

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
FEDERAL STATE AUTONOMOUS EDUCATIONAL
INSTITUTION OF HIGHER EDUCATION
"NORTH CAUCASUS FEDERAL UNIVERSITY"

SUGGESTIONS FOR TEACHING
for the performance of practical work
on the discipline "**Complex Variable Function and Integral Transform**"
for students of the field of study
11.03.02 Infocommunication Technologies and Communication Systems
Profile
"Intelligent Data Processing in Infocommunication Systems and Networks"

Stavropol 2026

INTRODUCTION

The purpose of mastering the discipline is the formation of general professional competencies of the future academic bachelor in the field of training 11.03.02 Infocommunication Technologies and Communication Systems.

The implementation of the goal determines the following tasks of mastering the discipline: the formation of systematized knowledge in the field of complex analysis; the expansion of the basic concepts used in real analysis to the complex area: function, limit, continuity, differentiability, integrability; to familiarize students with the practical applications of complex analysis methods in various sections of mathematics.

The discipline is optional. Its development takes place in the 1st semester.

To master the discipline, a student must master the basic concepts of mathematical analysis (limit, continuity of functions, derivative, Riemann integral, functional sequences and series), the basic concepts of algebra and geometry.

Content of practical classes

No Topics of the discipline	Naming of the topics of practical classes	Hours	Interactive form of the event
1 semester			
Topic 1. Complex numbers and operations on them			
1	Geometric representation of complex numbers. Arithmetic operations on complex numbers. Elevation to the power of a complex number and extraction of the root from it.	4	Solving typical tasks
Topic 2. Complex Variable Function			
2	Identification of the real and imaginary parts of the FKP; finding the values of the basic elementary functions of a complex variable	4	Solving typical tasks
Topic 3. Differentiation of functions of a complex variable			
3	Verification of the fulfillment of Cauchy-Riemann conditions and, if they are met, the finding of the derivative of the FKP; restoration of the analytical function for a given real or imaginary part of it.	4	Solving typical tasks
Topic 4. Integral of a function of a complex variable			
4	Calculation of the integral of a function of a complex variable by the curve and using the Cauchy theorem and the integral Cauchy formula.	4	Solving typical tasks
Topic 5. Numerical and functional complex series			
5	Finding the convergence interval of a power series.	6	Solving typical tasks
Topic 6. Taylor and Laurent Rows			
6	The decomposition of the analytic function into the Taylor and Laurent series.	6	Solving typical tasks
Topic 7. Isolated special points			
7	Finding Zeros of an Analytic Function and Isolated Singular Points	6	Solving typical tasks
Topic 8. Elements of the theory of deductions			
8	Finding the deduction of an analytic function at a singular point; Calculating an integral using Cauchy's theorem on deductions.	4	Solving typical tasks
Topic 9. Application of deductions to the calculation of integrals			
9	Calculating integrals using deductions.	2	Solving typical tasks
Total for the semester		40	
Total		40	

PRACTICAL LESSON PLANS

Topic 1. Complex numbers and operations on them

Objective: to consolidate the basic concepts of complex numbers and to practice the skills of working on complex numbers.

Knowledge and skills acquired by a student as a result of mastering the topic:

1. Basic concepts of a complex number.
2. Forms of writing complex numbers.
3. Operations on complex numbers.

When studying this topic, it is necessary to pay attention to the following points.

A *complex number* is an ordered pair of real numbers $z = (x, y)$. The first component of x this pair is called the real part, and the second component is called y the imaginary part (or coefficient for the imaginary part) of the complex number z , and for them the notation is accepted: $x = \operatorname{Re} z$, $y = \operatorname{Im} z$.

When $y = 0$ a complex number $z = (x, 0)$ is identified with a real number x . When $x = 0$ a complex number $z = (0, y)$ is called imaginary. Numbers $(0, 0) = 0$, $(1, 0) = 1$ and $(0, 1) = i$ are called zero, one, and imaginary unity, respectively.

By definition, the complex numbers $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ are equal if $x_1 = x_2$ and $y_1 = y_2$.

The sum of two complex numbers $z_1 = (x_1, y_1)$ is called the complex number $z_2 = (x_2, y_2)$, and their $z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$ product is the complex number $z_1 z_2 = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$.

Direct verification shows that the entered operations are commutative: $z_1 + z_2 = z_2 + z_1$, $z_1 z_2 = z_2 z_1$. For arbitrary three complex numbers, z_1, z_2, z_3 the equations are valid, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$ i.e. the law of associativity is fulfilled with respect to addition and multiplication, respectively. Operations are subject to the law of distributivity: $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$.

From the definitions of addition and multiplication operations, equations follow $z + 0 = z$, $z \cdot 0 = 0$, $z \cdot 1 = z$, $i \cdot i = i^2 = -1$.

On the basis of an easily verifiable equation $(0, 1) \cdot (y, 0) = (0, y)$, we conclude that a complex number $z = (x, y)$ can be represented in Cartesian (algebraic) form $z = x + iy$. In fact,

$$z = (x, y) = (x, 0) + (0, y) = (x, 0) + (0, 1) \cdot (y, 0) = x + iy.$$

Subtraction is defined as the inverse of addition, i.e. the difference $z_1 - z_2$ is a complex number z_3 that has the property $z_2 + z_3 = z_1$. If $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2)$, then $z_1 - z_2 = z_3 = (x_1 - x_2, y_1 - y_2)$.

A complex number $x - iy$ is called *conjugate with a complex number* $z = x + iy$, and the notation is adopted for it $\bar{z}$. The transition from a given complex number to a conjugate number is called the conjugate operation. It is obvious that

$$\overline{(\overline{z})} = z, \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}, \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}, z + \overline{z} = 2x, z - \overline{z} = 2iy.$$

Division by a complex number other than zero is defined as the inverse of multiplication, i.e. the number $\frac{z_1}{z_2}, z_2 \neq 0$ is understood as a complex number z_4 that has the property $z_2 \cdot z_4 = z_1$. To represent z_4 it in the form $x_4 + iy_4$, a simple calculation should be performed

$$\begin{aligned} z_4 = \frac{z_1}{z_2} &= \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \\ &= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}. \end{aligned}$$

Taking advantage of the relationship between Cartesian and polar coordinates $x = \rho \cos \varphi, y = \rho \sin \varphi$,

We get the so-called *trigonometric form* of the complex number : $z = x + iy$
 $z = \rho(\cos \varphi + i \sin \varphi)$ (1)

In this case, it ρ is usually called a modulus, and is φ an argument of a complex number and is denoted by $\rho = |z|, \varphi = \text{Arg} z$.

Using the well-known Euler formula

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

and equation (1), we get the so-called *exponential form of writing a complex number*

$$z = \rho e^{i\varphi}.$$

Control questions and tasks:

Define a complex number

Which complex number is called conjugate to a complex number $z = x + iy$?

Which of the following equations are fair

$$\overline{(\overline{z})} = z, \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}, \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}, z + \overline{z} = 2xi, z - \overline{z} = 2y ?$$

Write down the complex number in indicative form $z = \rho(\cos \varphi + i \sin \varphi)$.

1. Prove: If the complex number z is representable

$$z = r(\cos \psi + i \sin \psi) \quad (r > 0, \psi \text{ — Actual Number}), \text{ then}$$

$r = |z|$, and is ψ one of the values of the argument z .

2. By counting $z = x + iy \in \mathbb{C} \setminus \{0\}$, prove:

$$\arg z = \arctg(y/x) \text{ at } x > 0;$$

$$\arg z = \arctg(y/x) + \pi \text{ with } x < 0, y \geq 0;$$

$$\arg z = \arctg(y/x) - \pi \text{ with } x < 0, y < 0.$$

3. Follow these steps:

a) $(2 - 3i) - (i - 2)$;

b) $(2 - i)(i + 1) - (1,5i + 1)4i$;

c) $\frac{1}{1+2i} + \frac{i}{2-i}$;

d) $\frac{2}{1+i} - \frac{1-i}{1+2i}(2-i)$;

e) $(\sqrt{2} + i\sqrt{2})^2$.

Topic 2. Functions of a complex variable

Objective: to consolidate the basic definitions of the function of a complex variable and to work out finding the limit of the function of a complex variable.

Knowledge and skills acquired by a student as a result of mastering the topic:

1. Basic definitions of the function of a complex variable.
2. Limit and continuity of the function of a complex variable.

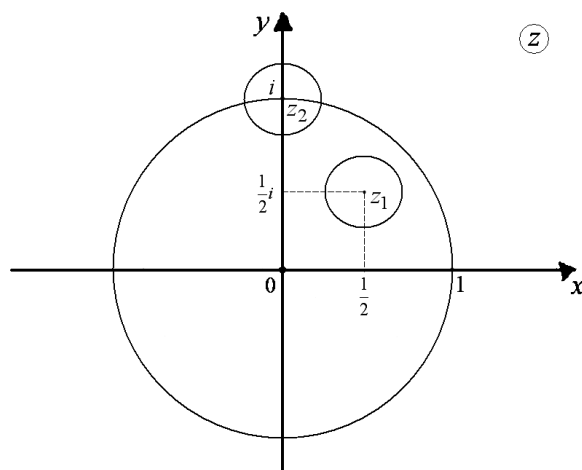
When studying this topic, it is necessary to pay attention to the following points.

Let $D \subset C$. Let us introduce the concept of the limit point of the set D .

Definition. A point $z_0 \in C$ is called *the bounding point of the set D* if there is a point other than $z \in D, z \neq z_0$ in any of its arbitrarily small δ neighborhoods.

A set can have limit points that belong to this set and limit points that do not belong to it.

For example, suppose $D = \{z \in C : |z| < 1\}$ – an open circle centered at the point $z = 0$ of radius $R = 1$. Then $z_1 = \frac{1}{2} + \frac{1}{2}i$, $z_2 = i$ – the limiting points of the set D , at the same time $z_1 \in D$, $z_2 \notin D$.



A closed set contains all its limiting points. A set of complex numbers is called bounded if it is contained entirely in a circle $|z| < R$.

Let be given an unambiguous function $w = f(z)$, $z \in D$ and z_0 is the limit point of the set D .

Definition. The $f(z)$ complex number A is called the limit of the complex function at z_0 , (or if z tends to z_0), $\varepsilon > 0$ if for any arbitrarily small positive number

$\delta = \delta(\varepsilon)$ there exists a number such that for any $z \in D$ of the conditions $0 < |z - z_0| < \delta$ it follows that $|f(z) - A| < \varepsilon$.

$$\lim_{z \rightarrow z_0} f(z) = A \Leftrightarrow \forall \varepsilon > 0 \exists \delta = \delta(\varepsilon) \mid \forall z \in D : 0 < |z - z_0| < \delta \Rightarrow |f(z) - A| < \varepsilon.$$

In other words, for all z points from a circle with a center z_0 and radius δ ($z \neq z_0$), the function points $f(z)$ are in a circle with a center A and radius ε .

The theorem (a sign of the existence of the limit of a function of a complex variable at a point). The Existence of a Limit $A = \alpha + \beta i$ Features $f(z) = u(x, y) + iv(x, y)$ complex variable $z = x + iy$ at the point $z_0 = x_0 + iy_0$ is equivalent to the existence of a limit α of its actual part $u(x, y)$ and the limit β of its imaginary part $v(x, y)$ at the point (x_0, y_0) .

For the functions of the complex variable $f(z)$ and $g(z)$, the following properties are true:

$$\lim_{z \rightarrow z_0} [f(z) \pm g(z)] = \lim_{z \rightarrow z_0} f(z) \pm \lim_{z \rightarrow z_0} g(z)$$

$$\lim_{z \rightarrow z_0} [f(z) \cdot g(z)] = \lim_{z \rightarrow z_0} f(z) \cdot \lim_{z \rightarrow z_0} g(z)$$

$$\lim_{z \rightarrow z_0} \frac{f(z)}{g(z)} = \frac{\lim_{z \rightarrow z_0} f(z)}{\lim_{z \rightarrow z_0} g(z)}; \quad \lim_{z \rightarrow z_0} g(z) \neq 0.$$

Definition. The function $f(z)$ is called *an infinitesimal quantity* (b. m. c.) at z_0 (or at $z \rightarrow z_0$), if its limit at that point is zero: $\lim_{z \rightarrow z_0} f(z) = 0$, i. e. by the definition of the limit,

$$\forall \varepsilon > 0 \exists \delta = \delta(\varepsilon) \mid \forall z \in D : 0 < |z - z_0| < \delta \Rightarrow |f(z)| < \varepsilon.$$

This condition means that $\lim_{z \rightarrow z_0} |f(z)| = 0$. Thus,

$$\lim_{z \rightarrow z_0} f(z) = 0 \Leftrightarrow \lim_{z \rightarrow z_0} |f(z)| = 0.$$

Definition. An improper complex number ∞ is called the limit of the function $f(z)$ at z_0 (or at $z \rightarrow z_0$) if, for any arbitrarily large positive number E , there is a positive number δ , defined with a function of the taken number E , such that for any $z \in D$, not equal to z_0 and other than z_0 modulo by an amount less than δ , the modulus of the corresponding value of the function $f(z)$ is greater than the taken number E .

$$\lim_{z \rightarrow z_0} f(z) = \infty \Leftrightarrow \forall E > 0 \exists \delta = \delta(E) \mid \forall z \in D : 0 < |z - z_0| < \delta \Rightarrow |f(z)| > E.$$

Such a function is called *an infinitely large quantity* (b.b.v.) when $z \rightarrow z_0$.

$$\lim_{z \rightarrow z_0} f(z) = \infty \Leftrightarrow \lim_{z \rightarrow z_0} |f(z)| = +\infty.$$

If $f(z)$ – b. m. c. at point z_0 , then $1/f(z)$ – b. b. c. at this point (and vice versa).

Definition. A function $w = f(z)$ is said **to be continuous** at point z_0 if there is a limit of this function at point z_0 that is equal to its value at this point:

$$\lim_{z \rightarrow z_0} f(z) = f(z_0).$$

Let the function $w = f(z)$ be continuous at z_0 , i.e. $\lim_{z \rightarrow z_0} f(z) = f(z_0)$ or

$$\lim_{z \rightarrow z_0} |f(z) - f(z_0)| = 0.$$

Let us assume $\Delta z = z - z_0$ that the $\Delta w = w - w_0 = f(z) - f(z_0)$ complex number Δz is called *the increment of the independent variable z at z_0* , and the complex number Δw is called *the increment of the function $w = f(z)$ at z_0* corresponding to the increment Δz of z . In these notations, the previous equation takes the form

$$\lim_{\Delta z \rightarrow 0} \Delta w = 0.$$

And the definition of the continuity of a function at a point can be formulated as follows.

Definition. The function $w = f(z)$ is said *to be continuous at z_0* if the increment of Δw this function at a given point corresponding to the increment Δz of the independent variable z is b.m.v. at $\Delta z \rightarrow 0$.

A function $w = f(z)$ is said *to be continuous on a set* if it is continuous at every point of that set.

If the function $f(z)$ is continuous at z_0 , then the function $|f(z)|$ is also continuous at z_0 .

Theorem (a sign of the continuity of a function of a complex variable at a point). The continuity of the function of $f(z) = u(x, y) + iv(x, y)$ a complex variable $z = x + iy$ at a point $z_0 = x_0 + iy_0$ is equivalent to the continuity of its real and imaginary parts at point (x_0, y_0) .

Control questions and tasks:

What is called a complex variable?

What is called a complex plane?

What is meant by a function of a complex variable?

How is the limit of the function $w = f(z)$ determined?

Which function is called continuous in the region?

Find values $\sqrt[n]{Z}$ if:

1) $Z = -1, n = 3,$

2) $Z = 8i, n = 3,$

3) $Z = 1, n = 5,$

4) $Z = 1 + i, n = 8.$

Solve the equations:

1) $Z^3 = 1 + i,$

2) $Z^4 + 1 = 0,$

3) $Z^5 = 1 + \sqrt{3}i,$

4) $Z^6 + 64 = 0,$

$$5) Z^2 = \overline{Z}^3.$$

Topic 3. Differentiation of the function of a complex variable

Objective: to consolidate the definition of the derivative function of a complex variable and to work out the conditions under which the function is analytical.

Knowledge and skills acquired by a student as a result of mastering the topic:

1. Derivative of a function of a complex variable.
2. The criterion of differentiability at the point.
3. Harmonic functions.

When studying this topic, it is necessary to pay attention to the following points.

Definition 1. A function $f(z)$ is said to be differentiable or monogenic at a point z_0 , if at any sufficiently small $|\Delta z|$ its increment at a point z_0 allows representation

$$\Delta f(z_0) = A\Delta z + \eta\Delta z,$$

where the number $A, A \neq \infty$, does not depend on $\Delta z, \eta \rightarrow 0$ the given $\Delta z \rightarrow 0$; A derived function $f(z)$ at a point z_0 and is denoted $f'(z_0)$ by $\frac{df(z_0)}{dz}, f'$.

Definition 1 is equivalent to the following definition.

Definition 2. A function $f(z)$ is called differentiable or monogenic at the point z_0 , if there is a finite limit

$$\lim_{\Delta z \rightarrow 0} \frac{\Delta f(z_0)}{\Delta z} = A.$$

A function $f(z)$ is said to be differentiable or monogenic in a region $D \subset C_z (\overline{C}_z)$ if it is differentiable at every point in that region.

A differential $f(z)$ at a point z is a linear part Δz of $f'(z)\Delta z$ the increment of $\Delta f(z)$ a function at a point z and is denoted $df(z)$ by:

$$df(z) = f'(z)\Delta z.$$

Theorem (Cauchy, Riemann). Let the function

$$f(z) = u(x, y) + iv(x, y), z = x + iy,$$

is defined in the domain $D, D \subset C_z$. For differentiability (monogeneity) $f(z)$ at a point $z_0 = x_0 + iy_0 \in D$, it is necessary and sufficient that the functions $u(x, y)$ and $v(x, y)$ are differentiable at the point (x_0, y_0) as functions of two real variables x, y , and that, in addition, the equations at this point are fulfilled

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}. \quad (1)$$

In this case, the derivative $f'(z_0)$ can be calculated using one of the formulas

$$f'(z_0) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} + i \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}.$$

Note 1. Equations (1) are called Cauchy-Riemann conditions or d'Alembert-Euler conditions, or CRED conditions.

A function $f(z)$ is said *to be analytic* at a point z_0 if it is differentiable in the vicinity of $\cup(z_0)$ that point, and is said *to be analytic in an area* if it is analytic at every point in that region.

Solving typical tasks

Example. Find all the points at which functions are differentiable:

1) $f(z) = \operatorname{Re} z$.

Solution. $f(z) = \operatorname{Re} z = x, \Rightarrow u = x, v = 0$.

Let's find private derivatives

$$\frac{\partial u}{\partial x} = 1; \quad \frac{\partial u}{\partial y} = 0; \quad \frac{\partial v}{\partial x} = 0; \quad \frac{\partial v}{\partial y} = 0.$$

The answer is: it is not differentiated anywhere.

2) $f(z) = x^2 + iy^2, \quad z = x + iy$.

Solution. $u = x^2; v = y^2$.

Let's find private derivatives

$$\frac{\partial u}{\partial x} = 2x; \quad \frac{\partial u}{\partial y} = 0; \quad \frac{\partial v}{\partial x} = 0; \quad \frac{\partial v}{\partial y} = 2y.$$

$$\begin{cases} x = y \\ 0 = 0 \end{cases} \Leftrightarrow y = x.$$

Answer: The function is differentiable on the line $\operatorname{Re} z = \operatorname{Im} z$.

Control questions and tasks:

1. Give a definition of a differentiable function at the point z_0 .
2. Write down the Cauchy-Riemann conditions.
3. Formulate the Cauchy-Riemann theorem.
4. Which function is called *analytical* in a point, *in a region*?

Task 1. Distinguish the real and imaginary parts of this function. Find the value of the function at a given point.

$$w = \frac{1}{z} + \frac{2i}{z-1}; \quad w(2).$$

Task 2. Find the points at which the function is differentiable. Calculate the derivative at a given point.

$$w = z^2 + i|z|^2; \quad w'(1+i).$$

Task 3. Restore the analytical function according to the specified conditions.

$$v(x, y) = \ln(x^2 + y^2) + x - 2y.$$

Topic 4. Integral of a function of a complex variable

Objective: to practice the skills of finding the integral of a function of a complex variable, the Cauchy theorem.

Knowledge and skills acquired by a student as a result of mastering the topic:

1. Determination of the integral from the function of a complex variable. Basic properties of the integral.

2. Cauchy's theorem.

When studying this topic, it is necessary to pay attention to the following points. Let not be Γ given a complex arc $f(z)$. Let's choose γ_k one point at random on each arc $\zeta_k = \xi_k + i\eta_k$ and consider the sum

$$S_\Gamma = S_\Gamma(f, T) = \sum_{k=0}^{n-1} f(\zeta_k)(z_{k+1} - z_k).$$

This sum is called the integral sum, for a function $f(z)$ along the curve Γ corresponding to its partition T .

Definition. A function $f(z)$ is said to be *integrable with respect to a curve Γ* if there exists a number $I \in \mathbb{C}$ that has the following property: for any $\varepsilon > 0$ there is a number $\delta > 0$ such that for each partition of T the curve Γ with diameter $\alpha(T) < \delta$, the inequality holds:

$$|I - S_\Gamma(f, T)| < \varepsilon.$$

Number I is called in this case *integral* from the function $f(z)$ along the curve Γ and is denoted by

$$\int_{\Gamma} f(z) dz.$$

Theorem. If the function $f(z) = u(x, y) + i v(x, y)$, $z = x + iy$, is continuous on a rectified curve Γ , and the equality is true

$$\int_{\Gamma} f(z) dz = \int_{\Gamma} u dx - v dy + i \int_{\Gamma} v dx + u dy, \quad (1)$$

where in the right side of equation (1) there are curvilinear integrals (of the II-kind, by coordinates).

The following properties of integrals are checked directly by the definition of the integral:

- a) $\int_{-\Gamma} f(z) dz = - \int_{\Gamma} f(z) dz;$
- b) $\int_{\Gamma} c f(z) dz = c \int_{\Gamma} f(z) dz, \quad c = \text{const};$
- c) $\int_{\bigcup_{j=1}^m \Gamma_j} f(z) dz = \sum_{j=1}^m \int_{\Gamma_j} f(z) dz.$

Let Γ -smooth curve with equation $z = z(t) = x(t) + iy(t)$, $\alpha \leq t \leq \beta$, then

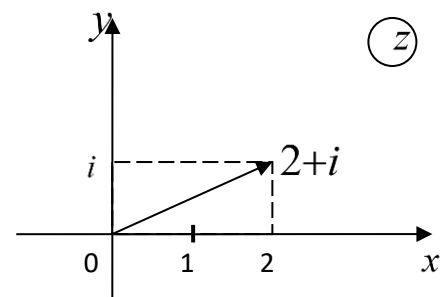
$$\int_{\Gamma} f(z) dz = \int_{\alpha}^{\beta} f(z(t)) z'(t) dt.$$

Example 1. Calculate the integral $\int_{\Gamma} z^2 dz$, where G is the radius-vector of the point

$$z = 2 + i.$$

Solution. The integrative function has the form

$$f(z) = z^2 = x^2 + i2xy - y^2 = x^2 - y^2 + i2xy.$$



Let's draw a straight line $y = \frac{1}{2}x$.

Using formula (1), we get

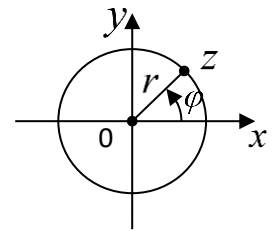
$$\begin{aligned}\int_{\Gamma} z^2 dz &= \int_0^2 \left(x^2 - \frac{x^2}{4} - 2x \cdot \frac{x}{2} \cdot \frac{1}{2} \right) dx + i \int_0^2 \left[2x \cdot \frac{x}{2} + \left(x^2 - \frac{x^2}{4} \right) \cdot \frac{1}{2} \right] dx = \\ &= \int_0^2 \left(x^2 - \frac{x^2}{4} - \frac{x^2}{2} \right) dx + i \int_0^2 \left(x^2 + \frac{3x^2}{8} \right) dx = \int_0^2 \frac{x^2}{4} dx + i \int_0^2 \frac{11x^2}{8} dx = \frac{x^3}{12} \Big|_0^2 + i \frac{11}{8} \frac{x^3}{3} \Big|_0^2 = \\ &= \frac{8}{12} + i \frac{11}{8} \cdot \frac{8}{3} = \frac{2}{3} + i \frac{11}{3} = \frac{1}{3}(2 + 11i).\end{aligned}$$

Answer: $\frac{1}{3}(2 + 11i)$.

Example 2. Calculate the integral $\int_{\Gamma} \frac{dz}{z}$, where D is the circle $|z| = r$.

Solution.
$$\int_{\Gamma} \frac{dz}{z} = \int_{\Gamma} \frac{dx + i dy}{x + iy} = \int_0^{2\pi} \frac{-r \sin \varphi + i r \cos \varphi}{r(\cos \varphi + i \sin \varphi)} d\varphi =$$
$$= \int_0^{2\pi} \frac{r(-\sin \varphi + i \cos \varphi)}{r(\cos \varphi + i \sin \varphi)} d\varphi = i \int_0^{2\pi} d\varphi = i\varphi \Big|_0^{2\pi} = 2\pi i.$$

Answer: $2\pi i$.



Control questions and tasks:

1. Give a definition of the integral over a complex variable.
2. Write down the formula that establishes the relationship between the integral of a complex variable and curvilinear integrals.
3. List the main properties of the integral of a complex variable.
4. Write down the formula for calculating the integral on a smooth curve.

Calculate Integral from Curve

a) $\int_L z \operatorname{Im} z dz, \quad L: |z| = 2;$

b) $\int_L \operatorname{Re} z dz, \quad L: (2+i)t, \quad 0 \leq t \leq 1.$

Topic 5. Numerical and functional complex series

Expression of the form

$$\sum_{n=1}^{\infty} z_n = z_1 + z_2 + \dots + z_n + \dots, \quad (1)$$

where $z_n = x_n + iy_n$ are complex numbers, is called a numerical complex series (a row in a complex domain).

Numerical series with complex terms have properties similar to many properties of numerical series with real terms.

Partial amounts of the series:

$$S_1 = z_1; \quad S_2 = z_1 + z_2; \quad \dots; \quad S_n = z_1 + \dots + z_n; \quad \dots$$

form a numerical complex sequence $\{S_n\}$.

Definition 1. Series (1) is said to converge if the sequence of partial sums corresponding to it $\{S_n\}$ converges, i.e. there is a finite limit

$$\lim_{n \rightarrow \infty} S_n = S. \quad (2)$$

The number S is called the sum of this series. If the limit (2) does not exist or is infinite, then the series (1) is called *divergent*.

Theorem 1. In order for the series (1) to be convergent and for its sum to be a number $S = S' + iS''$, it is necessary and sufficient that the numerical series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ are convergent and that their sums are respectively S', S'' . At the same time,

$$\sum_{n=1}^{\infty} z_n = \sum_{n=1}^{\infty} x_n + i \sum_{n=1}^{\infty} y_n = S' + iS'' = S.$$

With the help of this theorem, many important properties and statements that are valid for series with real terms are immediately transferred to series with complex terms. Let us list some of these properties:

1°. *A necessary sign of convergence*. If the series $\sum_{n=1}^{\infty} z_n$ converges, then $\lim_{n \rightarrow \infty} z_n = 0$ (The inverse statement is incorrect: it does not follow from the fact that

$\lim_{n \rightarrow \infty} |z_n| = 0$, it does not follow that the series $\sum_{n=1}^{\infty} z_n$ converges).

It follows that if the common term of series (1) does not tend to zero, then the series diverges (*a sufficient condition of divergence*).

2° Let the series $\sum_{n=1}^{\infty} z_n$ with $\sum_{n=1}^{\infty} w_n$ complex terms converge and their sums are equal S and Q correspondingly. Then the series $\sum_{n=1}^{\infty} (z_n \pm w_n)$ also converges and its sum is equal $S \pm Q$ to.

3° Let the series $\sum_{n=1}^{\infty} z_n$ converge and its sum is equal S to. Then, for any complex λ series, $\sum_{n=1}^{\infty} \lambda z_n$ it also converges and its sum is equal to λS .

4°. If a finite number of terms is discarded or added to a converging series, a converging series is also obtained.

Along with series (1), consider a series made up of the modules of its members:

$$\sum_{n=1}^{\infty} |z_n| = |z_1| + |z_2| + \dots + |z_n| + \dots, \quad (3)$$

which is numeric next to the full members.

Theorem 2. If series (3) converges, then series (1) also converges, which in this case is called *absolutely convergent*.

Converging, but not absolutely converging series are called *conventionally converging*.

Note 1. If the series converge absolutely, then with any rearrangement of its terms, the new series will also be absolutely converged, and to the same sum.

Note 2. The study of the convergence of series (3) as a positive numerical series can be carried out with the help of the known sufficient features of d'Alembert, Cauchy, etc.:

1) D'Alembert's sign. If there is $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$, then with $l < 1$ – the series converges, and with $l > 1$ – diverges.

2) Cauchy's sign. If there is $\lim_{n \rightarrow \infty} \sqrt[n]{u_n} = l$, then with $l < 1$ – the series converges, and with $l > 1$ – diverges.

3) The limiting feature of comparison. If $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are series with positive terms and there is a finite limit to the ratio of their common terms: $\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = k \neq 0$, then the series converge or diverge simultaneously. (As a reference series, the generalized harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$, which at $\alpha > 1$ – converge, and at $\alpha \leq 1$ – diverge).

Control questions

1. To give the concept of a numerical series with complex numbers.
2. Which series is called converging (diverging)?
3. Formulate the necessary criterion for the convergence of a numerical series with complex numbers.
4. Formulate a necessary and sufficient condition for the convergence of a numerical series with complex numbers.
5. List the main properties of converging series.
6. Give the concept of an absolutely converging numerical series with complex numbers. List their main properties.

Topic 11. Taylor and Laurent Rows

Objective: to practice the skills of decomposing functions into a Taylor series.

Knowledge and skills acquired by a student as a result of mastering the topic:

The concept of a power series.

Abel's theorem.

Determination of the radius of convergence of a power series.

Taylor's theorem on the decomposition of a holomorphic function into a power series.

When studying this topic, it is necessary to pay attention to the following points.

Definition. Functional series of the type

$$\sum_{n=0}^{\infty} a_n (\zeta - \zeta_0)^n, \quad (1)$$

where $\zeta_0, a_0, a_1, \dots$ are given complex numbers, ζ is a complex variable, called a *power series*.

Entering the notation $\zeta - \zeta_0 = z$, rewrite the row (1) in the form

$$\sum_{n=0}^{\infty} a_n z^n. \quad (2)$$

Theorem (Abel). *If the power series (2) converges at the point $z_0 \neq 0$, then it absolutely converges in the circle*

$$K_0 = \{z: |z| < |z_0|\},$$

And in the circle

$$K_1 = \{z: |z| \leq \rho\}, \text{ where } 0 < \rho < |z_0|,$$

This series converges evenly.

Corollary. *For each power series (2) there is a number $R > 0$ such at $|z| < R$ ($z = 0$ at $R = 0$) the series converges, and at $|z| > R$ - diverges.*

The number R is called the radius of convergence of the series (2), and the circle $\{z: |z| < R\}$ is called the circle of convergence of this series.

Theorem (Cauchy–Hadamard). *Radius of convergence of a power series*

$$\sum_{n=0}^{\infty} a_n z^n$$

is found by the formula

$$R = \frac{1}{l}, \text{ where } l = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|},$$

and it is implied that $R = 0$ at $l = +\infty$ and $R = +\infty$ at $l = 0$.

Theorem (on the decomposition of a holomorphic function into a power series; Taylor). *A holomorphic function in the region $D \subset C_z$ $f(z)$ in the vicinity of each point $z_0 \in D$ is representable as the sum of the power series*

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n, \quad (3)$$

converging in a circle $\cup(z_0, d) = \{z: |z - z_0| < d\}$ with a radius d equal to the distance from the point to the z_0 boundary ∂D of the region D . The coefficients of a_n the series (3) are expressed by the formulas

$$a_n = \frac{1}{2\pi i} \int_{\gamma_\rho} \frac{f(\zeta) d\zeta}{(\zeta - z_0)^{n+1}} \quad (n = 0, 1, 2, \dots),$$

where γ_ρ is an arbitrary circle $\{\zeta : |\zeta - z_0| = \rho\}$, $0 < \rho < d$.

Taylor series are adapted to represent holomorphic functions in a circle. We will now consider more general series in terms of positive and negative powers $(z - a)$.

Such series represent holomorphic functions in concentric rings

$$V = \{z \in C_z : r < |z - a| < R\}, \quad r \geq 0, R \leq \infty.$$

Especially important are the decompositions in rings with a zero inner radius ($r = 0$), i.e. in the punctured neighborhoods of points where the functions lose their holomorphicity (singular points).

Theorem (Laurent). Any function $f(z)$, holomorphic in a ring $V = \{r < |z - a| < R\}$, can be represented in this ring as the sum of a converging series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z - a)^n, \quad (1)$$

the coefficients of which are determined by the formulas

$$c_n = \frac{1}{2\pi i} \int_{|\zeta - a| = \rho} \frac{f(\zeta) d\zeta}{(\zeta - a)^{n+1}} \quad (n = 0, \pm 1, \pm 2, \dots),$$

where $r < \rho < R$.

Definition. The series (1), the coefficients of which are calculated according to formulas (2), is called *the Laurent series* of the function $f(z)$ in the ring V . The set of terms of this series with non-negative powers $(z - a)$ is called the regular part, and the set of terms with negative powers is called $(z - a)$ the principal part:

$$\sum_{n=-\infty}^{\infty} c_n (z - a)^n = \sum_{n=0}^{\infty} c_n (z - a)^n + \sum_{n=-1}^{\infty} c_n (z - a)^n.$$

Example 1. Decompose a function $f(z) = z^3 \cdot e^{1/z}$ into a Laurent series in a ring $0 < |z| < \infty$.

$$\textbf{Solution.} \quad z^3 \cdot e^{1/z} = z^3 \cdot \sum_{n=0}^{\infty} \frac{(1/z)^n}{n!} = z^3 \cdot \sum_{n=0}^{\infty} \frac{1}{z^n \cdot n!} = \sum_{n=0}^{\infty} \frac{1}{z^{n-3} \cdot n!}.$$

Example 2. Decompose a function $f(z) = \frac{z}{(z-i)(z+3)}$ into a Laurent series in a ring $1 < |z| < 3$.

Solution.

$$\frac{1}{z-i} = \frac{1}{z \left(1 - \frac{i}{z}\right)} = \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{i}{z}\right)^n = \sum_{n=0}^{\infty} \frac{i^n}{z^{n+1}}; \quad |z| > 1.$$

$$\frac{1}{z+3} = \frac{1}{3\left(1+\frac{z}{3}\right)} = \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^n \cdot (-1)^n; |z| < 3.$$

Consequently,

$$f(z) = \frac{z}{(z-i)(z+3)} = \frac{z}{3+i} \cdot \left(\sum_{n=0}^{\infty} \frac{i^n}{z^{n+1}} - \sum_{n=0}^{\infty} (-1)^n \cdot \frac{z^n}{3^{n+1}} \right).$$

Control questions and tasks:

Which functional rad is called a power series?

Formulate Abel's theorem.

What can be the convergence set of a power series?

Write down the Cauchy–Hadamard formula to determine the radius of convergence of a power series.

Formulate Taylor's theorem on the decomposition of a holomorphic function into a power series.

Find the radii of convergence of a power series $\sum_{n=0}^{\infty} \frac{z^n}{3^n}$.

Find the circles of convergence of a power series

$$\sum_{n=0}^{\infty} 2^n z^n.$$

Topic 7. Isolated special points

Objective: to practice the skills of finding zeros of functions and determining their order, to investigate special points of the function.

Knowledge and skills acquired by a student as a result of mastering the topic:

Classification of isolated singular points of analytic functions; Laurent series in the vicinity of a singular point; Sohotzky's theorem; infinitely distant isolated singular point.

When studying this topic, it is necessary to pay attention to the following points.

Definition 1. A point $a \in \overline{C}$ is called *an isolated singular point of a* function

$f(z)$ if there exists such a punctured neighborhood of this point (i.e., a set

$\{0 < |z - a| < R\}$ if the point a is finite, or a set $\{R < |z| < \infty\}$ if $a = \infty$), in which the function $f(z)$ is holomorphic.

Depending on the behavior $f(z)$ when approaching such a point, three types of special points are distinguished.

Definition 2. An isolated singular point of a a function $f(z)$ is called

(i) *a removable point*, if there is a finite point

$$\lim_{z \rightarrow a} f(z) = A \neq \infty;$$

(II) *pole*, if there is a

$$\lim_{z \rightarrow a} f(z) = \infty;$$

(III) is essentially a singular point, if $f(z)$ it has neither a finite nor an infinite limit at $z \rightarrow a$.

Theorem. An isolated singular point of a $a \in \mathbb{C}$ function $f(z)$ is removable if and only if the Laurent expansion $f(z)$ in the neighborhood a does not contain a principal part:

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n.$$

Theorem. An isolated singular point of a $a \in \mathbb{C}$ function $f(z)$ is a pole if and only if the principal part of the Laurent expansion $f(z)$ in the vicinity a of the point contains only a finite (and positive) number of non-zero terms:

$$f(z) = \sum_{n=-N}^{\infty} c_n (z-a)^n = \frac{c_{-N}}{(z-a)^N} + \frac{c_{-N+1}}{(z-a)^{N-1}} + \dots + \frac{c_{-1}}{z-a} + \sum_{n=0}^{\infty} c_n (z-a)^n, c_{-N} \neq 0. \quad (1)$$

Definition 3. The number of N the senior term of the main part of the Laurent decomposition (1) $f(z)$ of the function in the punctured neighborhood of the pole is called the order of the pole.

Theorem. An isolated singular point of a function $f(z)$ is essentially special if and only if the principal part of the Laurent expansion $f(z)$ in the vicinity a of the point contains infinitely many non-zero terms.

Control questions and tasks:

1. Define an isolated singular point of an analytic function.
2. Which isolated singular point is called an avoidable point, a pole, an essentially-singular point?
3. Write down the decomposition of the function in the Laurent series in the vicinity of the pole.
4. Write down the decomposition of the function in the Laurent series in the vicinity with an essentially-special point.

Topic 8. Elements of the theory of deductions

Goal: to practice the skills of finding deductions of functions.

Knowledge and skills acquired by a student as a result of mastering the topic:

Definition of deduction; Cauchy's theorem on deductions, calculation of deductions

When studying this topic, it is necessary to pay attention to the following points.

Definition. The subtraction of the function $f(z)$ at the endpoint a is the magnitude

$$\frac{1}{2\pi i} \int_{\gamma_\rho} f(z) dz, \text{ where is } \gamma_\rho \text{ a circle with a center at a point } a \text{ of sufficiently small}$$

radius ρ (so small that γ_ρ there are no other singular points $f(z)$ inside the circle except a for the circumference).

The deduction is indicated by one of the following symbols:

$$\operatorname{res}_{z=a} f(z),$$

$$\operatorname{Res}_{z=a} f(z),$$

$$[f(z); a].$$

We will write

$$\operatorname{Res}_{z=a} f(z) = \frac{1}{2\pi i} \int_{\gamma_p} f(z) dz.$$

A theorem (Cauchy, on deductions, or the basic theorem on deductions). *Let the function $f(z)$ be holomorphic everywhere in the domain D , except for no more than a countable set of isolated singular points. And let the region G , together D with its boundary ∂G , containing no singular points, then*

$$\int_{\partial G} f(z) dz = 2\pi i \sum_{a_v \in G} \operatorname{Res}_{z=a_v} f(z),$$

where a_v are the singular points $f(z)$, lying in G (their finite number).

Theorem. *The subtraction of a function f of an isolated singular point $a \in C$ is equal to the coefficient of the primus of the first degree $(z - a)$ in its Laurent decomposition in the neighborhood a :*

$$\operatorname{res}_a f(z) = c_{-1}.$$

Definition. Let a function f have infinity as its isolated singular point; its deduction at infinity is the magnitude

$$\operatorname{res}_{\infty} f(z) = \frac{1}{2\pi i} \int_{\gamma_R^-} f(z) dz,$$

where γ_R^- is a circle $\{|z| = R\}$ of a sufficiently large radius, passed clockwise.

Theorem (on the full sum of deductions). *Let the function f be holomorphic everywhere in the plane C_z except for a finite number of points a_v ($v = 1, \dots, n$); then the sum of its deductions at all finite singular points and the deduction at infinity is zero:*

$$\sum_{v=1}^n \operatorname{res}_{a_v} f(z) + \operatorname{res}_{\infty} f(z) = 0.$$

Example. Compute the deductions of the function relative to $f(z) = \frac{z^2 + z - 1}{z^3 - z}$

each of their singular points.

Solution. Transform the

$$f(z) = \frac{z^2 + z - 1}{z^3 - z} = \frac{z^2 + z - 1}{z(z-1)(z+1)}.$$

Let's use the formula

$$\operatorname{Res}_{z=a} f(z) = \lim_{z \rightarrow a} (z - a) f(z).$$

We have

$$\operatorname{Res} f(z) = \lim_{z \rightarrow 0} z f(z) = \lim_{z \rightarrow 0} z \cdot \frac{z^2 + z - 1}{z(z-1)(z+1)} = \frac{-1}{-1} = 1;$$

$$\operatorname{Res} f(z) = \lim_{z \rightarrow 1} (z-1) f(z) = \lim_{z \rightarrow 1} (z-1) \cdot \frac{z^2 + z - 1}{z(z-1)(z+1)} = \frac{1}{2};$$

$$\operatorname{Res} f(z) = \lim_{z \rightarrow -1} (z+1) \cdot \frac{z^2 + z - 1}{z(z-1)(z+1)} = \frac{1}{-2} = -\frac{1}{2}.$$

$$\text{Answer: } \operatorname{Res} f(z) = 1; \operatorname{Res} f(z) = \frac{1}{2}; \operatorname{Res} f(z) = -\frac{1}{2}.$$

Control questions and tasks:

1. Define the deduction at the final isolated singular point.
2. Formulate Cauchy's theorem on deductions.
3. What is the deduction of a function at an avoidable singular point?
4. Define the deduction at infinity.
5. Formulate a theorem about the full amount of deductions.

Topic 9. Application of deductions to the calculation of integrals

Objective: to practice the skills of applying deductions to the calculation of integrals.

Knowledge and skills acquired by a student as a result of mastering the topic:

Methodology for calculating improper integrals using deductions.

When studying this topic, it is necessary to pay attention to the following points.

Theorem. Let be $R(x, y)$ such a rational function of variables x, y that the rational function

$$R_1(z) = \frac{1}{z} R\left(\frac{1}{2}\left(z + \frac{1}{z}\right), \frac{1}{2i}\left(z - \frac{1}{z}\right)\right)$$

has no poles on a unit circle $\{z: |z|=1\}$. Then the values of the integral

$$I = \int_0^{2\pi} R(\cos \varphi, \sin \varphi) d\varphi \text{ are a number}$$

$$I = 2\pi \sum_{k=1}^n \operatorname{Res}_{z=z_k} R_1(z),$$

where $z_1, \dots, z_n, n \in \mathbb{N}$ are all the poles of $R_1(z)$ the function in the circle

$$E = \{z: |z| < 1\}.$$

Let the function $f(x)$ be defined on the real axis and integrable according to Riemann (or Lebesgue) on any interval $[a, b]$, $a < b$. Then, as is known from the course of mathematical analysis, the limit

$$\lim_{\substack{a \rightarrow -\infty \\ b \rightarrow \infty}} \int_a^b f(x) dx = \int_{-\infty}^{\infty} f(x) dx$$

is called the improper integral of $f(x)$. Here, a and b independently of each other, they tend to their limit values.

If $a = -R, b = R, (R > 0)$, then the limit

$$\lim_{R \rightarrow \infty} \int_{-R}^R f(x) dx = V.P. \int_{-\infty}^{\infty} f(x) dx$$

is called the improper integral of $f(x)$ in the sense of the main value.

The existence of an improper integral implies the existence of an integral in the sense of the principal value, and obviously there is equality of integrals. The integral of $f(x)$ in the sense of the principal value can exist for divergent improper integrals.

By Π^+ and Π^- denote respectively the upper and lower half-planes of the complex plane C :

$$\Pi^+ = \{z : \operatorname{Im} z > 0\}, \Pi^- = \{z : \operatorname{Im} z < 0\}.$$

Theorem. Let the function $f(z)$ be holomorphic in Π^+ , except for the set

$V = \{z_1, \dots, z_n\}, n \in N$, points $z_k \in \Pi^+$, and continuous in $\overline{\Pi^+} \setminus V$. If $\lim_{\substack{z \rightarrow \infty \\ z \in \Pi^+}} [zf(z)] = 0$, then

$$V.P. \int_{-\infty}^{\infty} f(x) dx = 2\pi i \sum_{k=1}^n \operatorname{Res}_{z=z_k} f(z).$$

The consequence. Let

$$f(z) = \frac{P_l(z)}{Q_s(z)} = \frac{a_l z^l + \dots + a_0}{b_s z^s + \dots + b_0}, a_l \neq 0, b_s \neq 0, -$$

a rational function whose denominator has no real roots, and its power is at least two units higher than the power of the numerator, i.e. $s \geq l + 2$.

Then

$$V.P. \int_{-\infty}^{\infty} \frac{P_l(x)}{Q_s(x)} dx = 2\pi i \sum_{k=1}^n \operatorname{Res}_{z=z_k} \frac{P_l(z)}{Q_s(z)},$$

where $z_1, \dots, z_n, n \leq s$ are the different zeros of the functions in pairs $Q_s(z)$, lying in Π^+ .

Theorem. Let the function $f(z)$ be holomorphic in the half-plane Π^+ , except for the finite set $V = \{z_1, \dots, z_n\}, n \in N$, points $z_k \in \Pi^+$ and be continuous in $\overline{\Pi^+} \setminus V$. If

$\lim_{\substack{z \rightarrow \infty \\ z \in \Pi^+}} f(z) = 0$ and $\alpha > 0$, then

$$V.P. \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx = 2\pi i \sum_{k=1}^n \operatorname{Res}_{z=z_k} \{f(z) e^{i\alpha z}\}.$$

A function f that is holomorphic in C_z , except for the points at which it has poles, is called *meromorphic*.

Meromorphic functions include constant and integer functions (which have no special points at all C_z), rational functions, the quotient of dividing an integer function by an integer function, and the product of a finite number of meromorphic functions, which is also a meromorphic function.

Let us denote the poles of the meromorphic function f , other than $z = 0$ through $a_1, a_2, \dots$. In this case, if there are an infinite number of poles, then we will number them in the order of non-decreasing moduli: $0 < |a_k| < |a_{k+1}|, k \in \mathbb{N}$. Through $T_k = T_k(z, a_k)$ we will denote the principal part of the expansion f into the Laurent series in the punctured neighborhood a_k of the point. If f has a $z = 0$ pole with the principal part $T_0(z, 0)$ at a point $g = f - T_0$, then there is a meromorphic function for which $z = 0$ there is an eliminable singularity. This remark makes it possible in a number of cases to consider meromorphic functions that do not have a pole at the beginning for the convenience of notation.

Theorem. A meromorphic function f with a finite number of poles in C_z is a rational function.

Let us denote in terms T of the principal part of the Laurent series expansion f at infinity ($T = 0$ if infinity is an eliminable singular). Given all the poles, $a_0, a_1, \dots, a_n$ ($n \in \mathbb{N} \cup \{0\}$) consider the C_n function

$$\varphi(z) = f(z) - T(z) - \sum_{k=0}^n T_k(z, a_k), \text{ holomorphic in } C_z. \text{ According to}$$

Liouville's theorem, it is constant: $\varphi = c_0$ ($c_0 \in \mathbb{C}$). Thus,

$$f(z) = c_0 + T(z) + \sum_{k=0}^n T_k(z, a_k)$$

The resulting representation f gives the decomposition of the rational function into the simplest fractions; the sum $c_0 + T(z)$ is an integer part f . With the help of the same formula, the inverse problem of finding a rational function by arbitrarily given principal parts of its decomposition is solved.

Example. Calculate the specified integrals:

$$1) \int_{|z|=3} \frac{zdz}{(z-1)(z-2)^2}.$$

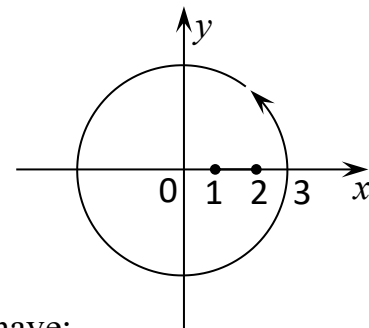
Solution. $f(z) = \frac{z}{(z-1)(z-2)^2};$

$z = 1$ – 1st order pole

$z = 2$ – 2nd order pole

According to Cauchy's theorem on deductions, we have:

$$I = \int_{|z|=3} \frac{zdz}{(z-1)(z-2)^2} = 2\pi i \left(\operatorname{Res}_{z=1} f(z) + \operatorname{Res}_{z=2} f(z) \right).$$



$$\operatorname{Res} f(z) = \lim_{z \rightarrow 1} (z-1) \frac{z}{(z-1)(z-2)^2} = 1.$$

$$\operatorname{Res} f(z) = \lim_{z \rightarrow 2} \left[(z-2)^2 \frac{z}{(z-1)(z-2)^2} \right]' = \lim_{z \rightarrow 2} \left(\frac{z}{z-1} \right)' = -\lim_{z \rightarrow 2} \frac{1}{(z-1)^2} = -1;$$

$$I = 2\pi i(1-1) = 0.$$

Answer: 0.

Control questions and tasks:

1. Write down a formula to calculate the integral $\int_0^{2\pi} R(\cos \phi, \sin \phi) d\phi$ using deductions.
2. Explain the methodology for calculating improper integrals using deductions.

Educational, methodological and information support of the discipline List of basic and additional literature necessary for mastering the discipline

List of basic literature:

- 1 Minkova R. M. Funktsii kompleksnogo peremezhnogo v primerakh i zadach [Functions of a complex variable in examples and tasks]. - Yekaterinburg: Ural University Publishing House, 2014. - 57 p. - ISBN 978-5-7996-1216-0
- 2 Sinkevich, G.I. Functions of a Complex Variable. Theory and Practice Elektronnyi resurs : uchebnik / G.I. Sinkevich. - St. Petersburg : St. Petersburg State University of Architecture and Civil Engineering, EBS ASV, 2016. - 184 p. - The book is in the basic version of the EBS IPRbooks. - ISBN 978-5-9227-0595-0

List of additional literature:

- 1 Practicum in mathematics. Educational and methodological materials of practical classes on the topic "Elements of the theory of functions of a complex variable": Direction of training 11.03.04 Electronics and nanoelectronics. Profile - Microelectronics and solid-state electronics. - Stavropol : NCFU, 2013. - 78 p.
- 2 Popov V. N. Prikladnyye voprosy teorii funktsii kompleksnogo peremerennogo [Applied issues of the theory of functions of a complex variable]. - Arkhangelsk: IPTs NArFU, 2013. - 164 p. - ISBN 978-5-261-00850-7
- 3 Special chapters of the geometric theory of functions of a complex variable. Manual : (spec. 010101 - Mathematics) / author. S. V. Nikitin; Federal Agency for Education, GOU VPO "Stavrop. State University". - Stavropol : SSU Publ., 2009. - 64 p. - References: p. 62

List of Educational and Methodological Support for Students' Independent Work in a Discipline (Module)

- 1 Lavrentyev M. A., Shabat B. V. Metody teorii funktsii kompleksnogo variable [Methods of the theory of functions of a complex variable]. - Izd. 6-e, ster. - M. : Lan, 2002. - 688 p. - (Textbooks for universities. Special literature). - References: p. 681-684. - Object. Decree: p. 685-688. - ISBN 5-9511-0014-3