $\delta = 0,56\%$. 3) a) $3,643$; b) $72,385$.
Option #14 1) $7/22 = 0.318$; $\sqrt{13} = 3,60$. 2) a) 27.1548 ($\pm 0,0016$) b) 0.3945 ; $\delta = 0,16\%$ 3) a) 0.3648 ; b) 21.7	Option #29 1) $7/3 = 2.33$; $\sqrt{58} = 7,61$; 2) a) 3.87683 ; $\delta = 0,33\%$ b) 13.5726 ($\pm 0,0072$). 3) a) 26.3 ; b) 4.8556 .
Option #15 1) $17/11 = 1.545$; $\sqrt{18} = 4,243$. 2) a) 0.8647 ($\pm 0,0013$) b) 24.3618 ; $\delta = 0,22\%$ 3) a) 2.4516 ; b) 0.863 .	Option #30 1) $14/17 = 0.823$; $\sqrt{53} = 7,28$ 2) a) 0.66835 ($\pm 0,00115$); b) 23.3748 ; $\delta = 0,27\%$; 3) a) 43.813 ; b) 0.645 .

II. Exercise.

Calculate the value of the expression. Find the absolute and relative errors of the function. As a result, leave *the y * values correct in the narrow sense*.

Options.

$$1. \quad y = \frac{a^2 b^3}{6\sqrt{c}}; \quad a = 2.32, \quad b = 7.325, \quad c = 12.923.$$

$$2. \quad y = 6a^2 + 2 \cdot \frac{a b^3}{\sqrt{c}}; \quad a = 3.23, \quad b = 9.128, \quad c = 10.72.$$

$$3. \quad y = \frac{h^3 \sqrt{l}}{3a^3}; \quad h = 3.81, \quad l = 11.85, \quad a = 2.728.$$

$$4. \quad y = \frac{\sqrt{r} \cdot s^2}{3t^3}; \quad r = 15.824, \quad s = 2.82, \quad t = 1.374.$$

$$5. \quad y = \frac{\sqrt{t} \cdot s^2}{2h^2}; \quad t = 0.984, \quad s = 2.185, \quad h = 1.37.$$

$$6. \quad y = \frac{2\sqrt{b} \cdot a^3}{c^2}; \quad a = 2.1, \quad b = 8.175, \quad c = 1.1415.$$

$$7. \quad y = \frac{4\sqrt{a} \cdot b^2}{h^3}; \quad a = 3.4, \quad b = 8.171, \quad h = 1.08.$$

$$8. \quad y = \frac{\sqrt{st}}{3h^2}; \quad h = 4.245, \quad s = 1.8, \quad t = 1.91.$$

$$9. \quad y = 3\sqrt{sa^2 + 2sb^3}; \quad a = 8.71, \quad b = 1.93, \quad s = 2.255.$$

$$10. \quad y = \frac{4s\sqrt{s} \cdot b^2}{t^2}; \quad b = 9.25, \quad s = 11.91, \quad t = 2.08.$$

$$11. \quad y = \frac{\sqrt{a} \cdot b^2}{2c^3} + \sqrt{ab}; \quad a = 3.84, \quad b = 9.1, \quad c = 10.1.$$

$$12. \quad y = \frac{h^2 \cdot s^2}{\sqrt{t}} + h \cdot s; \quad h = 0.25, \quad s = 3.725, \quad t = 0.528.$$

$$13. \quad y = \frac{\sqrt{n} \cdot m^2}{\sqrt{m}} + mn; \quad m = 15.34, \quad n = 17.01.$$

$$14. \quad y = \frac{\sqrt{q} \cdot p^2}{2t^2} + \sqrt{ptq}; \quad p = 0.25, \quad t = 1.272, \quad q = 4.82.$$

$$15. \quad y = \frac{\sqrt{b} \cdot a^3}{s^2} + \sqrt{as}; \quad a = 13.1, \quad b = 15.42, \quad s = 4.25.$$

$$16. \quad y = \frac{\sqrt{rs}}{2t} + s\sqrt{t}; \quad r = 0.47, \quad s = 2.425, \quad t = 3.72.$$

$$17. \quad y = \frac{\sqrt{abc}}{m}; \quad a = 1.2, \quad b = 3.52, \quad c = 8.4, \quad m = 1.24.$$

$$18. \quad y = 5r^3 + 2 \cdot \frac{r^2 \sqrt{s}}{t^2}; \quad r = 1.32, \quad s = 2.41, \quad t = 3.2.$$

$$19. \quad y = \frac{3h^2 \sqrt{l}}{2a^3}; \quad a = 2.1, \quad h = 1.47, \quad l = 1.41.$$

$$20. \quad y = \frac{s^3 t^2}{2\sqrt{h}}; \quad s = 1.71, \quad t = 1.1, \quad h = 2.44.$$

$$21. \quad y = \frac{3\sqrt{b} \cdot a^3}{c^2}; \quad a = 2.6, \quad b = 1.45, \quad c = 2.7.$$

$$22. \quad y = \frac{4\sqrt{a} \cdot b^2}{h^3}; \quad a = 1.6, \quad b = 1.11, \quad h = 0.28.$$

$$23. y = \frac{\sqrt{r} \cdot s^2}{3h^2}; \quad r = 3.44, \quad s = 2.1, \quad h = 2.3.$$

$$24. y = 3\sqrt{s}a^3 + \frac{2s}{b^2}; \quad a = 2.1, \quad s = 4.1, \quad b = 1.41.$$

$$25. y = \frac{3\sqrt{t} \cdot b^3}{r^2}; \quad b = 1.1, \quad t = 4.12, \quad r = 1.2.$$

$$26. y = \frac{a^2 b^2}{2\sqrt{c}}; \quad a = 2.51, \quad b = 8.325, \quad c = 12.723.$$

$$27. y = 6\ln(a^2) + 2 \cdot \frac{a b^2}{\sqrt{c}}; \quad a = 3.23, \quad b = 9.128, \quad c = 10.72.$$

$$28. y = \frac{h^2 \sqrt{l}}{3t^3}; \quad h = 4.81, \quad l = 1.85, \quad t = 2.748.$$

$$29. y = \frac{\ln(r^2) \cdot s^2}{t^3}; \quad r = 15.824, \quad s = 2.82, \quad t = 1.374.$$

$$30. y = \frac{2\sqrt{t} \cdot s^2}{\ln(h^2)}; \quad t = 3.52, \quad s = 1.1, \quad h = 3.1.$$

Laboratory work No. 2

Topic: Interpolation by unequal-distance nodes

I. Test questions and tasks.

1. What polynomial is called an interpolation polynomial?
2. What are called interpolation nodes?
3. How many interpolation polynomials can be constructed for a given function and a given system of nodes?
4. What is the geometric meaning of interpolation?
5. What does the graph of the interpolation polynomial represent for the following data:

a) $(x_0, y_0), (x_1, y_1);$

b) $(x_0, y_0), (x_1, y_1), (x_2, y_2).$

6. Write out the formulas for the error (remainder term) of interpolation and the estimate of the interpolation error.
7. Construct the Lagrange and Newton interpolation polynomials for a function given in a table:

x	1	2	3	5
y	1	5	17	89

Make a check.

8. The function $f(x)$ is defined in a table:

x	0	1	2	6
y	-1	-3	3	1187

Using the Lagrange and Newton interpolation polynomials, find its value at $x = 4$.

9. The Lagrange interpolation polynomial is given by:

$$P_n(x) = \varphi_0(x) \cdot y_0 + \varphi_1(x) \cdot y_1 + \dots + \varphi_n(x) \cdot y_n,$$

$$\text{Where } \varphi_i(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}.$$

Prove that $\varphi_0(x) + \varphi_1(x) + \dots + \varphi_n(x) = 1$.

10. Prove the theorem: Divided differences n -th order of a polynomial of degree n are constant.
11. Using the table of function values $y = e^x$, calculate and estimate the error of the result using the Lagrange and Newton interpolation formulas. $e^{3.62}$

x	3.60	3.65	3.70
y	35,598	38,475	40.447

12. The function $f(x)$ is given by the table

x	10	15	17	20
$f(x)$	3	7	11	17

Find the value x for which $f(x) = 10$.

I. Exercise.

Find the intermediate value of the function at point x^* using the table of its values, taking the points $x_i, i = 0, 1, 2, 3$, as interpolation nodes, using the Lagrange and Newtonian interpolation method. Estimate the error of the obtained result.

II. Options.

Option	$f(x)$	x_0	x_1	x_2	x_3	x^*
1	$\sin x$	0.16	0.18	0.20	0.25	0.187
2	$\sin x$	0.20	0.23	0.25	0.27	0.225
3	$\cos x$	0.15	0.16	0.18	0.20	0.173
4	$\sin x$	0.30	0.32	0.34	0.35	0.337
5	$\cos x$	0.21	0.22	0.24	0.25	0.215
6	$\cos x$	0.31	0.32	0.34	0.35	0.322
7	$\sin x$	0.40	0.41	0.43	0.44	0.421
8	e^x	0.15	0.16	0.18	0.20	0.167
9	e^x	0.20	0.21	0.23	0.24	0.215
10	e^x	0.30	0.31	0.32	0.34	0.315
11	$\sin x$	0.21	0.23	0.24	0.25	0.233
12	$\cos x$	0.16	0.18	0.19	0.22	0.165
13	$\ln x$	0.20	0.22	0.23	0.25	0.223
14	$\ln x$	0.15	0.17	0.18	0.20	0.171
15	$\ln x$	0.30	0.32	0.33	0.35	0.325
16	$\ln x$	1.12	1.13	1.15	1.16	1.135
17	$\sqrt{x}$	0.10	0.11	0.12	0.14	0.105
18	$\sqrt{x}$	0.15	0.16	0.18	0.20	0.154
19	$\sqrt{x}$	0.20	0.21	0.23	0.24	0.214
20	$\ln x$	1.21	1.22	1.24	1.25	1.225
21	$\sin x$	0.12	0.13	0.15	0.16	0.134
22	$\sqrt{x}$	0.22	0.23	0.25	0.26	0.234
23	$\sin x$	0.10	0.30	0.40	0.60	0.116
24	$\ln x$	1.20	1.21	1.23	1.25	1.125
25	$\cos x$	0.15	0.16	0.18	0.20	0.156
26	$\sqrt{x}$	1.10	1.11	1.13	1.15	0.106

Laboratory work No. 3

Topic: Interpolation by algebraic polynomials at equidistant nodes

I. Test questions and tasks.

1. Definition of finite differences of different orders.
2. Expressing finite differences of any order in terms of function values. Give examples.
3. Write out Newton's interpolation formulas and their remainder terms.
4. Expression of derivatives through finite differences.
5. Residual terms of Newton's interpolation formulas without using derivatives of the corresponding order.
6. Estimate the accuracy with which it is possible to calculate using Newton's formulas $\ln 100.5$ and $\ln 102.6$, if the values are known: $\ln 100$, $\ln 101$, $\ln 102$, $\ln 103$.
7. Prove that finite differences of order n from a polynomial of degree n are equal to a constant value.
8. Sum the final rows:
 - 1) $1^2 + 2^2 + 3^2 + \dots + (n-1)^2 + n^2$; Answer: $\left(\frac{n(n+1)(2n+1)}{6}\right)$.
 - 2) $1^3 + 2^3 + 3^3 + \dots + (n-1)^3 + n^3$; Answer: $\left(\frac{n^2(n+1)^2}{4}\right)$.
 - 3) $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2$; Answer: $\left(\frac{n(4n^2-1)}{3}\right)$.
 - 4) $1^3 + 3^3 + 5^3 + \dots + (2n-1)^3$. Answer: $n^2(2n^2-1)$.

II. Exercise.

For a function defined in a table, find its values at points t_1, t_2, t_3 . Obtain results with the accuracy allowed by the table definition of the function.

Function A

X	f(x)	x	f(x)
0.0	0.946083	0.5	1.324684
0.1	1.028685	0.6	1.389181
0.2	1.108047	0.7	1.449592
0.3	1.183958	0.8	1.505817
0.4	1.256227	0.9	1.557775

Interpolation points

No.	t_1	t_2	t_3	a
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1	0.175418	0.715878	0.464331	1.344273
2	0.090566	0.826611	0.395142	1.534355
3	0.157369	0.826216	0.445135	1.273228
4	0.053526	0.866027	0.610308	1.435993
5	0.109658	0.710092	0.468515	1.026087
6	0.094189	0.755627	0.343888	1.058530
7	0.132747	0.755108	0.447232	1.390474
8	0.058421	0.731564	0.429815	1.417970
9	0.063946	0.907395	0.455020	1.373809
10	0.182461	0.717093	0.525286	1.197522
11	0.173762	0.759231	0.482714	1.470054
12	0.129140	0.766507	0.238812	1.183944

Function B

x	f(x)	x	f(x)
0.0	0.962852	0.5	1.337904
0.1	1.044824	0.6	1.401593
0.2	1.123512	0.7	1.461175
0.3	1.198709	0.8	1.516552
0.4	1.270228	0.9	1.567650

Interpolation points

No.	t ₁	t ₂	t ₃	a
13	0.052914	0.882964	0.391986	1.189116
14	0.147327	0.721404	0.355038	1.564167
15	0.148489	0.822272	0.544808	1.573997
16	0.070926	0.901307	0.213966	1.547732
17	0.124548	0.769617	0.303341	1.504725
18	0.009510	0.944211	0.417157	1.994108
19	0.087996	0.778350	0.546174	1.110465
20	0.051598	0.709740	0.542017	1.404176
21	0.083288	0.887929	0.517459	1.484642
22	0.174607	0.902430	0.240569	1.275640
23	0.172537	0.819656	0.698720	1.331830
24	0.160270	0.851787	0.463563	1.307859

Function C

x	f(x)	x	f(x)
0.0	0.979498	0.5	1.350965
0.1	1.060831	0.6	1.413842
0.2	1.138837	0.7	1.472590
0.3	1.213312	0.8	1.527116
0.4	1.284076	0.9	1.577351

Interpolation points

No.	t ₁	t ₂	t ₃	a
25	0.047475	0.805753	0.653213	1.423549
26	0.104834	0.865748	0.634411	1.361976
27	0.095092	0.861356	0.490247	1.314712
28	0.133412	0.879781	0.473981	1.472912
29	0.069174	0.793965	0.465591	1.498836
30	0.076449	0.702410	0.552699	1.580131
31	0.086081	0.711768	0.446462	1.479177
32	0.045216	0.721249	0.670968	1.429159
33	0.182628	0.729050	0.433820	1.097459
34	0.155091	0.892695	0.403838	0.981061
35	0.118535	0.884719	0.589080	1.399221
36	0.041623	0.839124	0.294268	1.048028

Function D

x	f(x)	x	f(x)
0.0	0.996021	0.5	1.363865
0.1	1.076705	0.6	1.425925
0.2	1.154020	0.7	1.483835
0.3	1.227768	0.8	1.537508
0.4	1.297768	0.9	1.586879

Interpolation points

No.	t ₁	t ₂	t ₃	a
37	0.143689	0.915777	0.599657	1.018375
38	0.042666	0.825305	0.584323	1.262410
39	0.165626	0.705625	0.456150	1.445709
40	0.126742	0.748197	0.450881	1.467961
41	0.096649	0.891334	0.692973	1.342491
42	0.064256	0.843814	0.373073	1.362658
43	0.071993	0.742732	0.474374	1.567605
44	0.105362	0.711920	0.524242	1.585812
45	0.159355	0.819366	0.337411	1.427805
46	0.161147	0.900531	0.252948	1.146269
47	0.104874	0.852465	0.470731	1.033643
48	0.175694	0.779153	0.306156	1.551797

Laboratory work No. 4

Topic: Interpolation with cubic splines

I. Test questions and tasks.

1. Give the definition of a spline. Linear spline.
2. Give the definition of a cubic spline.
3. Types of boundary conditions when constructing splines.
4. For table-defined functions

a)

x	-1	0	1	2
y	3	2	5	1

b)

x	1	2	4	5
y	3	4	6	5

construct linear and parabolic splines.

5. Provide estimates of the error in interpolation using cubic splines for the function $f(x)$ and its derivatives $f'(x)$ and $f''(x)$.

II. Exercise .

For a function given by a table, find its value at points z_1, z_2, z_3 , using spline interpolation with the condition $S_3''(a) = S_3''(b) = 0$, $a = x_0$, $b = x_5$.

III. Options.

No.	$f(x)$	$x_0, x_1, x_2, x_3, x_4, x_5$	z_1, z_2, z_3
1	$\sin x$	0.16, 0.18, 0.20, 0.23, 0.25, 0.28	0.165, 0.121, 0.252
2		0.20, 0.23, 0.25, 0.27, 0.28, 0.30	0.201, 0.245, 0.203
3		0.30, 0.32, 0.34, 0.35, 0.37, 0.38	0.315, 0.342, 0.371
4		0.40, 0.41, 0.43, 0.44, 0.46, 0.47	0.407, 0.437, 0.465
5		0.21, 0.23, 0.24, 0.25, 0.27, 0.28	0.215, 0.253, 0.274
6		0.80, 0.83, 0.84, 0.85, 0.87, 0.88	0.806, 0.845, 0.876
7	$\cos x$	0.15, 0.16, 0.18, 0.20, 0.21, 0.22	0.155, 0.181, 0.214
8		0.21, 0.22, 0.24, 0.27, 0.28, 0.29	0.215, 0.246, 0.285
9		0.31, 0.32, 0.35, 0.37, 0.38, 0.40	0.315, 0.354, 0.856
10		0.16, 0.18, 0.19, 0.25, 0.27, 0.29	0.172, 0.195, 0.274
11	e^x	0.00, 0.25, 0.30, 0.40, 0.50, 0.55	0.155, 0.355, 0.530
12		0.20, 0.40, 0.45, 0.60, 0.70, 0.75	0.250, 0.442, 0.730
13		0.30, 0.40, 0.60, 0.65, 0.70, 0.80	0.350, 0.630, 0.740
14		0.70, 0.80, 0.95, 1.00, 1.10, 1.15	0.706, 0.905, 1.134
15		1.00, 1.11, 1.13, 1.23, 1.24, 1.34	1.02, 1.131, 1.128
16		0.23, 0.25, 0.40, 0.50, 0.65, 0.70	0.236, 0.457, 0.658

17	$\sqrt{x}$	0.10, 0.25, 0.45, 0.50, 0.70, 0.85	0.108, 0.456, 0.755
18		0.15, 0.26, 0.28, 0.30, 0.35, 0.45	0.154, 0.281, 0.386
19		0.20, 0.25, 0.35, 0.40, 0.55, 0.60	0.245, 0.355, 0.558
20		1.50, 1.65, 1.70, 1.75, 1.85, 1.90	1.154, 1.736, 1.855
21	$Ln\ x$	1.50, 1.65, 1.70, 1.85, 1.90, 1.95	1.502, 1.762, 1.923
22		1.20, 1.21, 1.23, 1.26, 1.28, 1.30	1.205, 1.232, 1.286
23		0.35, 0.45, 0.50, 0.60, 0.70, 0.85	0.368, 0.555, 0.775
24		0.90, 0.92, 0.95, 1.00, 1.10, 1.12	0.915, 0.955, 1.075
25		0.85, 0.95, 1.00, 1.20, 1.25, 1.27	0.875, 1.185, 1.252

Laboratory work No. 5

Topic: Approximation of functions using the least squares method

I. Test questions and tasks.

1. Formulate the problem of approximating a tabulated function by a polynomial using the least squares method.
2. What is the difference between constructing an interpolation polynomial and constructing a polynomial using the least squares method?
3. Provide a condition under which the interpolation polynomial and the polynomial constructed using the least squares method coincide.
4. Describe the methods for selecting the approximating function in the least squares method.
5. How can we go from the approximating function $F(a, m, x) = a \cdot \exp(m \cdot x)$, $a > 0$ to the approximating function of the form $f(b_0, b_1, x) = b_0 + b_1 \cdot x$?

II. Exercise.

Task 1.

- 1) Approximate a tabulated function by a second-degree polynomial.
- 2) Compare the original values of the function y_i with those obtained from the constructed polynomial.
- 3) Find the standard deviation using the formula:

$$S = \left[\sum_{i=0}^n (P_2(x_i) - y_i)^2 \right]^{\frac{1}{2}}.$$

Task 2 (for all students).

Next table

t	14.5	30.0	64.5	74.5	86.7	94.5	98.9
k	0	0.004	0.018	0.029	0.051	0.073	0.090

gives the values of specific electrical conductivity of glass depending on the temperature in degrees C .

Find for function

$$k = f(t),$$

given in a table, approximating function of the form

$$k = b \cdot e^{at}.$$

III. Task options.

Values $x_i = i \cdot 0.1$, $i = 0, 2, \dots, 19$, the same for all options.

i	Values $y_i = y(x_i)$					
	Option 1	Option 2	Option 3	Option 4	Option 5	Option 6
0	2.05	2.09	2.02	1.99	2.23	2.07
1	1.94	2.05	1.98	2.03	2.29	2.17
2	1.92	2.19	1.67	2.20	2.27	2.21
3	1.87	2.18	1.65	2.39	2.62	2.31
4	1.77	2.17	1.57	2.19	2.72	2.10
5	1.88	2.27	1.42	2.61	2.82	2.09
6	1.71	2.58	1.37	2.35	3.13	2.12
7	1.60	2.73	1.07	2.60	3.49	1.63
8	1.56	2.82	0.85	2.55	3.82	1.78
9	1.40	3.04	0.48	2.49	3.95	1.52
10	1.50	3.03	0.35	2.50	4.22	1.16
11	1.26	3.45	-0.30	2.52	4.48	1.07
12	0.99	3.62	-0.61	2.44	5.06	0.85
13	0.97	3.85	-1.20	2.35	5.50	0.56
14	0.91	4.19	-1.39	2.26	5.68	0.10
15	0.71	4.45	-1.76	2.19	6.19	-0.25
16	0.43	4.89	-2.28	2.24	6.42	-0.65
17	0.54	5.06	-2.81	2.34	7.04	-1.06
18	0.19	5.63	-3.57	1.96	7.57	-1.66
19	0.01	5.91	-4.06	2.19	8.10	-2.01

i	Values $y_i = y(x_i)$					
	Option 7	Option 8	Option 9	Option 10	Option 11	Option 12
0	2.18	-0.10	-0.16	2.09	2.15	0.10
1	2.43	-0.21	0.01	2.31	2.41	-0.01
2	2.40	0.01	0.10	2.72	2.58	-0.19
3	2.43	0.05	0.16	2.77	2.84	-0.11
4	2.65	-0.13	0.05	2.78	3.28	-0.31
5	2.75	-0.23	0.35	2.97	3.46	-0.78
6	2.67	-0.21	0.19	3.00	4.02	-0.64
7	2.66	-0.43	0.50	3.51	4.11	-0.85
8	2.63	-0.57	0.74	3.43	4.61	-1.18
9	2.75	-0.44	1.03	3.58	5.03	-1.39
10	2.41	-0.44	1.06	3.58	5.34	-1.79
11	2.24	-0.83	1.49	3.54	5.86	-2.02
12	2.12	-0.78	1.79	3.82	6.33	-2.48
13	1.74	-0.81	2.03	3.90	6.81	-2.93
14	1.57	-1.06	2.22	3.77	7.21	-3.26
15	1.17	-1.41	2.50	3.81	7.67	-3.91
16	0.96	-1.40	2.88	4.00	8.23	-4.41
17	0.63	-1.70	3.21	3.97	8.68	-4.91
18	0.25	-1.96	3.63	4.08	9.35	-5.30
19	-0.01	-1.91	3.90	4.08	9.93	-6.00

i	Values $y_i = y(x_i)$					
	Option 13	Option 14	Option 15	Option 16	Option 17	Option 18
0	0.17	0.80	0.04	0.08	-0.02	0.14
1	0.07	0.29	0.47	0.14	0.44	0.23
2	0.17	0.52	0.78	0.37	0.51	0.44
3	0.05	0.77	1.01	0.36	0.67	0.54
4	0.12	0.93	1.19	0.44	0.69	0.72
5	0.00	1.20	1.60	0.48	1.04	0.76
6	0.01	1.20	1.93	0.27	1.14	0.37
7	-0.05	1.35	2.22	0.39	1.37	0.64
8	-0.21	1.39	2.50	0.50	1.77	0.57
9	-0.50	1.48	3.01	0.48	2.00	0.44
10	-0.50	1.52	3.22	0.69	2.12	0.41
11	-0.86	1.71	3.71	0.50	2.47	0.30
12	-1.24	1.72	4.23	0.31	2.90	-0.01
13	-1.47	1.87	4.78	0.37	3.50	-0.03
14	-1.79	1.86	5.27	0.43	3.99	-0.47
15	-2.25	1.89	5.75	0.33	4.06	-0.68
16	-2.55	2.04	6.16	0.31	4.54	-0.93
17	-3.18	1.73	6.76	0.09	4.99	-1.28
18	-3.60	2.04	7:30	0.08	5.36	-1.53
19	-3.93	2.03	8.00	0.03	5.99	-1.93

i	Values $y_i = y(x_i)$					
	Option 19	Option 20	Option 21	Option 22	Option 23	Option 24
0	-1.86	-1.65	-1.89	-1.84	-1.92	-1.90
1	-1.95	-2.00	-2.07	-1.98	-1.60	-1.80
2	-2.12	-1.87	-2.30	-1.72	-1.57	-1.82
3	-2.06	-1.89	-2.26	-1.58	-1.41	-1.86
4	-2.15	-1.75	-2.34	-1.59	-1.36	-1.83
5	-2.00	-1.59	-2.66	-1.59	-0.97	-2.02
6	-2.12	-1.44	-2.88	-1.58	-0.59	-2.01
7	-2.31	-1.51	-2.85	-1.64	-0.71	-2.05
8	-2.29	-1.00	-3.16	-1.55	-0.15	-2.46
9	-2.57	-1.17	-3.49	-1.35	0.01	-2.68
10	-2.56	-0.87	-3.88	-1.33	0.22	-2.85
11	-2.86	-0.47	-4.22	-1.47	0.63	-2.98
12	-2.85	-0.33	-4.45	-1.50	1.07	-3.30
13	-3.03	-0.00	-4.99	-1.65	1.42	-3.40
14	-3.25	0.34	-5.36	-1.62	1.68	-3.90
15	-3.08	0.49	-5.71	-1.87	2.49	-4.37
16	-3.29	0.81	-6.51	-1.61	2.57	-4.65
17	-3.67	1.37	-6.76	-1.86	3.09	-5.00
18	-3.70	1.72	-7.35	-1.84	3.40	-5.42
19	-3.85	2.03	-8.02	-1.91	4.00	-6.13

Laboratory work No. 6

Topic: Numerical differentiation

I. Test questions and tasks.

1. Formulate the problem of numerical differentiation.
2. Give the simplest formulas for numerical differentiation.
3. Describe the known methods for constructing numerical differentiation formulas.

4. Prove the representation

$$f'(x_i) = \frac{f(x_i + h) - f(x_i)}{h} - \frac{h}{2} \cdot f''(\xi), \quad \xi \in (x_i, x_{i+1}), \quad h > 0.$$

5. Prove the representation

$$f'(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2 \cdot h} - \frac{h^2}{6} \cdot f'''(\xi), \quad \xi \in (x_i - h, x_i + h), \quad h > 0.$$

6. Prove the representation

$$f''(x_i) = \frac{f(x_i - h) - 2 \cdot f(x_i) + f(x_i + h)}{h^2} - \frac{h^2}{12} \cdot f^{IV}(\xi), \quad \xi \in (x_i - h, x_i + h), \quad h > 0.$$

7. What is the ill-posedness of the numerical differentiation problem?

8. Show that the algorithm

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{h}$$

The calculation of the approximate value of the derivative at a point x_i is incorrect.

9. Show that the algorithm

$$f'(x) \approx \frac{f(x + h) - f(x - h)}{2 \cdot h}$$

The calculation of the approximate value of the derivative at a point x_i is incorrect.

10. Show that the algorithm

$$f''(x) \approx \frac{f(x - h) - 2 \cdot f(x) + f(x + h)}{h^2}$$

The calculation of the approximate value of the derivative at a point x_i is incorrect.

11. Provide a definition of the concept of “optimal step of numerical differentiation formulas”.
12. Find the optimal steps h for calculating the approximate values of derivatives at a point x using the formulas given in problems 8, 9, 10.
13. Provide practical estimates of the errors of numerical differentiation formulas.
14. Find the error of the method

$$f'(x_i) \approx \frac{f_{i+1} - f_i}{h}, \quad h > 0.$$

using Runge's principle.

15. Find the error of the method

$$f'(x_i) \approx \frac{f_{i+1} - f_{i-1}}{2h}, \quad h > 0.$$

using Runge's principle.

II. Exercise.

Task 1. Construct an interpolation polynomial based on the given information $p_n(x)$, and then set $y'(x) \approx p'_n(x)$, $y''(x) \approx p''_n(x)$.

Task 2. A table of values of the function $y = f(x)$ is given.

1. Create a table of values of the derivative $y'(x)$ at the grid nodes.
2. Create a table of values of the second derivative $y''(x)$ at the grid nodes.
3. Write a computer program to calculate $y'(x)$ and $y''(x)$ at grid nodes.

Hint: In task 2, use second-order approximations of derivatives.

Table 1	
x	f (x)
0.05	1,05127
0.10	1,10517
0.15	1,16183
0.20	1,22140
0.25	1,28403
0.30	1.34986
0.35	1.41907
0.40	1.49182
0.45	1,56831
0.50	1.64872
Table 4	
x	f (x)
0.45	1,56831
0.50	1.64872
0.55	1.73325
0.60	1.82212
0.65	1,91554
0.70	2,01375
0.75	2,11700
0.80	2.22554
0.85	2.33965

Table 2	
x	f (x)
0.05	0.02119
0.10	0.04139
0.15	0.06070
0.20	0.07918
0.25	0.09691
0.30	0.11394
0.35	0.13033
0.40	0.14613
0.45	0.16137
0.50	0.17609
Table 5	
x	f (x)
0.45	0.43497
0.50	0.47943
0.55	0.52269
0.60	0.56464
0.65	0.60519
0.70	0.64422
0.75	0.68164
0.80	0.71736
0.85	0.75128

Table 3	
x	f (x)
0.05	0.99875
0.10	0.99500
0.15	0.98877
0.20	0.98007
0.25	0.96891
0.30	0.95534
0.35	0.93937
0.40	0.92106
0.45	0.90045
0.50	0.87758
Table 6	
x	f (x)
0.45	0.90045
0.50	0.87758
0.55	0.85252
0.60	0.82534
0.65	0.79608
0.70	0.76484
0.75	0.73169
0.80	0.69671
0.85	0.65998

0.9	2,45960
Table 7	
x	<i>f</i>(x)
1.05	2,85765
1.10	3,00417
1.15	3,15819
1.20	3.32012
1.25	3,49034
1.30	3,66930
1.35	3,85743
1.40	4,05520
1.45	4,26311
1.50	4,48169
Table 10	
x	<i>f</i>(x)
0.85	0.65998
0.90	0.62161
0.95	0.58168
1.00	0.54030
1.05	0.49757
1.10	0.45360
1.15	0.40849
1.20	0.36236
1.25	0.31532
1.30	0.26750
Table 13	
x	<i>f</i>(x)
1.20	3.32012
1.25	3,49034
1.30	3,66930
1.35	3,85743
1.40	4,05520
1.45	4,26311
1.50	4,48169
1.55	4.71147

0.90	0.78333
Table 8	
x	<i>f</i>(x)
1.05	0.86742
1.10	0.89121
1.15	0.91276
1.20	0.93204
1.25	0.94898
1.30	0.96356
1.35	0.97572
1.40	0.98545
1.45	0.99271
1.50	0.99749
Table 11	
x	<i>f</i>(x)
0.85	1,13833
0.90	1,26016
0.95	1,39838
1.00	1,55741
1.05	1.74332
1.10	1,96476
1.15	2,23450
1.20	2,57215
1.25	3,00957
1.30	3.60210
Table 14	
x	<i>f</i>(x)
1.20	0.93204
1.25	0.94898
1.30	0.96356
1.35	0.97572
1.40	0.98545
1.45	0.99271
1.50	0.99749
1.55	0.99978

0.90	0.62161
Table 9	
x	<i>f</i>(x)
1.05	0.49757
1.10	0.45360
1.15	0.40849
1.20	0.36236
1.25	0.31532
1.30	0.26750
1.35	0.21901
1.40	0.16997
1.45	0.12050
1.50	0.07074
Table 12	
x	<i>f</i>(x)
0.85	0.75128
0.90	0.78333
0.95	0.81342
1.00	0.84147
1.05	0.86742
1.10	0.89121
1.15	0.91276
1.20	0.93204
1.25	0.94898
1.30	0.96356
Table 15	
x	<i>f</i>(x)
1.20	1,07292
1.25	1,05376
1.30	1,03782
1.35	1,02488
1.40	1,01477
1.45	1,00734
1.50	1.00251
1.55	1,00022

1.60	4,95303
1.65	5,20698
Table 16	
x	<i>f</i>(x)
2.10	0.86321
2.15	0.83690
2.20	0.80850
2.25	0.77807
2.30	0.74571
2.35	0.71147
2.40	0.67546
2.45	0.63776
2.50	0.59847
2.55	0.55768
Table 19	
x	<i>f</i>(x)
1.40	0.16997
1.45	0.12050
1.50	0.07074
1.55	0.02079
1.60	0.02920
1.65	0.07912
1.70	0.12884
1.75	0.17825
1.80	0.22720
1.85	0.27559
Table 22	
x	<i>f</i>(x)
0.25	0.24740
0.30	0.29552
0.35	0.34290
0.40	0.38942
0.45	0.43497
0.50	0.47943
0.55	0.52269

1.60	0.99957
1.65	0.99687
Table 17	
x	<i>f</i>(x)
2.10	0.58485
2.15	0.65403
2.20	0.72790
2.25	0.80735
2.30	0.89348
2.35	0.98769
2.40	1,09169
2.45	1,20770
2.50	1.33865
2.55	1,48839
Table 20	
x	<i>f</i>(x)
1.40	4,05520
1.45	4,26311
1.50	4,48169
1.55	4.71147
1.60	4.95303
1.65	5,20698
1.70	5,47395
1.75	5,75460
1.80	6,04965
1.85	6,35982
Table 23	
x	<i>f</i>(x)
0.25	0.96891
0.30	0.95534
0.35	0.93937
0.40	0.92106
0.45	0.90045
0.50	0.87758
0.55	0.85252

1.60	1,00043
1.65	1.00314
Table 18	
x	<i>f</i>(x)
2.10	0.50485
2.15	0.54736
2.20	0.58850
2.25	0.62817
2.30	0.66628
2.35	0.70271
2.40	0.73739
2.45	0.77023
2.50	0.80114
2.55	0.83005
Table 21	
x	<i>f</i>(x)
1.40	0.98545
1.45	0.99271
1.50	0.99749
1.55	0.99978
1.60	0.99957
1.65	0.99687
1.70	0.99166
1.75	0.98399
1.80	0.97385
1.85	0.96128
Table 24	
x	<i>f</i>(x)
0.25	0.60206
0.30	0.52288
0.35	0.45593
0.40	0.39794
0.45	0.34679
0.50	0.30103
0.55	0.25964

0.60	0.56464
0.65	0.60519
0.70	0.64422
Table 25	
x	<i>f</i>(x)
1.25	0.09691
1.30	0.11394
1.35	0.13033
1.40	0.14613
1.45	0.16137
1.50	0.17609
1.55	0.19033
1.60	0.20412
1.65	0.21748
1.70	0.23045

0.60	0.82534
0.65	0.79608
0.70	0.76484
Table 26	
x	<i>f</i>(x)
1.25	0.22314
1.30	0.26236
1.35	0.30010
1.40	0.33647
1.45	0.37156
1.50	0.40547
1.55	0.43825
1.60	0.47000
1.65	0.50078
1.70	0.53063

0.60	0.22185
0.65	0.18709
0.70	0.15490
Table 27	
x	<i>f</i>(x)
1.25	0.31532
1.30	0.26750
1.35	0.21901
1.40	0.16997
1.45	0.12050
1.50	0.07074
1.55	0.02079
1.60	0.02920
1.65	0.07912
1.70	0.12884

Laboratory work No. 7

Topic: Numerical integration

I. Test questions and tasks.

1. Briefly outline the statement of the numerical integration problem.
2. Provide definitions of the basic concepts: quadrature sum, quadrature formula, nodes and coefficients of the quadrature formula, error of the quadrature formula.
3. Briefly outline the essence of the rectangle method.
4. Briefly outline the essence of the trapezoid method.
5. Briefly outline the essence of Simpson's method.
6. Write out the compound formulas of rectangles, trapezoids, and Simpson's formulas.
7. Write out the errors and their estimates for the composite formulas of rectangles, trapezoids, and Simpson's equations.
8. Briefly outline the methods for choosing the integration step.
9. Provide practical estimates of errors in numerical integration formulas (Runge's principle).
10. Briefly outline the essence of the Newton- Cotes formulas .
11. Briefly outline the essence of Gauss-type formulas.
12. List the most important properties of Legendre polynomials.
13. Calculate the integral

$$\int_0^1 \frac{dx}{1+x}$$

using the trapezoidal formula for $n = 10$ and estimate the remainder term.

14. Calculate the integral

$$\int_4^5 \ln x dx$$

using Simpson's formula $n = 10$ and estimate the remainder term.

15. Calculate the integral

$$\int_0^{0.8} f(x) dx$$

using Simpson's formula and estimate the remainder term if the integrand is given by the table:

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	1.0000	0.9950	0.9801	0.9553	0.9211	0.8756	0.8553	0.7648	0.6967

16. Find the integration step h for calculating the definite integral

$$\int_2^3 \ln x dx$$

according to the trapezoidal formula with an accuracy of 0.001.

17. Find the integration step h for calculating the definite integral

$$\int_2^3 \ln x dx$$

according to Simpson's formula with an accuracy of 0.0001.

II. Exercise.

1. Calculate the definite integral using the rectangle and trapezoid formula for $n = 10$. Estimate the error of the result by compiling a table of finite differences.
2. Calculate the definite integral using Simpson's formula for $n = 10$. Estimate the error of the result by compiling a table of finite differences.
3. Develop computer programs for implementing the trapezoid and Simpson methods.

III . Options.

1) a) $\int_{0.8\sqrt{2x^2+1}}^{1.6} dx$

b) $\int_{1.2}^2 \frac{\ln(x+2)}{x} dx$

2) a) $\int_{1.2\sqrt{x^2+3.2}}^{2.7} dx$

b) $\int_{1.6}^{2.4} (x+1) \cdot \sin x dx$

3) a) $\int_{0.8\sqrt{2x^2+1}}^{1.6} dx$

b) $\int_{1.2}^2 \frac{\ln(x+2)}{x} dx$

4) a) $\int_{1.2\sqrt{x^2+1.2}}^2 dx$

b) $\int_{1.2}^2 \frac{\ln(x^2+3)}{2x} dx$

5) a) $\int_{2.2\sqrt{x^2+0.5}}^{2.6} dx$

b) $\int_{1.4}^3 x^2 \ln x dx$

6) a) $\int_{1.4\sqrt{3x^2+1}}^{2.2} dx$

b) $\int_{1.4}^{2.2} \frac{\ln(x^2+2)}{x+1} dx$

7) a) $\int_{0.8\sqrt{x^2+4}}^{1.8} dx$

b) $\int_{0.4}^{1.2} \frac{\cos(x^2)}{x+1} dx$

8) a) $\int_{1.6\sqrt{x^2+2.5}}^{2.2} dx$

b) $\int_{0.8}^{1.6} (x^2+1) \sin(x-0.5) dx$

9) a) $\int_{0.6\sqrt{x^2+0.8}}^{1.6} dx$

b) $\int_{0.6}^{1.4} x^2 \cos x dx$

- 10) a) $\int_{1.2}^2 \frac{dx}{\sqrt{x^2 + 1.2}}$ b) $\int_{1.2}^2 \frac{\ln(x^2 + 3)}{2x} dx$
- 11) a) $\int_{1.2}^2 \frac{dx}{\sqrt{2x^2 + 0.7}}$ b) $\int_{2.5}^{3.3} \frac{\ln(x^2 + 0.8)}{x-1} dx$
- 12) a) $\int_{3.2}^4 \frac{dx}{\sqrt{0.5x^2 + 1}}$ b) $\int_{0.5}^{1.2} \frac{tg(x^2)}{x+1} dx$
- 13) a) $\int_{0.8}^{1.7} \frac{dx}{\sqrt{2x^2 + 0.3}}$ b) $\int_{1.3}^{2.2} \frac{\sin(x^2 - 1)}{2\sqrt{x}} dx$
- 14) a) $\int_{1.2}^2 \frac{dx}{\sqrt{0.5x^2 + 1.5}}$ b) $\int_{0.2}^{1.0} (x+1)\cos(x^2) dx$
- 15) a) $\int_{2.1}^{3.6} \frac{dx}{\sqrt{x^2 + 3}}$ b) $\int_{0.8}^{1.2} \frac{\sin(x^2 - 0.4)}{x+2} dx$
- 16) a) $\int_{1.3}^{2.5} \frac{dx}{\sqrt{0.2x^2 + 1}}$ b) $\int_{0.15}^{0.63} \sqrt{x+1} \ln(x+3) dx$
- 17) a) $\int_{0.6}^{1.4} \frac{dx}{\sqrt{1.2x^2 + 0.5}}$ b) $\int_{1.2}^{2.8} \frac{\ln(x^2 + 1)}{2x+1} dx$
- 18) a) $\int_{1.6}^{2.2} \frac{dx}{\sqrt{x^2 + 0.5}}$ b) $\int_{0.8}^{1.6} (x^2 + 1.6)\cos(x-0.5) dx$
- 19) a) $\int_{0.6}^{1.6} \frac{dx}{\sqrt{0.2x^2 + 2.5}}$ b) $\int_{0.6}^{1.4} x^2 \sin(x) dx$
- 20) a) $\int_{1.2}^2 \frac{dx}{\sqrt{x^2 + 5.2}}$ b) $\int_{1.2}^2 \frac{\ln(x^2 + 0.3)}{2x+1} dx$
- 21) a) $\int_{1.2}^2 \frac{dx}{\sqrt{2x^2 + 0.7}}$ b) $\int_{2.5}^{3.3} \frac{\ln(x^2 + 0.8)}{x+1} dx$
- 22) a) $\int_{3.2}^4 \frac{dx}{\sqrt{0.1x^2 + 2.5}}$ b) $\int_{0.5}^{1.2} \frac{tg(x^2)}{x+1} dx$
- 23) a) $\int_{0.8}^{1.7} \frac{dx}{\sqrt{0.2x^2 + 3}}$ b) $\int_{1.3}^{2.2} \frac{\sin(x^2 + 1)}{\sqrt{x}} dx$
- 24) a) $\int_{1.2}^2 \frac{dx}{\sqrt{1.5x^2 + 0.5}}$ b) $\int_{0.2}^{1.0} (x+2)\cos(x^2 - 1) dx$
- 25) a) $\int_{2.1}^{3.6} \frac{dx}{\sqrt{0.3x^2 + 1.3}}$ b) $\int_{0.8}^{1.2} \frac{\sin(x^2 + 0.4)}{2\sqrt{x}} dx$

2. a). $0.3 \sin 0.1 x + 0.15 x = 1$;
3. a). $0.4 \sin 0.2 x - 0.25 x = 1$;
4. a). $0.3 \cos 0.2 x - 0.4 x = 0$;
5. a). $0.2 \cos 0.1 x - 0.24 x = 0$;
6. a). $0.2 \cos 0.2 x + 0.1 x = 0$;
7. a). $0.5 \sin x - 0.8 x + 1 = 0$;
8. a). $2 \sin 2 x + 8.2 x - 1 = 0$;
9. a). $1.2 \sin 0.5 x - 1.5 x + 1 = 0$;
10. a). $2.4 x - 1.2 \cos 0.3 x = 0$;
11. a). $0.3 x + 0.5 \cos 0.1 x = 0$;
12. a). $5 x - \cos x - 1 = 0$;
13. a). $3 x + \cos 0.2 x - 2 = 0$;
14. a). $2.1 x - \sin 0.1 x = 0.25$;
15. a). $2.5 x - \sin 0.3 x = 3.2$;
16. a). $0.1 x + 0.5 \cos 0.1 x = 0.3$;
17. a). $\cos x - 3 x - 1 = 0$;
18. a). $\sin x - 3 x + 1 = 0$;
19. a). $4 x - \sin 2 x - 1.2 = 0$;
20. a). $1 - 2.5 x - \sin x = 0$;
21. a). $\sin x - 3.2 x - 1 = 0$;
22. a). $\cos x + 1.8 x - 1.2 = 0$;
23. a). $0.8 - 2.5 x + \cos x = 0$;
24. a). $0.5 - \sin x + 2.9 x = 0$;
25. a). $2.8 x - \sin x = 1.2$;

- b). $10 x - e^x = 0$.
- b). $e^{0.5 x} - 16 x = 0$.
- b). $11 x - e^x = 0$.
- b). $e^{0.2 x} - 5 x = 0$.
- b). $4 x - e^{0.1 x} = 0$.
- b). $e^{0.1 x} - 4.5 x = 0$.
- b). $e^{0.3 x} - 4 x = 0$.
- b). $e^{0.2 x} - 5.1 x = 0$.
- b). $e^{0.3 x} - 7.2 x = 0$.
- b). $6 x - e^{0.1 x} = 0$.
- b). $5.8 x - e^{0.2 x} = 0$.
- b). $e^{0.6 x} - 12 x = 0$.
- b). $e^{0.5 x} - 9.8 x = 0$.
- b). $e^{0.4 x} - 12.2 x = 0$.
- b). $9.8 x - e^{0.6 x} = 0$.
- b). $e^{0.7 x} - 13.2 x = 0$.
- b). $10.2 x - e^x = 0$.
- b). $e^{0.9 x} - 10.4 x = 0$.
- b). $e^x - 11.4 x = 0$.
- b). $e^{-0.2 x} - 1.2 x = 0$.
- b). $10 x - e^x = 0$.
- b). $e^{1.1 x} - 12.2 x = 0$.
- b). $8.2 x - e^{0.8 x} = 0$.
- b). $5.6 x - e^{0.6 x} = 0$.

Laboratory work No. 9

Topic: Numerical solution of systems of linear algebraic equations .

I. **Test questions and tasks.**

1. Is the vector $[1; 1; 1]^T$ a solution to the system $A * X = F$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad F = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} ?$$

2. Is the vector $[1; 1; 1]^T$ a solution to the system $A * X = F$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad F = \begin{bmatrix} 6 \\ 15 \\ 23 \end{bmatrix} ?$$

3. To what type of methods - direct (exact) or iterative - does the Gauss method belong?
4. Briefly outline the essence of the Gauss method.
5. What is the forward and backward motion in Gauss's method?
6. State the essence of Khaletsky's scheme .
7. Explain the essence of the square root method.
8. Explain the essence of the running method.
9. Let the system of equations $A * X = F$ be given :

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 4 & 3 & 1 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 8 \end{bmatrix} .$$

Find: 1). All norms of the vector F;

2). $\| A \|_1, \| A \|_2$.

10. A system of equations is given

$$\begin{cases} 4x_1 + 0.24x_2 - 0.08x_3 = 8, \\ 0.09x_1 + 3x_2 - 0.15x_3 = 9, \\ 0.04x_1 - 0.08x_2 + 4x_3 = 20. \end{cases}$$

To solve this system, construct an iterative process. Are the convergence conditions for the iterative process satisfied?

11. Solve the system from task 9 using the iteration method with an accuracy ϵ of 0.0001. Find the termination criterion for the iteration process.
12. Solve the system from task 9 using the iteration method with an accuracy ϵ of 0.001. Estimate the number of iterations (steps) required.
13. Indicate how the Seidel method differs from the simple iteration method?

II. **Exercise.**

1. System

$$\begin{pmatrix} 2+\alpha & 0.5 & 0 & 0 & 0 \\ 0.5 & 6.1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 2.5 & 0.5 & 0 \\ 0 & 0 & 0.5 & 3.4 & 0.5 \\ 0 & 0 & 0 & 0.5 & 7.1+\beta \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2.6 \\ 7.0 \\ 4.1 \\ 4.5 \\ 6.9 \end{pmatrix}$$

$\alpha = 0.02 \cdot n$, $\beta = 0.01 \cdot n$, n is the variant number, solve by the run method.

2. A system of linear equations is given $A \cdot X = B$, where

$$A = \begin{bmatrix} 9.8 + \alpha & 1.02 & 0.75 & 0.5 \\ 1.05 & 8.9 & 0.45 & 1.22 \\ 0.73 & 0.55 & 9.95 & 0.51 \\ 0.35 & 0.45 & 1.18 & 9.75 + \alpha \end{bmatrix}$$

$$B = [15.655; 22.705; 25.805; 23.480 - \beta]^T$$

$$\alpha = 0.02 \cdot n, \beta = 0.01 \cdot n$$

n - option number .

Solve this system using simple iteration and Seidel's method. Continue iterations until the inequality is satisfied.

$$\|X^{(k)} - X^{(k-1)}\| < 0.0001.$$

Estimate the error of the obtained results.

3. System

$$\begin{bmatrix} 12.72 + \alpha & 2.45 & 0.35 & 1.54 \\ 3.44 & 15.82 & 1.84 & 0.35 \\ 4.55 & 0.78 & 20.75 + \alpha & 0.85 \\ 3.44 & 0.55 & 0.35 & 20.75 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 20.755 \\ 25.455 \\ 18.470 + \beta \\ 19.680 \end{bmatrix}$$

$\alpha = 0.02 \cdot n$, $\beta = 0.01 \cdot n$, n is the variant number, solve using the simple iteration method and the Seidel method with an error $\epsilon_{of} = 0.0001$.

4. System

$$\begin{bmatrix} 10.25 + \alpha & 1.85 & 0.35 \\ 2.22 & 12.54 & 1.48 \\ 3.08 & 2.44 & 15.85 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12.445 + \alpha \\ 15.550 \\ 18.445 \end{bmatrix}$$

$\alpha = 0.02 \cdot n$, n is the variant number, solve using the simple iteration method and the Seidel method, performing 5 iterations. Estimate the errors in the obtained results.

5. Calculate the number of steps required to solve the system from task 3 with an error $\epsilon_{of} = 0.0001$.

5. Solve the systems from all tasks using computer software.

6. **Note** : To solve systems of equations, develop appropriate computer programs.

Laboratory work No. 10

Topic: Numerical solution of systems of nonlinear equations

I. Test questions and tasks.

1. Is a pair of numbers (1,1) a solution to the system of equations

$$x^2 + y^2 = 2,$$

$$xy = 1 ?$$

2. Is a pair of numbers (1,0) a solution to the system of equations

$$x^2 + y^2 = 1,$$

$$2x + \sin y = 1 ?$$

3. System

$$\sin(x+y) + 2x = 0.5,$$

$$\cos(2x + y) + 5y = 0.1$$

bring to form

$$x = f(x, y),$$

$$y = g(x, y).$$

4. Find the domain of solutions of the system

$$\sin(x+y) + 2x = 0.5,$$

$$\cos(2x+y) + 5y = 0.1 .$$

5. A system of nonlinear equations reduced to the form

$$x_1 = f_1(x_1, x_2, \dots, x_n),$$

$$x_2 = f_2(x_1, x_2, \dots, x_n),$$

$$\dots\dots\dots$$

$$x_n = f_n(x_1, x_2, \dots, x_n),$$

is solved by the iterative method. Formulate the conditions for convergence of this method.

6. A display of the form is given

$$x = f(x), \quad \text{where } x, f - n - \text{dimensional vectors.}$$

Give a condition under which this mapping is contractive.

7. Zoom in on functions

$$f_1(x, y) = \sin(x+y) + 2x - 0.5,$$

$$f_2(x, y) = \cos(2x+y) + 5y - 0.1$$

Taylor polynomials of the first order in the neighborhood of the point (0, 0).

II. Exercise.

1. Using the iteration method, solve the system of nonlinear equations with an accuracy of $\varepsilon = 0.001$.
2. Using Newton's method, solve a system of nonlinear equations with an accuracy of $\varepsilon = 0.001$.
3. To solve systems of nonlinear equations using iteration and Newton methods, create appropriate programs.
4. Solve these systems in the task using computer software.

III. Options.

1. a). $\begin{cases} \sin(x+1) - y = 1.2; \\ 2x + \cos y = 2. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy + 0.4) = x^2; \\ 0.6x^2 + 2y^2 = 1, \quad x > 0, y > 0. \end{cases}$
2. a). $\begin{cases} \cos(x-1) + y = 0.5; \\ x - \cos y = 3. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.6x = 0; \\ x^2 + y^2 = 1, \quad x > 0, y > 0. \end{cases}$
3. a). $\begin{cases} \sin x + 2y = 2; \\ \cos(y-1) + x = 0.7. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy + 0.1) = x^2; \\ x^2 + 2y^2 = 1. \end{cases}$
4. a). $\begin{cases} \cos x + y = 1.5; \\ 2x - \sin(y-0.5) = 1. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.2x = 0.2; \\ x^2 + y^2 = 1. \end{cases}$
5. a). $\begin{cases} \sin(x+0.5) - y = 1; \\ \cos(y-2) + x = 0. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy + 0.3) = x^2; \\ 0.9x^2 + 2y^2 = 1. \end{cases}$
6. a). $\begin{cases} \cos(x+0.5) + y = 0.8; \\ \sin y - 2x = 1.6. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.3x = 0; \\ x^2 + y^2 = 1. \end{cases}$
7. a). $\begin{cases} \sin(x-1) = 1.3 - y; \\ x - \sin(y+1) = 0.8. \end{cases}$ b). $\begin{cases} \operatorname{tg}xy = x^2; \\ 0.8x^2 + 2y^2 = 1. \end{cases}$
8. a). $\begin{cases} 2y - \cos(x+1) = 0; \\ x + \sin y = -0.4. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.5x = 0.1; \\ x^2 + y^2 = 1. \end{cases}$
9. a). $\begin{cases} \cos(x+0.5) - y = 2; \\ \sin y - 2x = 1. \end{cases}$ b). $\begin{cases} \operatorname{tg}xy = x^2; \\ 0.7x^2 + 2y^2 = 1. \end{cases}$
10. a). $\begin{cases} \sin(x+2) - y = 1.5; \\ x + \cos(y-2) = 0.5. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.2x = 0.1; \\ x^2 + y^2 = 1. \end{cases}$
11. a). $\begin{cases} \sin(y+1) - x = 1.2; \\ 2y + \cos x = 2. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy + 0.2) = x^2; \\ 0.6x^2 + 2y^2 = 1. \end{cases}$
12. a). $\begin{cases} \cos(y-1) + x = 0.5; \\ y - \cos x = 3. \end{cases}$ b). $\begin{cases} \sin(x+y) = 1.5x - 0.1; \\ x^2 + y^2 = 1. \end{cases}$
13. a). $\begin{cases} \sin y + 2x = 2; \\ \cos(x-1) + y = 0.7. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy + 0.4) = x^2; \\ 0.8x^2 + 2y^2 = 1. \end{cases}$
14. a). $\begin{cases} \cos y + x = 1.5; \\ 2y - \sin(x-0.5) = 1. \end{cases}$ b). $\begin{cases} \sin(x+y) = 1.2x - 0.1; \\ x^2 + y^2 = 1. \end{cases}$

15. a). $\begin{cases} \sin(y+0.5) - x = 1; \\ \cos(x-2) + y = 0. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy+0.1) = x^2; \\ 0.9x^2 + 2y^2 = 1. \end{cases}$
16. a). $\begin{cases} \cos(y+0.5) + x = 0.8; \\ \sin x - 2y = 1.6. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.4x = 0; \\ x^2 + y^2 = 1. \end{cases}$
17. a). $\begin{cases} \sin(y-1) + x = 1.3; \\ y - \sin(x+1) = 0.8. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy+0.1) = x^2; \\ 0.5x^2 + 2y^2 = 1. \end{cases}$
18. a). $\begin{cases} 2x - \cos(y+1) = 0; \\ y + \sin x = -0.4. \end{cases}$ b). $\begin{cases} \sin(x+y) = 1.1x - 0.1; \\ x^2 + y^2 = 1. \end{cases}$
19. a). $\begin{cases} \cos(y+0.5) - x = 2; \\ \sin x - 2y = 1. \end{cases}$ b). $\begin{cases} \operatorname{tg}(x-y) - xy = 0; \\ x^2 + 2y^2 = 1. \end{cases}$
20. a). $\begin{cases} \sin(y+2) - x = 1.5; \\ y + \cos(x-2) = 0.5. \end{cases}$ b). $\begin{cases} \sin(x-y) - xy = -1; \\ x^2 - y^2 = \frac{3}{4}. \end{cases}$
21. a). $\begin{cases} \sin(x+1) - y = 1; \\ 2x + \cos y = 2. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy+0.2) = x^2; \\ x^2 + 2y^2 = 1. \end{cases}$
22. a). $\begin{cases} \cos(x-1) + y = 0.8; \\ x - \cos y = 2. \end{cases}$ b). $\begin{cases} \sin(x+y) - 1.5x = 0; \\ x^2 + y^2 = 1. \end{cases}$
23. a). $\begin{cases} \sin x + 2y = 1.6; \\ \cos(y-1) + x = 1. \end{cases}$ b). $\begin{cases} \operatorname{tg} xy = x^2; \\ 0.5x^2 + 2y^2 = 1. \end{cases}$
24. a). $\begin{cases} \cos x + y = 1.2; \\ 2x - \sin(y-0.5) = 2. \end{cases}$ b). $\begin{cases} \sin(x+y) = 1.2x - 0.2; \\ x^2 + y^2 = 1. \end{cases}$
25. a). $\begin{cases} \sin(x+0.5) - y = 1.2; \\ \cos(y-2) + x = 0. \end{cases}$ b). $\begin{cases} \operatorname{tg}(xy+0.1) = x^2; \\ 0.7x^2 + 2y^2 = 1. \end{cases}$

Laboratory work NO. 11

Topic: Numerical solution of the Cauchy problem

I. Test questions and tasks.

1. Is the function $\exp(x)$ a solution to the differential equation $y'(x) = y(x)$?
2. Is the function $\exp(x)$ a solution to the differential equation $y'(x) = 2y(x)$?
3. Formulate the Cauchy problem for an ODE. Give an example.
4. Specify methods for solving the Cauchy problem for ODE.
5. Describe the essence of the numerical solution of the Cauchy problem for ODE.
6. Solve the Cauchy problem for DE
$$y'(x) = y(x), y(0) = 1$$
by the method of series.
7. Write down the solution to the Cauchy problem for the differential equation
$$y'(x) = f(x, y), y(x_0) = y_0$$
in integral form.
8. Give the formula for the Euler-Rhenium method of the Cauchy problem
$$y'(x) = f(x, y), y(x_0) = y_0.$$
9. Find a solution to the Cauchy problem for differential equations
$$y'(x) = y(x), y(0) = 1$$
using the Euler method with a step of 0.1 on the interval $[0, 0.5]$. Compare the obtained solution with the analytical one. Indicate the errors at each grid point.
10. the fourth-order Runge-Kutta method for solving the Cauchy problem
$$y'(x) = f(x, y), y(x_0) = y_0.$$
11. Provide an approximate formula for the error at each grid point for the Runge-Kutta methods.
12. Find a solution to the Cauchy problem for differential equations
$$y'(x) = y(x), y(0) = 1$$
13. Solve the second-order Runge-Kutta problem for the differential equation
$$y'(x) = y(x), y(0) = 1$$
with a step of 0.1 on the interval $[0, 0.5]$. Compare the obtained solution with the analytical one. Indicate the errors at each grid point.
14. Formulate the idea of constructing Adams' methods.
15. Describe the advantages and disadvantages of Adams methods in comparison with Runge-Kutta methods.

II. Exercise.

Using the methods of Euler, Runge-Kutta and Adams, solve the given differential equation $u' = f(x, u)$ with the initial condition $u(x_0) = u_0$ on the interval $[a, b]$ with an accuracy $\varepsilon = 0.001$.

1. To solve this problem, develop appropriate computer programs.

2. To solve this problem, use computer software.

III. Options.

1. $u' = x + \cos \frac{u}{\sqrt{5}}, \quad u_0(1.8) = 2.6, \quad x \in [1.8; 2.8].$
2. $u' = x + \cos \frac{u}{3}, \quad u_0(1.6) = 4.6, \quad x \in [1.6; 2.6].$
3. $u' = x + \cos \frac{u}{\sqrt{10}}, \quad u_0(0.6) = 0.8, \quad x \in [0.6; 1.6].$
4. $u' = x + \cos \frac{u}{\sqrt{7}}, \quad u_0(0.5) = 0.6, \quad x \in [0.5; 1.5].$
5. $u' = x + \cos \frac{u}{\pi}, \quad u_0(1.7) = 5.3, \quad x \in [1.7; 2.7].$
6. $u' = x + \cos \frac{u}{2.25}, \quad u_0(1.4) = 2.2, \quad x \in [1.4; 2.4].$
7. $u' = x + \cos \frac{u}{e}, \quad u_0(1.4) = 2.5, \quad x \in [1.4; 2.4].$
8. $u' = x + \cos \frac{u}{\sqrt{2}}, \quad u_0(0.8) = 1.4, \quad x \in [0.8; 1.8].$
9. $u' = x + \cos \frac{u}{\sqrt{3}}, \quad u_0(1.2) = 2.1, \quad x \in [1.2; 2.2].$
10. $u' = x + \cos \frac{u}{\sqrt{11}}, \quad u_0(2.1) = 2.5, \quad x \in [2.1; 3.1].$
11. $u' = x + \cos \frac{u}{1.25}, \quad u_0(0.4) = 0.8, \quad x \in [0.4; 1.4].$
12. $u' = x + \cos \frac{u}{\sqrt{15}}, \quad u_0(0.3) = 0.9, \quad x \in [0.3; 1.3].$
13. $u' = x + \cos \frac{u}{\sqrt{0.3}}, \quad u_0(0.7) = 2.1, \quad x \in [0.7; 1.7].$
14. $u' = x + \sin \frac{u}{\sqrt{5}}, \quad u_0(1.8) = 2.6, \quad x \in [1.8; 2.8].$
15. $u' = x + \sin \frac{u}{3}, \quad u_0(1.6) = 4.6, \quad x \in [1.6; 2.6].$
16. $u' = x + \sin \frac{u}{\sqrt{10}}, \quad u_0(0.6) = 0.8, \quad x \in [0.6; 1.6].$
17. $u' = x + \sin \frac{u}{\sqrt{7}}, \quad u_0(0.5) = 0.6, \quad x \in [0.5; 1.5].$
18. $u' = x + \sin \frac{u}{\pi}, \quad u_0(1.7) = 5.3, \quad x \in [1.7; 2.7].$
19. $u' = x + \sin \frac{u}{\sqrt{2.8}}, \quad u_0(1.4) = 2.2, \quad x \in [1.4; 2.4].$
20. $u' = x + \sin \frac{u}{e}, \quad u_0(1.4) = 2.5, \quad x \in [1.4; 2.4].$

21. $u' = x + \sin \frac{u}{\sqrt{2}}, \quad u_0(0.8) = 1.3, \quad x \in [0.8; 1.8].$
22. $u' = x + \sin \frac{u}{\sqrt{3}}, \quad u_0(1.1) = 1.5, \quad x \in [1.1; 2.1].$
23. $u' = x + \sin \frac{u}{\sqrt{11}}, \quad u_0(0.6) = 1.2, \quad x \in [0.6; 1.6].$
24. $u' = x + \sin \frac{u}{1.25}, \quad u_0(0.5) = 1.8, \quad x \in [0.5; 1.5].$
25. $u' = x + \sin \frac{u}{\sqrt{15}}, \quad u_0(0.3) = 0.9, \quad x \in [0.3; 1.3].$

Laboratory work No. 12

Topic: Numerical solution of boundary value problems for ODEs

I. Exercise.

Find the solution of the boundary value problem using the finite difference method and the sweep method on the segment $[a, b]$ with an accuracy $\varepsilon_{\text{of}} = 0.001$.

1. To solve this problem, develop appropriate computer programs.
2. To solve this problem, use computer software.

II. Options.

- | | | | |
|-----|--|-----|---|
| 1. | $y'' + \frac{y'}{x} + 2y = x,$ $\begin{cases} y(0.7) = 0.5, \\ 2y(1) + 3y'(1) = 1.2. \end{cases}$ | 2. | $y'' - xy' + 2y = x + 1,$ $\begin{cases} y(0.9) - 0.5y'(0.9) = 2, \\ y(1.2) = 1. \end{cases}$ |
| 3. | $y'' + xy' + y = x + 1,$ $\begin{cases} y(0.5) + 2y'(0.5) = 1, \\ y'(0.8) = 1.2. \end{cases}$ | 4. | $y'' + 2y' - \frac{y}{x} = 3,$ $\begin{cases} y(0.2) = 2, \\ 0.5y(0.5) - y'(0.5) = 1. \end{cases}$ |
| 5. | $y'' + 2y' - xy = x^2,$ $\begin{cases} y'(0.6) = 0.7, \\ y(0.9) - 0.5y'(0.9) = 1. \end{cases}$ | 6. | $y'' - y' + \frac{2y}{x} = x + 0.4,$ $\begin{cases} y(1.1) - 0.5y'(1.1) = 2, \\ y'(1.4) = 4. \end{cases}$ |
| 7. | $y'' - 3y' + \frac{y}{x} = 1,$ $\begin{cases} y(0.4) = 2, \\ y(0.7) + 2y'(0.7) = 0.7. \end{cases}$ | 8. | $y'' + 3y' - \frac{y}{x} = x + 1,$ $\begin{cases} y'(1.2) = 1, \\ 2y(1.5) - y'(1.5) = 0.5. \end{cases}$ |
| 9. | $y'' - \frac{y'}{2} + 3y = 2x^2,$ $\begin{cases} y(1) + 2y'(1) = 0.6, \\ y(1.3) = 1. \end{cases}$ | 10. | $y'' + 1.5y' - xy = 0.5,$ $\begin{cases} 2y(1.3) - y'(1.3) = 1, \\ y(1.6) = 3. \end{cases}$ |
| 11. | $y'' + 2xy' - y = 0.4,$ $\begin{cases} 2y(0.3) + y'(0.3) = 1, \\ y'(0.6) = 2. \end{cases}$ | 12. | $y'' - 0.5xy' + y = 2,$ $\begin{cases} y(0.4) = 1.2, \\ y(0.7) + 2y'(0.7) = 1.4. \end{cases}$ |

$$13. \quad \begin{cases} y'' + \frac{2y'}{x} - 3y = 2, \\ y'(0.8) = 1.5, \\ 2y(1.1) + y'(1.1) = 3. \end{cases}$$

$$14. \quad \begin{cases} y'' + 2x^2 y' + y = x, \\ 2y(0.5) - y'(0.5) = 1, \\ y(0.8) = 3. \end{cases}$$

$$15. \quad \begin{cases} y'' - 3xy' + 2y = 1.5, \\ y'(0.7) = 1.3, \\ 0.5y(1) + y'(1) = 2. \end{cases}$$

$$16. \quad \begin{cases} y'' + 2xy' - 2y = 0.6, \\ y'(2) = 1, \\ 0.4y(2.3) - y'(2.3) = 1. \end{cases}$$

$$17. \quad \begin{cases} y'' + \frac{y'}{x} - 0.4y = 2x, \\ y(0.6) - 0.3y'(0.6) = 0.6, \\ y'(0.9) = 1.7. \end{cases}$$

$$18. \quad \begin{cases} y'' - \frac{y'}{2x} + 0.8y = x, \\ y(1.7) + 1.2y'(1.7) = 2, \\ y(2) = 1. \end{cases}$$

$$19. \quad \begin{cases} y'' - \frac{y'}{3} + xy = 2, \\ y(0.8) = 1.6, \\ 3y(1.1) - 0.5y'(1.1) = 1. \end{cases}$$

$$20. \quad \begin{cases} y'' + 0.8y' - xy = 1.4, \\ y(1.8) = 0.5, \\ 2y(2.1) + y'(2.1) = 1.7. \end{cases}$$

$$21. \quad \begin{cases} y'' + 2y' - \frac{y}{x} = \frac{1}{x}, \\ 0.5y(0.9) + y'(0.9) = 1, \\ y(1.2) = 0.8. \end{cases}$$

$$22. \quad \begin{cases} y'' - \frac{y'}{4} + \frac{2y}{x} = \frac{x}{2}, \\ 1.5y(1.3) - y'(1.3) = 0.6, \\ 2y(1.6) = 0.3. \end{cases}$$

$$23. \quad \begin{cases} y'' - 0.5y' + 0.5xy = 2x, \\ y'(1) = 0.5, \\ 2y(1.3) - y'(1.3) = 2. \end{cases}$$

$$24. \quad \begin{cases} y'' - 0.5x^2 y' + 2y = x^2, \\ y(1.6) + 0.7y'(1.6) = 2, \\ y(1.9) = 0.8. \end{cases}$$

$$25. \quad \begin{cases} y'' + \frac{y'}{2x} - y = \frac{2}{x}, \\ y(0.6) = 1.3, \\ 0.5y(0.9) - 1.2y'(0.9) = 1. \end{cases}$$

Laboratory work No. 13

Topic: Solution of a mixed problem for a hyperbolic equation using the grid method

I. Exercise.

Solve a mixed problem for the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with initial conditions

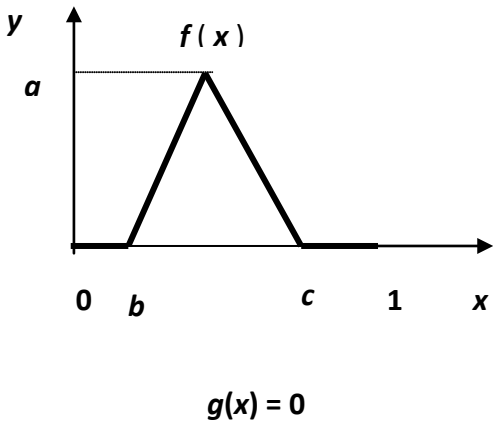
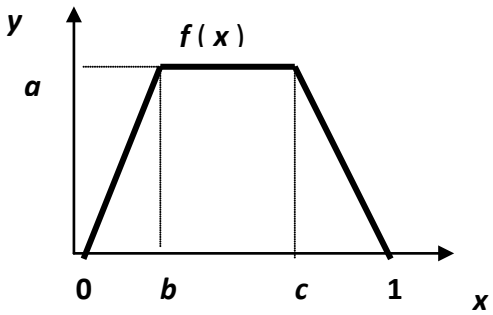
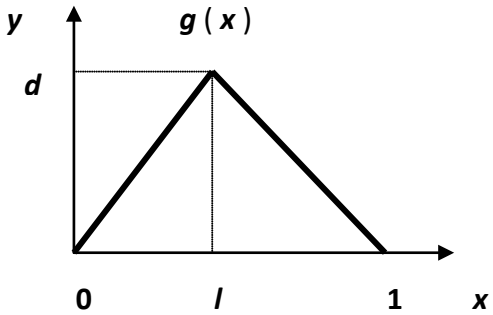
$$u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = g(x), \quad 0 \leq x \leq 1 \text{ and zero boundary conditions}$$

$$u(0, t) = u(1, t) = 0.$$

3. To solve this problem, develop appropriate computer programs.
4. To solve this problem, use computer software.

II. Options.

Number Variants	$f(x)$ $g(x)$	a	b	c	d	l
1	The figure shows two graphs on the interval [0, 1]. The top graph, labeled $f(x)$, is a triangle with vertices at (0,0), (b,a), and (1,0). The bottom graph, labeled $g(x)$, is a rectangle with height c on the interval $[d, l]$ and zero elsewhere.	1.0	0.05	1.5	0.05	0.45
2		2.0	0.10	1.6	0.10	0.50
3		3.0	0.15	1.7	0.15	0.55
4		4.0	0.20	1.8	0.20	0.60
5		5.0	0.25	1.9	0.25	0.65
6		6.0	0.30	-2.0	0.30	0.70
7		7.0	0.35	-2.1	0.35	0.75
8		8.0	0.40	-2.2	0.40	0.80

9	 $g(x) = 0$	1.0	0.05	0.45		
10		2.0	0.10	0.50		
11		3.0	0.15	0.55		
12		4.0	0.20	0.60		
13		5.0	0.25	0.65		
14		6.0	0.30	0.70		
15		7.0	0.35	0.75		
16		8.0	0.40	0.80		
17	 	1.0	0.10	0.20	-1.0	0.10
18		2.0	0.15	0.30	-3.0	0.20
19		3.0	0.20	0.40	-4.0	0.30
20		4.0	0.30	0.50	-5.0	0.40
21		5.0	0.40	0.60	-1.0	0.50
22		6.0	0.50	0.70	1.0	0.60
23		7.0	0.60	0.80	2.0	0.70
24		8.0	0.70	0.90	2.5	0.80
25		9.0	0.80	0.95	3.0	0.90

Laboratory work No. 14

Topic: Solution of a mixed problem for a parabolic equation using the grid method

I. Exercise.

Solve a mixed problem for the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the initial condition

$u(x,0) = f(x)$ and boundary conditions $u(0, t) = a$,

$u(1, t) = b$.

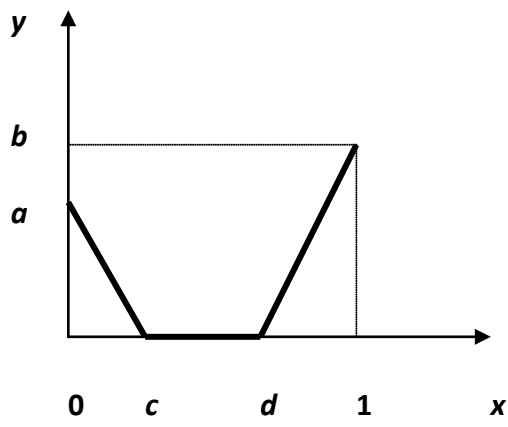
5. To solve this problem, develop appropriate computer programs.

6. To solve this problem, use computer software.

II. Options.

Number options	$f(x)$	a	b	c	d
1		1.1	3.0	0.05	
2		1.2	3.5	0.15	
3		1.3	4.0	0.25	
4		1.4	4.5	0.35	
5		1.5	5.0	0.45	
6		1.6	5.5	0.55	
7		1.7	6.0	0.65	
8		1.8	6.5	0.75	
9		8.0	3.0	8:00	0.15
10		9.0	4.0	PM	0.05
11		10.0	5.0	9:00	0.25
12		11.0	6.0	PM	0.35
13		12.0	7.0	10:0	0.45
14		13.0	8.0	0 PM	0.55
15		14.0	9.0	23:0	0.65
16		15.0	10.0	0	0.75
				24.0	
				0	
				25.0	
				0	
				26:0	
				0	
				27.0	
				0	

17
18
19
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24
25



10.0	14.0	0.05	0.30
11.0	14.5	0.10	0.35
8.0	15.0	0.15	0.40
7.0	15.5	0.20	0.45
6.0	16.0	0.25	0.50
5.0	17.0	0.30	0.55
4.0	17.5	0.35	0.60
3.0	18.0	0.40	0.65
2.0	19.0	0.45	0.70

Laboratory work No. 15-16

Topic: Solution of the Dirichlet problem for an elliptic equation using the grid method

I. Exercise.

Find a solution to the Dirichlet problem in a square with side 1 for the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with boundary conditions of the form $u(0, y) = f_1(y)$,

$$u(1, y) = f_2(y), 0 \leq y \leq 1, u(x, 0) = f_3(x), u(x, 1) = f_4(x), 0 \leq x \leq 1.$$

7. To solve this problem, develop appropriate computer programs.

8. To solve this problem, use computer software.

II. Options.

Number options	$f_1(y)$	$f_2(y)$	$f_3(x)$	$f_4(x)$
1	y^2	$\cos y + (2 -$	x^3	$x + 1$
2	$e^y - e y^2$	$\cos 1) y$	$-x^3 + 1$	x^2
3	$-y^2 + 1$	y	$\sin x - (1 + \sin 1)$	x
4	0	y	$x^3 + 1$	x
5	$e^y + (1 - e) y^2 -$	y	$\sin x - x^3 \sin 1$	x
6	1	y	0	$2x + 1$
7	y^2	$\cos y + (3 -$	x^3	x^2
8	0	$\cos 1) y$	$\sin x - x^3 \sin 1$	$x - 2$
9	$2e y - (1 + 2e)$	y	$-x^3 + 1$	$9x^2 - 15x - 12$
10	$y^2 - 1$	$-y$	$9x^2 + 7x + 6$	$6x^2 - 12x - 9$
11	$-10y^2 - 8y + 6$	$-10y^2 - 30y +$	$6x^2 + 4x + 3$	$4x^2 - 24x - 21$
12	$-7y^2 - 5y + 3$	22	$4x^2 + 2x + 1$	$18x^2 - 24x - 21$
13	$-5y^2 - 3y + 1$	$-7y^2 - 21y +$	$18x^2 + 16x +$	$3x^2 - 9x - 6$
14	$-19y^2 - 17y +$	13	15	$x + 1$
15	15	$-5y^2 - 15y + 7$	$3x^2 + x$	$x^2 + 1$
16	$-2y - 4y^2$	$-19y^2 - 57y +$	1	e^x
	1	49	1	
	1	$4 - 12y - 4y^2$	1	
	1	$y + 1$		
		$y + 1$		
		e^y		

17	$-y^3$	$1 - y^3$	x^2	$x^2 - 1$
18	$5y - y^2$	$4 - y^2 + 5y$	$x^2 + 3x$	$x^2 + 3x + 4$
19	$3 - 7y$	$7 - 6y$	$4x + 3$	$5x - 4$
20	y^2	$(1 - y)^2$	x^2	$(x - 1)^2$
21	y^2	$y^2 + 2y$	$x^2 - x$	$x^2 + x + 1$
22	y	$y + e$	e^x	$e^x + 1$
23	0	$\operatorname{tg} y$	0	$\operatorname{tg} x$
24	0	$\sin y$	0	$\sin x$
25	1	$1 + \cos y$	x	$x + \cos x$

Educational, methodological and informational support of the discipline

List of key literature:

1. Zenkov A.V. Numerical methods: textbook / A.V. Zenkov . - Ekaterinburg: Publishing house of the Ural. University, 2016. - 124 p.
2. Kalitkin, N. N. Numerical Methods: A Manual / N. N. Kalitkin ; edited by A. A. Samarskii. - 2nd ed. - St. Petersburg: BHV-Petersburg, 2014. - 586 p.: ill. - (Educational Literature for Universities). - Approved by the Scientific and Methodological Council. - Bibliography : pp. 575-581 (129 titles). - Index items : pp. 582-586. (10 copies): [Electronic resource].

List of additional literature:

1. Kireev V. I. Numerical methods in examples and problems: [a textbook for university students studying in the field of "Applied Mathematics"] / V. I. Kireev, A. V. Panteleev. - 4th ed., corrected . - St. Petersburg [and others]: Lan, 2015. - 448 p.
2. Pirumov , U. G. Numerical Methods: Theory and Practice: A Textbook for Bachelors / U. G. Pirumov ; Moscow Aviation Institute (National Research University). – 5th ed., revised and enlarged. – Moscow: Yurait , 2012. – 421 p. – (Bachelor. Basic Course). – Stamp: Add. MO. – Bibliography : p. 416.
3. Srochko V. A. Numerical methods: a lecture course: [for students of higher educational institutions studying in the specialty 010200 "Applied Mathematics and Computer Science" and in the direction 510200 "Applied Mathematics and Computer Science"] / V. A. Srochko . - St. Petersburg [and others]: Lan, 2016 .-- 208 p.

List of resources of the information and telecommunications network "Internet" necessary for mastering the discipline

1. Bazhenova I.Yu. Introduction to programming [Electronic resource]: a tutorial / I.Yu. Bazhenova, V.A. Sukhomlin . – Electronic text data. - Moscow, Saratov: Internet University of Information Technologies (INTUIT), University education, 2017. – 327 p. – URL : <http://www.iprbookshop.ru/67397.html>
2. Ivanov A.P., Poznyak L.T. Workshop on numerical methods. Calculation functions . URL: <http://www.apmath.spbu.ru/ru/structure / depts /is/task1-2013.pdf> :
3. Pimenov V.G. Numerical methods: in 2 parts. Part 2: [study guide] / V.G. Pimenov, A.B. Lozhnikov ; [scientific editor Yu.A. Melentsova]; Ministry of Education and Science of Russia. Federation, Ural. feder univ. – Ekaterinburg: Ural Publishing House. Univ., 2014. – 106 p. – URL: https://elar.urfu.ru/bitstream/10995/31219/1/978-5-7996-1342-6_2014.pdf .
4. Numerical Methods . Part 1. URL : <https://teach-in.ru/course/numerical-methods-part-1>

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
Federal State Autonomous
Educational Institution of Higher Education "North Caucasus Federal University"

GUIDELINES
on organizing and conducting independent work
in the discipline "**Engineering Mathematics / Инженерная математика**"
for students in the field of study
02.04.01 "Mathematics and Computer Science"
direction (profile)
"Training Program for Electronic Information"

INTRODUCTION

The discipline "Engineering Mathematics / Инженерная математика" is included in the bachelor's degree program in the field 02.04.01 - "Mathematics and Computer Science" and is a mandatory part of the block of disciplines.

This course explores the application of mathematical tools to solving applied problems, approximating functions, solving differential equations with boundary conditions, and numerically calculating definite integrals and derivatives. To reinforce the material covered, students will complete practical assignments using modern information technologies.

The purpose of this course is to ensure the fulfillment of the requirements of the State Educational Standard "Mathematics and Computer Science", according to which a bachelor's degree must have the ability to use fundamental knowledge in the field of mathematical analysis, complex and functional analysis, algebra, analytical geometry, differential geometry and topology, differential equations, discrete mathematics and mathematical logic, probability theory, mathematical statistics and random processes, numerical methods, theoretical mechanics in future professional activities.

To consolidate the material studied, it is expected that practical tasks will be completed using modern mathematical packages, MahCad , Maple , MatLab .

This discipline is designed to promote the fundamentalization of education, strengthen mathematical knowledge and develop students' systems thinking, and study theoretical, methodological and practical problems of the formation, functioning and development of information systems.

1 Requirements for the level of mastery of the discipline content

As a result of mastering the discipline, the student should possess the following competencies:

Index	Formulation:
OPK-2	Able to create and explore new mathematical models in the natural sciences, improve and develop concepts, theories and methods
PC-1	Able to demonstrate fundamental knowledge of mathematics and natural sciences, programming and information technology

2 Organizational and methodological recommendations for mastering the discipline

Students' independent work is a crucial prerequisite for developing a scientific approach to learning. It is conducted before each practical and laboratory lesson and involves completing a specific assignment.

The prepared material and practical assignment are presented in notes.

Independent study (IS) is one of the active forms of learning.

Independent studies in the discipline "Engineering Mathematics / Инженерная математика" are aimed at:

- to consolidate and deepen the knowledge acquired by students in lectures and laboratory classes;
- to instill practical skills in working with modern information technologies.

Independent studies are conducted in specialized classrooms equipped with computer technology and the ability to use Internet and library resources.

The proposed methodological recommendations contain information for students necessary for preparing and conducting laboratory classes in the discipline "Engineering Mathematics / Инженерная математика", and may also be useful for officials whose responsibilities include working with organizational and administrative documents (ORD).

GOST R 51141-08 defines the ORD as a type of written document that records the resolution of administrative and organizational issues, as well as issues of management, interaction, support and regulation of the activities of government bodies, institutions, enterprises, organizations, their divisions and officials.

According to the All-Russian Classifier of Management Documentation (OKUD), ORD are classified into four groups:

- organizational and legal documentation (charters, regulations, rules, staffing schedules, instructions);
- administrative documentation (orders, instructions, decrees, decisions, directions);
- information and reference documentation (letters, telegrams, faxes, acts, protocols, reports and memos, etc.);

- documents on personnel (applications, personal cards, orders on personnel, etc.).

The preparation of the ORD is carried out in accordance with the requirements of GOST R 6.30-2003 "Unified documentation systems. Unified system of organizational and administrative documentation. Requirements for document preparation."

This standard establishes: the composition of document details; requirements for the execution of document details; requirements for document forms.

3 Methodological guidelines for students' independent work

The guidelines include the following sections:

- the purpose of the work;
- a task that must be completed by a student as a result of independent work;
- task options;
- the main theoretical provisions necessary for completing the assignment, they should be brief and contain references to literature in which these provisions are set out in a volume sufficient for completing independent work;
- stages of task completion with indication of specific deadlines for completion of each stage and the entire task as a whole;
- requirements for the design of the graphic and text parts of independent work;
- an example of completing one of the task options and preparing a report;
- bibliographic list of used sources.

To help students and monitor their independent work, teacher consultations are used.

Consultations are one of the forms of work of the department to provide pedagogically appropriate assistance to students in their independent work.

Consultations should be individualized. If clarification of common issues is needed for several students, group consultations are held.

Consultations are conducted with the aim of:

- clarification of questions that arise among students during independent study of educational material and completion of homework;
- deepening and consolidating knowledge on individual issues and topics of the discipline;
- providing methodological assistance in choosing methods of independent work.

Attendance at student consultations is voluntary. However, the instructor may call for interviews those students who do not demonstrate solid knowledge during their studies and who do not voluntarily participate in the consultations.

During the consultation, the teacher explains the most complex issues that are unclear to students and points out literature that covers them.

When consulting students, the teacher is obliged to become familiar with how they study the recommended literature, give instructions and advice on

methods of working on the educational material, facilitating a deeper and more lasting assimilation of it

Consultations should not be additional classes or substitutes for them. Doing homework for students or with them is prohibited. Consultations should not become a form of cramming for tests and exams.

If during the consultation it becomes clear that the student's lack of understanding of the question is due to insufficient study of certain sections of the program, then the consultant is obliged to recommend that the student study them and monitor the degree of assimilation of these issues.

Consultations do not constitute a knowledge assessment. The student's knowledge of the subject matter demonstrated during the consultation should not influence their assessment of the course material in exams and tests.

4. Organization of students' knowledge control. Forms of students' knowledge control

Students' knowledge, skills, and abilities are assessed and monitored during laboratory classes, consultations, and exams. During assessments, the instructor evaluates the student's understanding of the course "Engineering Mathematics / Инженерная математика" and their ability to apply management methods to solve specific problems.

Students' knowledge assessment can be carried out in the following forms:

- current knowledge control;
- intermediate knowledge control.

Current monitoring of students' knowledge aims to:

- to evaluate the work of each student in assimilation of the educational material, to identify shortcomings in his preparation and to provide practical assistance in eliminating them;

The main forms of current monitoring of students' knowledge are:

- oral control survey;
- written control blitz survey;
- protection of laboratory work;
- intermediate testing;
- checking lecture notes.

Oral assessments of students are conducted during lectures (and lab sessions). Based on these assessments, the instructor evaluates the student's preparation for the class.

Students complete a written quiz during lectures and labs scheduled by the instructor. Students provide written responses to questions posed by the instructor. The results are recorded in the student's journal. During labs, students' knowledge and practical skills are assessed on a 4-point scale. The grades received are recorded in the journal.

When reviewing lecture notes, the quality of their writing is analyzed. Errors are noted, the instructor's review evaluates the quality of note-taking, and provides recommendations for improving the quality of lecture note-taking.

Interim assessment in the discipline is carried out in the form of a credit test and a credit test with a grade.

5 Recommendations for working with literature and sources

The study of literature and sources must begin with reading the relevant chapters of textbooks, teaching aids, or literature recommended as primary or secondary for the discipline "Information Systems Tools," which directly or indirectly relate to the topic being studied.

When studying literature and sources, students are advised to take brief notes. However, they should not rewrite the entire content of the topic being studied; they should only jot down the main ideas and, in their opinion, the most important thoughts. In certain cases, where important definitions, concepts, necessary factual material and examples, or statistical information related to the topic being studied are encountered, students should transcribe them as citations with full bibliographical references.

A summary of recommended literature and sources should be kept, with the collected materials divided into individual chapters and sections according to the curriculum. All references to the literature and sources used should be included.

An important step in working with recommended literature and sources is studying legislative and regulatory acts at the federal, regional, local, and departmental levels. When studying Decrees of the President of the Russian Federation, Laws and Codes of the Russian Federation, resolutions, regulations, recommendations, etc., students should identify any changes and additions that may have been made since their publication.

The basis of the technology for intensifying training on the platform of digital educational technologies are educational and illustrative materials (support notes) for the discipline "Information Systems Tools".

Working with educational and illustrative materials has the following stages.

1. Study of the theoretical foundations of the educational material in the classroom:

presentation of the material being studied by the teacher to students with an explanation based on the supporting notes;

2. Independent work:

Individual work of students on the supporting notes; frontal consolidation of the supporting notes' blocks.

3. First repetition - reproduction of the content of the given topic of the reference summary from memory.

4. Oral recitation of the supporting notes is a necessary stage of external speech activity in the assimilation of educational material.

5. The second repetition is mutual questioning and mutual assistance of students to each other.

The use of educational and illustrative materials allows us to generalize complex material and activate students' thinking.

It is important to remember that the most important thing for a student in independent work with recommended literature and sources is the formation of their own individual style, which can become the basis for future professional activity.

6 List of recommended literature

List of key literature:

1. Zenkov A.V. Numerical methods: textbook / A.V. Zenkov . - Ekaterinburg: Publishing house of the Ural. University, 2016. - 124 p.

3. Kalitkin, N. N. Numerical Methods: A Manual / N. N. Kalitkin ; edited by A. A. Samarskii. - 2nd ed. - St. Petersburg: BHV-Petersburg, 2014. - 586 p.: ill. - (Educational Literature for Universities). - Approved by the Scientific and Methodological Council. - Bibliography : pp. 575-581 (129 titles). - Index items : pp. 582-586. (10 copies): [Electronic resource].

List of additional literature:

1. Kireev V. I. Numerical methods in examples and problems: [a textbook for university students studying in the field of "Applied Mathematics"] / V. I. Kireev, A. V. Panteleev. - 4th ed., corrected . - St. Petersburg [and others]: Lan, 2015. - 448 p.

2. Pirumov , U. G. Numerical Methods: Theory and Practice: A Textbook for Bachelors / U. G. Pirumov ; Moscow Aviation Institute (National Research University). – 5th ed., revised and enlarged. – Moscow: Yurait , 2012. – 421 p. – (Bachelor. Basic Course). – Stamp: Add. MO. – Bibliography : p. 416.

3. Srochko V. A. Numerical methods: a lecture course: [for students of higher educational institutions studying in the specialty 010200 "Applied Mathematics and Computer Science" and in the direction 510200 "Applied Mathematics and Computer Science"] / V. A. Srochko . - St. Petersburg [and others]: Lan, 2016 .-- 208 p.

List of resources of the information and telecommunications network "Internet" necessary for mastering the discipline

1. Bazhenova I.Yu. Introduction to programming [Electronic resource]: a tutorial / I.Yu. Bazhenova, V.A. Sukhomlin . – Electronic text data. - Moscow, Saratov: Internet University of Information Technologies (INTUIT), University education, 2017. – 327 p. – URL : <http://www.iprbookshop.ru/67397.html>

2. Ivanov A.P., Poznyak L.T. Workshop on numerical methods. Calculation functions . URL: <http://www.apmath.spbu.ru/ru/structure / depts /is/task1-2013.pdf>

3. Pimenov V.G. Numerical methods: in 2 parts. Part 2: [study guide] / V.G. Pimenov, A.B. Lozhnikov ; [scientific editor Yu.A. Melentsova]; Ministry of Education and Science of Russia. Federation, Ural. feder univ. – Ekaterinburg: Ural Publishing House. University, 2014. – 106 p. – URL: http://elar.urfu.ru/bitstream/10995/31219/1/978-5-7996-1342-6_2014.pdf.

4. [Numerical Methods . Part 1.](https://teach-in.ru/course/numerical-methods-part-1) URL : <https://teach-in.ru/course/numerical-methods-part-1>