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SUGGESTIONS FOR TEACHING
for the performance of laboratory work
in the discipline "Information Theory and Coding"
for students of the direction of training
11.03.02 Infocommunication Technologies and Communication Systems
"Intelligent Data Processing in Infocommunication Systems and Networks"

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Practical work 1

Investigation of Characteristics of Signals Essential for Their Transmission via Communication Channels

The purpose of the work is to acquire skills in determining and studying the characteristics of signals that are essential for their transmission through communication channels.

Theoretical part

A deterministic signal $s(t)$ is a signal whose instantaneous values are known at any time. A deterministic signal can be set in an analytical form, as a function of time $s(t)$ or represented by a graph, table, or oscillogram.

The signal is also the object of transport from the transmitter to the receiver, and the communication technique is essentially the technique of transporting (transmitting) signals through communication channels. Therefore, first of all, in the laboratory work, the parameters of the signal are determined, which are the main from the point of view of its transmission through communication channels. Such parameters are the duration of the signal TS , s, the spectrum width ΔFS , Hz, and its dynamic range DS , dB.

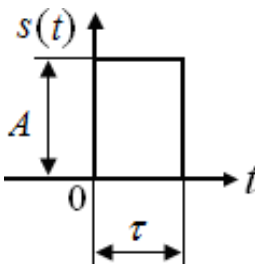
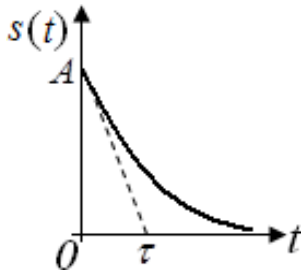
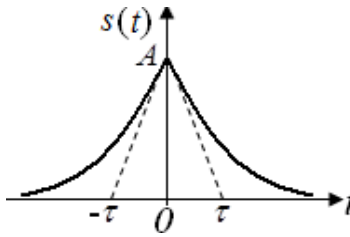
TS signal duration, s, is the time interval $(t1, t2)$ during which the signal exists (defined):

$$TS = t2 - t1. \quad (1.1)$$

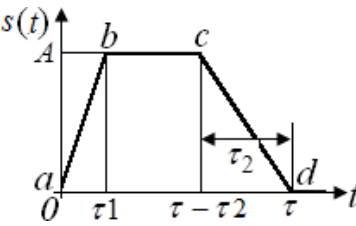
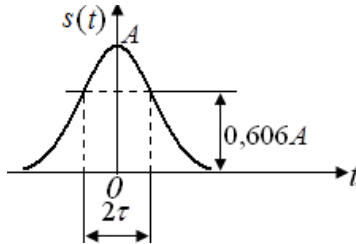
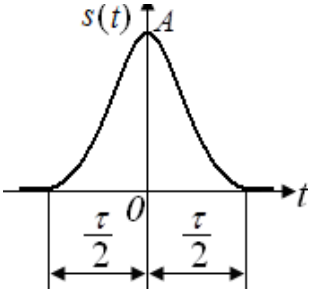
Despite the fact that in practice signals are always limited in time, along with signals specified at a limited time interval $(t1, t2)$, signals specified at a semi-infinite $(0, \infty)$ or infinite $(-\infty, \infty)$ time intervals are considered

exponential momentum. An example of a signal given on an infinite ($-\infty, \infty$) time interval is a two-way exponential momentum. Mathematical models and parameters of the listed signals are given in Table 1.1.

Table 1.1 – Name, graphical and analytical definition of signals studied in laboratory work

Name	Graphic Definition	Mathematical model in Mathcad and Parameters
Square Pulse		$s(t) := \begin{cases} A & \text{if } 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases}$ Pulse parameters: A is the height of the rectangular pulse; τ is the duration of the rectangular pulse. <i>Note</i> – Keyword "otherwise" is used in the context to specify the value of the signal "in all other cases"
One-way Exponential Momentum		$s(t) := \begin{cases} A \cdot e^{-\frac{t}{\tau}} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$ Parameters of imuls: A is the height of the one-sided exponential momentum; τ is the time constant one-way exponential momentum
Bilateral Exponential Momentum		$s(t) := A \cdot e^{-\frac{ t }{\tau}}$ Pulse parameters: A is the height of the bilateral exponential momentum; τ is the time constant of the two-way exponential momentum

Sawtooth Pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau} \right) & \text{if } 0 \leq t \leq \tau \\ 0 & \text{if } t \geq \tau \end{cases}$ Pulse parameters: A is the height of the sawtooth pulse; τ is the duration of the sawtooth pulse. <i>Note</i> – The segment ab is called the forward path of the sawtooth pulse, the segment bc is the reverse path of the sawtooth pulse
Triangular Pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau_1} \right) & \text{if } 0 \leq t \leq \tau_1 \\ \left[A \cdot \left(1 - \frac{t - \tau_1}{\tau_2} \right) \right] & \text{if } \tau_1 \leq t \leq \tau \\ 0 & \text{if } t > \tau \end{cases}$ Pulse parameters: A is the height of the triangular pulse; τ_1 and τ_2 are the duration of the edge and the duration of the triangular pulse cut; $\tau = \tau_1 + \tau_2$ is the duration of the triangular pulse. <i>Note</i> – The segment ab is called the front, and the segment bc is the slice of the triangular momentum

Trapezoidal pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau_1} \right) & \text{if } 0 \leq t \leq \tau_1 \\ A & \text{if } \tau_1 \leq t \leq \tau - \tau_2 \\ \left[A \cdot \left(1 - \frac{t - \tau + \tau_2}{\tau_2} \right) \right] & \text{if } \tau - \tau_2 \leq t \leq \tau \\ 0 & \text{if } t > \tau \end{cases}$ Pulse parameters: A is the height of the trapezoidal pulse; τ is the duration of the trapezoidal pulse; τ_1 is the duration of the trapezoidal pulse front; τ_2 is the duration of the trapezoidal pulse slice. <i>Note</i> – The segment ab is called the front of the trapezoidal pulse, the segment bc is the apex of the trapezoidal pulse, the segment cd is the slice of the trapezoidal pulse
Bell Pulse		$s(t) := A \cdot e^{-\frac{1}{2} \cdot \left(\frac{t}{\tau} \right)^2}$ Pulse parameters: A is the height of the bell-shaped impulse; 2τ is the time interval between the inflection points of the bell-shaped pulse. <i>Note</i> – The value of the parameter $2\tau K$ is also determined by the level of $0.606AK$.
Cosine-square momentum		$s(t) := \begin{cases} \left(A \cdot \cos\left(\frac{\pi}{\tau} \cdot t \right) \right)^2 & \text{if } -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0 & \text{if } t > \frac{\tau}{2} \end{cases}$ Pulse parameters: A is the height of the cosine-square impulse; τ is the duration of the cosine-square pulse. <i>NOTE</i> – The value of $\tau/2$ is also determined by the level of $0.5A$

Along with the concept of signal duration, the concept of effective signal duration TSE, c is widely used, i.e. the duration of the time interval within which 90...99% of the signal energy is concentrated. If the signal $s(t)$ is set on a semi-infinite $(0, \infty)$ time interval, then its active duration is calculated from the condition

$$\int_0^{TSE} s^2(t) dt = k \int_0^{\infty} s^2(t) dt, \quad (1.2)$$

where k is the coefficient of decrease in the signal energy due to the artificial limitation of its duration ($k = 0.9 - 0.99$).

The second most important characteristic from the point of view of signal transportation along the communication channel is the signal spectrum width ΔFS , Hz, or the effective signal spectrum width ΔFSE , Hz. The effective signal spectrum width (signal spectrum width) is the frequency range within which 90...99% of the signal energy is concentrated. This parameter gives an idea of the rate of change of the signal within the interval of its existence. The effective signal spectrum width is calculated using the formula

$$\Delta FSE = 2 \cdot fBE, \quad (1.3)$$

where fBE is the upper effective boundary frequency of the signal spectrum, which is determined from the condition

$$\int_{-fBE}^{fBE} |S(f)|^2 df = k \int_{-\infty}^{\infty} |S(f)|^2 df, \quad (1.4)$$

where k is the coefficient of signal energy reduction due to artificial limitation of its spectrum width ($k = 0.9 - 0.99$);

$|S(f)|$ is the module of the spectral function of the signal.

In electrical communication technology, the signal spectrum is often deliberately reduced. This is due to the fact that equipment and communication lines have a limited bandwidth of transmitted frequencies. Spectrum reduction is carried out on the basis of permissible signal distortions. For example, in telephone communication, it is required that speech is intelligible and that

Correspondents could recognize each other by voice. To fulfill these conditions, it is enough to transmit a voice signal in the band from 300 to 3400 Hz. Transmission of a wider range of speech in this case is impractical, since it leads to technical complications and increased costs.

Note – In determining the effective duration and width of the signal spectrum, two approaches are usually used: energy and information. The content of the energy approach to determining these parameters was considered earlier. The methodology for implementing the information approach to determining the effective duration and width of the signal spectrum is discussed further in the section "Methodology and Procedure for Performing Work".

The spectral function of *the* $S(f)$, V·Hz, signal is a complex function that is a direct Fourier transform from the signal

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{j2\pi ft} dt, \quad (1.5)$$

where f is the frequency, Hz;

$j = \sqrt{-1}$ – imaginary unit.

The spectral function is basic in terms of describing the properties of a signal in the frequency domain.

The modulus of the spectral function of the signal is calculated using the formula

$$|S(f)| = \sqrt{\text{Re}^2\{S(f)\} + \text{Im}^2 S(f)}, \quad (1.6)$$

$$\text{where } \text{Re}\{S(f)\} = \int_{-\infty}^{\infty} s(t)\cos(2\pi ft) dt \quad - \quad \text{Material} \quad \text{Part}$$

the spectral function of the signal;

$$\text{Im}\{S(f)\} = \int_{-\infty}^{\infty} s(t)\sin(2\pi ft) dt \quad - \quad \text{imaginary part of the spectral function}$$

In signal theory, the product of the duration of a *TS signal* by its spectrum width ΔF is denoted BS and is called the signal base:

$$BS = TS \cdot \Delta FS . \quad (1.7)$$

A signal characteristic that defines the bandwidth requirements of the transmitting device, receiver, and channel of the electrical communication system.

NOTE – If the signal base is of the order of one ($BS \approx 1$), then the signal is considered simple or narrowband. If $BS \gg 1$, then the signal is considered complex or wideband.

Dynamic range is the ratio of the highest instantaneous signal power to the lowest power that must be distinguished from zero for a given signal transmission quality over a communication channel:

$$D_S = 10 \lg \frac{P_{S \max}}{P_{S \min}} , \quad (1.8)$$

where $P_{S \max}$ – maximum instantaneous signal power, B2;

$P_{S \min}$ is the lowest signal power that must be distinguished from zero for a given signal transmission quality over the communication channel, B2.

Note – The dynamic range of the speaker's speech, for example, is 25... 30 dB, a small vocal ensemble 45...65 dB, a symphony orchestra 70...95 dB. In order to avoid channel overloads in radio broadcasting, the dynamic range is often reduced to 35...45 dB.

As can be seen from (1.8), in order to determine the dynamic range of the signal, it is necessary to first calculate the highest instantaneous signal power. In this regard, let us consider the energy characteristics of the signals:

1. Signal energy E , B2c, released at a resistance of one Ohm – integral of the square of the signal along the entire time axis:

$$E = \int_{-\infty}^{\infty} s^2(t) dt . \quad (1.9)$$

If the signal $s(t)$ is specified at a finite $(0, TS)$ time interval, then this characteristic is calculated from the condition

$$E = \int_0^{TS} s^2(t) dt. \quad (1.10)$$

The energy of a signal can be finite or infinite. For example, any signal of finite duration has a finite energy (unless it contains delta functions $\delta(t)$ or branches extending to infinity), while any periodic signal, such as a harmonic oscillation, on the contrary, has infinite energy.

2. Instantaneous power $p(t)$, B2, of the signal released at a resistance of one Ohm is the square of the instantaneous value of the signal:

$$p(t) = s^2(t). \quad (1.11)$$

3. Average power P_{cp} , B² signal released at resistance one Ohm, for a given period of time $(t1, t2)$ is the value of the signal energy released at the resistance of one Ohm, for a period of time $(t1, t2)$, related to the duration of this time interval:

$$P_{cp} = \frac{1}{T} \int_{t_1}^{t_2} s^2(t) dt, \quad (1.12)$$

where $T = t2 - t1$ is the averaging time interval.

If the signal $s(t)$ is given at a finite $(0, T)$ time interval and if the signal energy is infinite, then we can determine its average power over the entire time axis. To do this, we need to use the previous formula and perform a limit transition by directing the averaging interval to infinity:

$$P_{cp} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T s^2(t) dt. \quad (1.13)$$

4. RMS (effective) signal value square root of average signal strength: σ_S , B, –

$$\sigma_S = \sqrt{P_{cp}}. \quad (1.14)$$

Note – Parameters E , $p(t)$ and

P_{cp} sometimes called specific

power and energy to emphasize the implied unit value of load resistance.

Based on the duration of the TS signal, its dynamic range DS and the spectrum width ΔFS , a more general and illustrative characteristic is introduced – the volume of the VS signal, dB:

$$VS = TS \cdot \Delta FS \cdot DS. \quad (1.15)$$

The volume of the VS signal gives a general idea of the capabilities of a given set of signals as message carriers. The larger the volume of the signal, the more information can be "embedded" in this volume and the more difficult it is to transmit such a signal over the communication channel with the required quality.

Equipment and materials

Equipment. To perform laboratory work, you need a personal computer with the following characteristics: a Pentium/Celeron processor with a clock speed of 300 MHz and higher, RAM of at least 128 MB or more, free disk space of at least 100 MB, a CD reader, a monitor of the Super VGA type (number of colors – 256).

Software. To perform the laboratory work, you need the operating system WINDOWS 2000/ XP Professional, the Microsoft .NET Framework library version 1.1 or higher, the program Mathcad 13 and higher.

Safety Instructions

When performing laboratory work, **it is prohibited:**

- independently repair a personal computer, as well as install and remove existing software;

- violate generally accepted safety rules when working with electrical equipment, in particular, touch electrical outlets with metal objects, etc.;
- eat and drink and litter at the workplace of a personal computer user.

In the event of a malfunction of a personal computer, it is necessary to **immediately** inform the laboratory service personnel (system administrator, operator).

Instructions on the procedure for performing work

Let's consider the method of performing the work on the following example.

The signal $s(t)$ is given, which is a segment of a sine wave with an amplitude of $U = 2$ V, turning to zero at the ends of the time interval $[0, \tau]$, where $\tau = 1$ ms = 10^{-3} s.

$$s(t) = U \sin\left(\frac{\pi t}{\tau}\right), \quad 0 \leq t \leq \tau. \quad (1.16)$$

It is required for the $s(t)$ signal to determine:

- duration (effective duration);
- energy, instantaneous and average power;
- RMS (effective) value;
- dynamic range, considering that the lowest signal power that must be distinguished from zero for a given transmission quality is 10^{-6} V²;
- the upper boundary frequency of the spectrum and the width of the spectrum, using information and energy approaches;
- using information and energy approaches;
- volume, using information and energy approaches.

Let's consider the solution to the problem in the Mathcad mathematical package:

1. Run the math package Mathcad. According to the analytical definition and the specified parameters, enter the definition of the signal $s(t)$ and plot it (Figure 1.1). Obviously, the duration of the signal in question is 1 ms ($TS = 1$ ms).

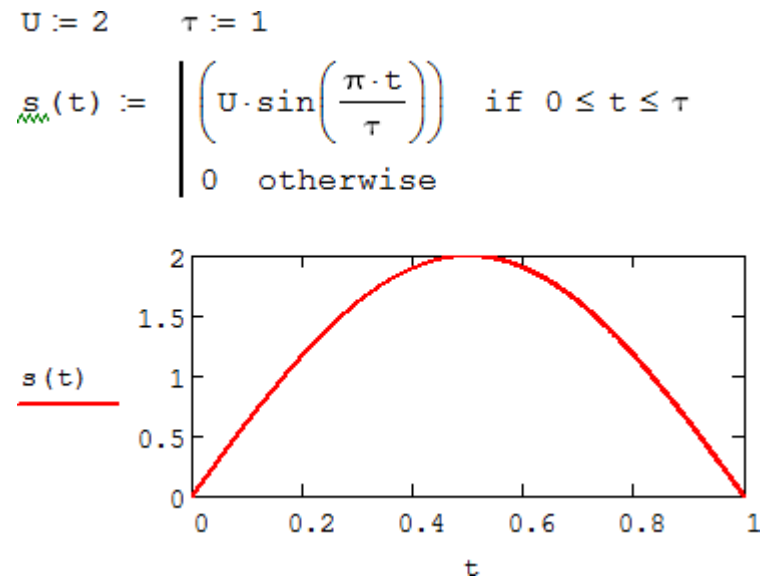


Figure 1.1 – Definition and graph of the $s(t)$ signal

2. Using the formulas (1.10) – (1.12), we determine the energy, instantaneous and average signal power $s(t)$ (Figure 1.2).

$$E := \int_0^{\tau} s(t)^2 dt = 2$$

$$p(t) := s(t)^2$$

$$P_{cp} := \frac{1}{\tau} \cdot \int_0^{\tau} s(t)^2 dt = 2$$

Figure 1.2 – Determination of energy, instantaneous and average signal power $s(t)$

Plotting the instantaneous signal strength $s(t)$ (Figure 1.3).

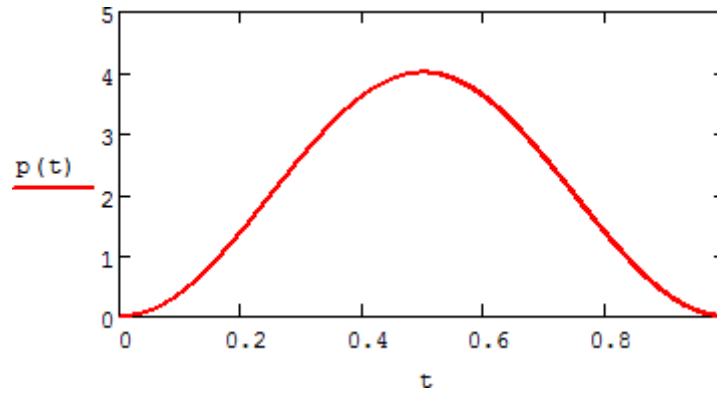


Figure 1.3 – Graph of instantaneous signal strength $s(t)$

4. Using formula (1.14), we calculate the root mean square (effective) value of the signal $s(t)$ (Figure 1.4).

$$\sigma_s := \sqrt{P_{cp}} = 1.414$$

Figure 1.4 – Determination of $s(t)$ RMS

5. From the analysis of the graph in Figure 1.3, it can be seen that the highest instantaneous signal strength $s(t)$ is reached at a time equal to half

Duration signal, t. e. $P_{s \max} = p(t) \Big|_{t=0.5}$ Q2. Then by

$$= p(0.5) = 4$$

In formula (1.18), we find the dynamic range DS, dB, of the *signal* $s(t)$, assuming that the lowest signal power that must be distinguished from zero for a given transmission quality is 10^{-6} V² (Figure 1.5).

$$\begin{aligned} P_{\min} &:= 10^{-6} \\ P_{\max} &:= p(0.5) = 4 \\ DS &:= 10 \cdot \log \left(\frac{P_{\max}}{P_{\min}} \right) = 66.021 \end{aligned}$$

Figure 1.5 – Determination of the dynamic range of *the* $s(t)$ signal

Therefore, the dynamic range of the $s(t)$ signal is 66.021 dB ($DS = 66.021$ dB).

6. Determining the upper effective boundary frequency of the *spectrum fBE signal* $s(t)$, using the information approach:

6.1) using the formula of the direct integral Fourier transform, we find the spectral function of the signal $s(t)$ (Figure 1.6);

$$j := \sqrt{-1}$$

$$S(f) := \int_0^{\tau} s(t) \cdot e^{-j \cdot 2 \cdot \pi \cdot f \cdot t} dt$$

Figure 1.6 – Determination of the spectral function of the $s(t)$ signal

6.2) Determine and plot the normalized amplitude spectrum of the signal $S(T)$ (Figure 1.7);

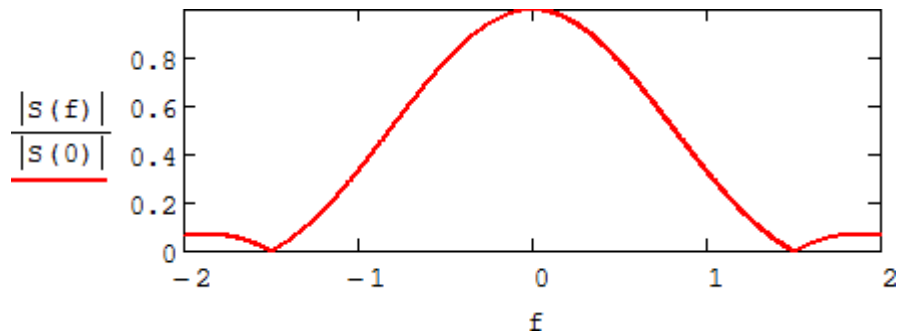


Figure 1.7 – Graph of the normalized amplitude spectrum *of the* $s(t)$ signal

6.3) On the graph of the normalized amplitude spectrum of the signal $S(T)$, draw a horizontal dotted line at the level of 0.1 (Figure 1.8);

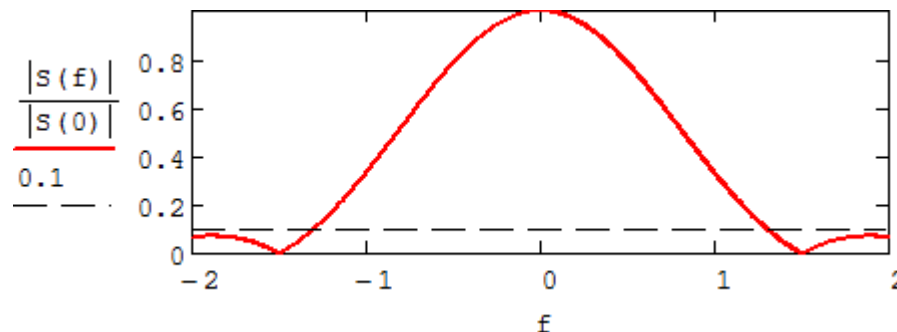


Figure 1.8 – On the graph of the normalized amplitude spectrum *of the signal* $s(t)$, a horizontal dotted line is drawn at the level of 0.1

6.4) Using the "X-Y Trace" graph trace window, we find the projections of the intersection point of the specified level 0.1 with the graph of the normalized amplitude spectrum of *the signal* $s(t)$ on the f -axis, determine the upper effective cut-off frequency of the spectrum f_{BE} , kHz, (Figure 1.9). As can be seen from Figure 1.9, the upper effective cut-off frequency of the signal spectrum $s(t)$ is 1.304 kHz ($f_{VE} = 1.304$ kHz).

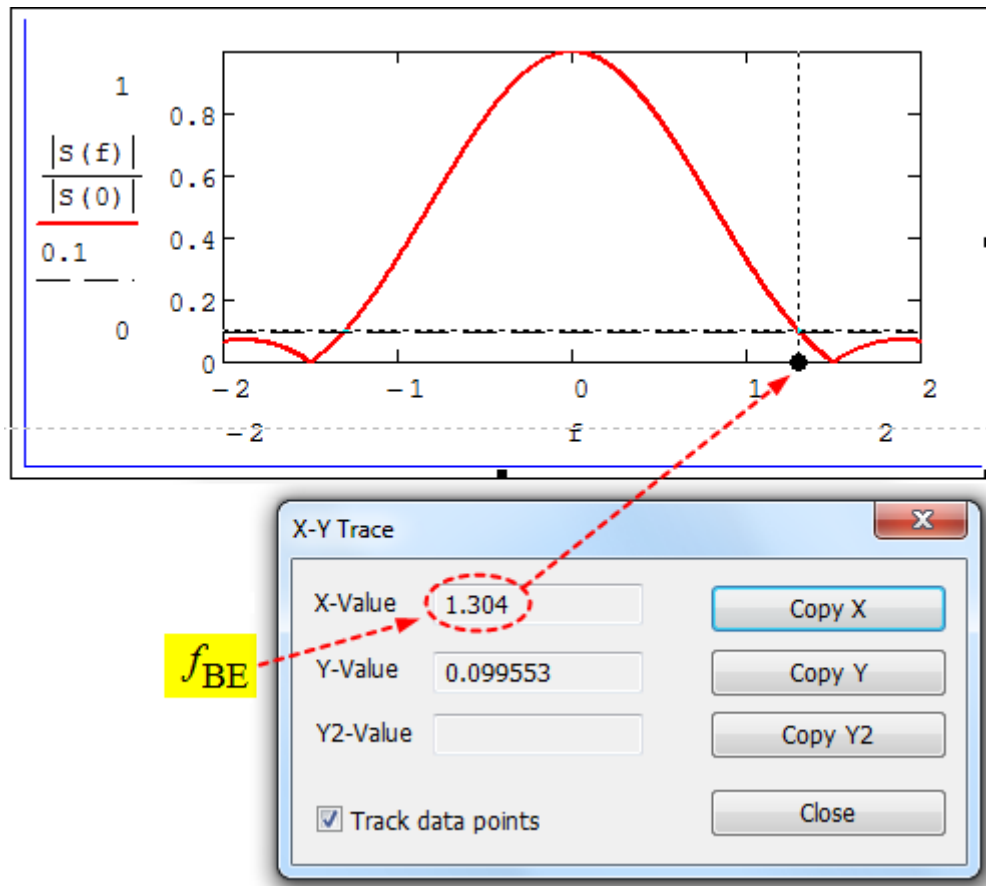


Figure 1.9 – Determining the upper effective cut-off frequency of the signal spectrum $s(t)$

7. Using formula (1.3), we calculate the effective spectrum width ΔF_{SE} of the $s(t)$ signal (Figure 1.10).

$$f_{be} := 1.304$$

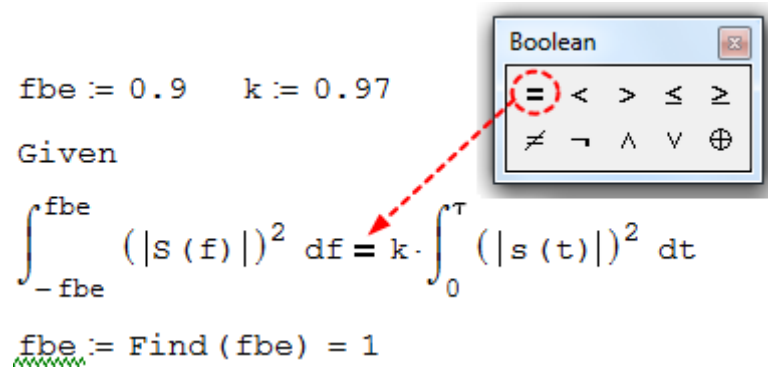
$$\Delta F_{se} := 2 \cdot f_{be} = 2.608$$

Figure 1.10 – Calculating the spectrum width ΔF_{SE} and ΔF_{SE2} of the $s(t)$ signal

As can be seen from the calculations in Figure 1.10, the effective spectrum width ΔF_{SE} of the *signal* $s(t)$, calculated using the information approach, is 2.608 kHz ($\Delta F_{SE} = 2.608$ kHz).

8. Using the method considered earlier, we determine the upper effective boundary frequency of the signal spectrum $s(t)$ using the energy approach to determining this parameter:

8.1) find the upper effective cut-off frequency of the signal $s(t)$ (Figure 1.11);



$f_{be} := 0.9 \quad k := 0.97$
 Given

$$\int_{-f_{be}}^{f_{be}} (|s(f)|)^2 df = k \cdot \int_0^{\tau} (|s(t)|)^2 dt$$
 $f_{be} := \text{Find}(f_{be}) = 1$

Figure 1.11 – Determining the upper cut-off frequency of the spectrum $f_{VE} = 1$ kHz

As can be seen from Figure 1.11, the -1 to 1 kHz frequency range of the $s(t)$ signal spectrum contains 97% of the total energy of this signal.

8.2) calculate the effective spectrum width ΔF_{SE} of the signal $s(t)$ using formula (1.3) (Figure 1.12);

$$\Delta F_{se} := 2 \cdot f_{be} = 1.999$$

Figure 1.12 – Calculating the spectrum width ΔF_{SE} of the $s(t)$ signal

As can be seen from the calculations in Figure 1.12, the effective spectrum width ΔF_{SE} of the $s(t)$ signal, calculated using the energy approach, is 1.999 kHz ($\Delta F_{SE} = 1.999$ kHz).

9. Using formula (1.7), we determine the signal base $s(t)$ using the information and energy approaches:

9.1) calculate the signal base $s(t)$ using the information approach (Figure 1.13);

$$B_s := \tau \cdot \Delta F_{se} = 2.608$$

Figure 1.13 – Calculating the signal base $s(t)$ using the information approach

9.2) calculate the signal base $s(t)$ using the energy approach (Figure 1.14);

$$B_s := \tau \cdot \Delta F_{se} = 1.999$$

Figure 1.14 – Calculating the signal base $s(t)$ using the energy approach

10. Using formula (1.15), we determine the volume of the signal $s(t)$ using the information and energy approaches:

10.1) determine the signal volume $s(t)$ using the information approach (Figure 1.15);

$$Vs := \tau \cdot Bs \cdot Ds = 172.182$$

Figure 1.15 – Calculating the signal volume $s(t)$ using the information approach

As can be seen from the calculations in Figure 1.15, the signal volume $s(t)$ calculated using the information approach is 172.182 dB ($VS = 172.182$ dB).

10.2) determine the volume of the signal $s(t)$ using the energy approach (Figure 1.16);

$$Vs := \tau \cdot Bs \cdot Ds = 131.984$$

Figure 1.16 – Calculating the signal volume $s(t)$ using the energy approach

As can be seen from the calculations given in Figure 1.16, the signal volume $s(t)$ calculated using the energy approach is 131.984 dB ($VS = 131.984$ dB), which is about 1.3 times less than the value of the same parameter found using the information approach.

Thus, one and the same signal can have different characteristics that are essential for its transmission through communication channels, using different methods of their determination. This fact must be taken into account in engineering practice when analyzing and synthesizing electrical communication systems.

Work procedure:

1. Run Mathcad.
2. Define in Mathcad for given signals

$s1(t)$ and $s2(t)$ in

in accordance with the version of the individual task (Table 1.2), their numerical parameters:

- the duration (effective duration) of signals;
- energy, instantaneous and average power;
- RMS (effective) value;
- dynamic range, considering that the lowest signal power that must be distinguished from zero for a given transmission quality is 10^{-6} V^2 ;
- the upper boundary frequency of the spectrum and the width of their spectra, using information and energy approaches;
- using information and energy approaches;
- volume, using information and energy approaches.

3. To study the effect of changing the duration of signals on the $s_1(t)$ and $s_2(t)$ effective width of their spectra.

4. Save the results of an individual task.
5. Generate a report on the laboratory work.
6. Protect the work.

Variants of tasks

The number of the variant (Table 1.2) corresponds to the number of the student's surname in the list of the study group (subgroup).

Table 1.2 – Variants of individual tasks

Variant	Designation signal	Signal Name	Signal Parameters
1	$s_1(t)$	Trapezoidal pulse	$A = 2 \text{ V}$, $\tau = 3 \text{ ms}$, $\tau_1 = 1 \text{ ms}$, $\tau_2 = 1 \text{ ms}$
	$s_2(t)$	Cosine-square Momentum	$A = 2 \text{ V}$, $\tau = 6 \text{ ms}$
2	$s_1(t)$	Double-sided Exponential momentum	$A = 2 \text{ V}$, $\tau = 2 \text{ ms}$
	$s_2(t)$	Sawtooth Pulse	$A = 2 \text{ V}$, $\tau = 2 \text{ ms}$
3	$s_1(t)$	Triangular Pulse	$A = 2 \text{ V}$, $\tau_1 = 1 \text{ ms}$, $\tau_2 = 1 \text{ ms}$
	$s_2(t)$	Bell Pulse	$A = 2 \text{ V}$, $\tau = 2 \text{ ms}$

4	$s1(t)$	Single-sided Exponential momentum	$A = 2 \text{ V}, \tau = 2 \text{ ms}$
	$s2(t)$	Cosine-square Momentum	$A = 2 \text{ V}, \tau = 6 \text{ ms}$
5	$s1(t)$	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 5 \text{ ms}$
	$s2(t)$	Triangular Pulse	$A = 2 \text{ V}, \tau_1 = 2 \text{ ms}, \tau_2 = 3 \text{ ms}$
6	$s1(t)$	Double-sided Exponential momentum	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
	$s2(t)$	Cosine-square Momentum	$A = 2 \text{ V}, \tau = 8 \text{ ms}$
7	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}, \tau_1 = 1 \text{ ms}, \tau_2 = 1 \text{ ms}$
	$s2(t)$	Triangular Pulse	$A = 4 \text{ V}, \tau_1 = 1 \text{ ms}, \tau_2 = 3 \text{ ms}$
8	$s1(t)$	Single-sided Exponential momentum	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
	$s2(t)$	Sawtooth Pulse	$A = 3 \text{ V}, \tau = 6 \text{ ms}$
9	$s1(t)$	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
10	$s1(t)$	Single-sided Exponential momentum	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
11	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 5 \text{ ms}, \tau_1 = 1 \text{ ms}, \tau_2 = 1 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
12	$s1(t)$	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Triangular Pulse	$A = 2 \text{ V}, \tau_1 = 2 \text{ ms}, \tau_2 = 2 \text{ ms}$
13	$s1(t)$	Trapezoidal pulse	$A = 4 \text{ V}, \tau = 5 \text{ ms}, \tau_1 = 2 \text{ ms}, \tau_2 = 2 \text{ ms}$
	$s2(t)$	Double-sided Exponential momentum	$A = 4 \text{ V}, \tau = 3 \text{ ms}$
14	$s1(t)$	Triangular Pulse	$A = 4 \text{ V}, \tau_1 = 1 \text{ ms}, \tau_2 = 3 \text{ ms}$
	$s2(t)$	Cosine-square Momentum	$A = 4 \text{ V}, \tau = 8 \text{ ms}$
15	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 5 \text{ ms}, \tau_1 = 2 \text{ ms}, \tau_2 = 2 \text{ ms}$
	$s2(t)$	Single-sided Exponential momentum	$A = 2 \text{ V}, \tau = 2 \text{ ms}$

Note – Graphical and analytical determination of the parameters of the signals under study is given in Table 1.1

Contents of the report

The report on the laboratory work is drawn up in the form of a Mathcad or Word document and must include:

1. Name of the laboratory work.
2. The purpose of laboratory work.
3. Wording individual Quests and Result its implementation.
4. Brief conclusions based on the results of laboratory work.

Control questions

1. What characteristics of signals are essential for their transmission through communication channels?
2. What energy characteristics of signals are studied in the course of laboratory work?
3. How to determine the signal strength?
4. How to determine the signal energy?
5. How do you determine instantaneous signal strength?
6. How do you determine the average signal strength if the signal energy is infinite?
7. How to Define RMS (active)
The meaning of the signal?
8. What is Effective Signal Duration?
9. How do I determine the effective signal duration?
10. Record the Forward Integral Fourier Transform for the Signal $s(t)$.
11. How to determine the amplitude spectrum of a signal?
12. What is Effective Spectrum Width?
13. How to Determine the Upper Effective Cutoff Frequency of the Spectrum signal?
14. How do you determine the effective spectrum width of a signal?

15. How to Affects Change Duration signal
(effective signal duration) by the effective width of its spectrum?
16. What is the methodology for performing an individual task?
17. What Conclusions Yes, you can do by Results
of an individual task?

Practical work 2

Study of Signal Characteristics in Their Vector Representation

The purpose of the work is to acquire the skills of determining and studying the characteristics of signals used in their vector representation.

Theoretical part

When solving a number of problems related to the analysis and transformation of signals, it is advisable to map these signals by vectors of some vector space. This, as a rule, makes it possible to simplify the solution.

Let us consider the main characteristics of the signals used in their vector representation.

One of the fundamental characteristics of the signals used in their vector representation is the signal norm. The most common assumptions are

that real analog signals have a norm $\|s\|$, B $\|s\| = \sqrt{c}$:

$$\|s\| = \sqrt{\int_{-\infty}^{\infty} s^2(t) dt} \quad (1.1)$$

(A positive value is selected from the two possible root values).

For complex signals, the norm is calculated using the formula

$$\|s\| = \sqrt{\int_{-\infty}^{\infty} s(t) s^*(t) dt}, \quad (1.2)$$

where $*$ is a symbol of a complex-conjugate quantity.

The square of the norm $\|s\|^2$, coincides with the previously considered concept of signal energy:

$$E = \|s\|^2 = \int_{-\infty}^{\infty} s^2(t) dt \quad (1.3)$$

For impulse signals, i.e., signals that exist only within a finite time interval $(0, T)$, the formulas (1.1) – (1.3) can be rewritten as:

$$\|s\| = \sqrt{\int_0^T s^2(t) dt} \quad (1.4)$$

$$\|s\| = \sqrt{\int_0^T s(t) s^*(t) dt}, \quad (1.5)$$

$$E = \|s\|^2 = \int_0^T s^2(t) dt \quad (1.6)$$

Now Consider more one fundamental the characteristics of the signals used in their vector representation – distance Between $s_1(T)$ and $s_2(T)$:

$$d[s_1(t), s_2(t)] = \sqrt{\int_0^T [s_1(t) - s_2(t)]^2 dt}. \quad (1.7)$$

It follows that the square of the distance between the signals $s_1(t)$ and $s_2(t)$:

$$d^2[s_1(t), s_2(t)] = \int_0^T [s_1(t) - s_2(t)]^2 dt. \quad (1.8)$$

Let's consider the concept of the energy of the sum of $s_1(t)$ and $s_2(t)$: two signals

$$\begin{aligned} E = \int_{-\infty}^{\infty} [s_1(t) + s_2(t)]^2 dt &= \int_{-\infty}^{\infty} s_1^2(t) dt + \int_{-\infty}^{\infty} s_2^2(t) dt + 2 \int_{-\infty}^{\infty} s_1(t) s_2(t) dt = \\ &= E_{s_1} + E_{s_2} + 2 \int_{-\infty}^{\infty} s_1(t) s_2(t) dt. \end{aligned} \quad (1.9)$$

From (1.9) should be that in Difference from themselves signals their energies are non-additive, i.e., the energy of the total signal contains the following

called mutual energy

E_{12} , B2C:

$$Es_{12} = 2 \int_{-\infty}^{\infty} s_1(t) s_2(t) dt . \tag{1.10}$$

An expression under the sign of the integral on the right side of the formula (1.10) is called the dot product of the signals $S1(t)$ and $S2(t)$:

$$(s1, s2) = \int_{-\infty}^{\infty} s1(t) s2(t) dt. \quad (1.11)$$

Dot product (s, s) signal $s(t)$ on himself equals

square of signal norm

$$(s, s) = \int_{-\infty}^{\infty} s(t) s(t) dt = \|s\|^2$$

The ratio (1.11) allows you to consider the cosine of the angle Between $S1(t)$ and $S2(t)$ $\sigma\iota\gamma\nu\alpha\lambda\sigma$ with the selected metric (1.7):

$$\cos\psi = \frac{(s1s2)}{\|s1\| \cdot \|s2\|}. \quad (1.12)$$

The angle of ψ between the two signals shall lie in the range from 0 to 180° if:

- angle $\psi = 90^\circ$, the signals $S1(t)$ and $S2(t)$ $\alpha\rho\epsilon$ orthogonal;
- angle $\psi = 180^\circ$, then the signals $S1(t)$ $\alpha\nu\delta$ $S2(t)$ $\alpha\rho\epsilon$ opposites.

Example 1. As an illustration, suppose there are two time-shifted exponential voltage pulses:

$$\begin{aligned} s1(t) &= 5\exp(-105t), \\ s2(t) &= 5\exp\left[-105\left(t - 2 \cdot 10^{-6}\right)\right]. \end{aligned}$$

Find the dot product of these signals, as well as the angle ψ between them.

The squares of the modules (energies) of these signals are the same

$$\|s1\|^2 = \|s2\|^2 = 25 \int_0^{\infty} \exp(-105t) dt = 1,25 \cdot 10^{-4} \text{ B2} \cdot \text{c}$$

The dot product calculated by formula (1.11) is:

$$(s1, s2) = 24 \int_0^{\infty} \exp(-105t) \cdot \exp[-105(t - 2 \cdot 10^{-6})] dt = 1,023 \cdot 10^{-4} \text{ B2} \cdot c$$

Hence $\cos\psi = 0.819$ and $\psi = 35^\circ$.

Equipment and materials

To perform the laboratory work, you need a personal computer with the following characteristics: a Pentium/Celeron processor with a clock speed of 300 MHz or higher, RAM of at least 128 MB or more, free disk space of at least 100 MB, a CD reader, a Super VGA monitor (number of colors – 256).

To perform the laboratory work, you need the operating system WINDOWS 2000/ XP Professional, Microsoft .NET Framework library version 1.1 or higher, Mathcad 13 and higher.

Safety Instructions

When performing laboratory work, it is prohibited:

- independently repair a personal computer, as well as install and remove existing software;
- violate generally accepted safety rules when working with electrical equipment, in particular, touch electrical outlets with metal objects, etc.;
- to eat, drink and litter at the workplace of the user of a personal computer.

In case of a malfunction of a personal computer, it is necessary to immediately inform the laboratory service personnel (system administrator, operator).

Instructions on the procedure for performing work

Let's consider the method of performing the work on the following example.

Example: Two signals, $s1(t)$ and $s2(t)$, $\alpha\rho\varepsilon\gamma\iota\varpi\varepsilon\nu$. Signal $s1(t)$ represents is a segment of a sine wave with an amplitude $U = 2$ V, which turns to zero at the ends of the time interval $[0, \tau]$, where $\tau = 1$ ms = 10^{-3} s (analytical description is represented by the formula (1.16)). Signal $s2(t)$ is a square pulse with a height of $A = 1.5$ V and a duration of $\tau = 1$ ms.

It is required for the signals $s1(t)$ and $s2(t)$ to determine:

- norms and squares of norms;
- distance and square of the distance between signals;
- dot product;
- the cosine of the angle and the angle between the signals;
- the energy of the sum and mutual energy;

Let's consider the solution to the problem in the Mathcad mathematical package:

1. Run the Mathcad math package. According to the analytical definition and the specified parameters, enter the definition of the first signal and plot it (Figure 1.1).

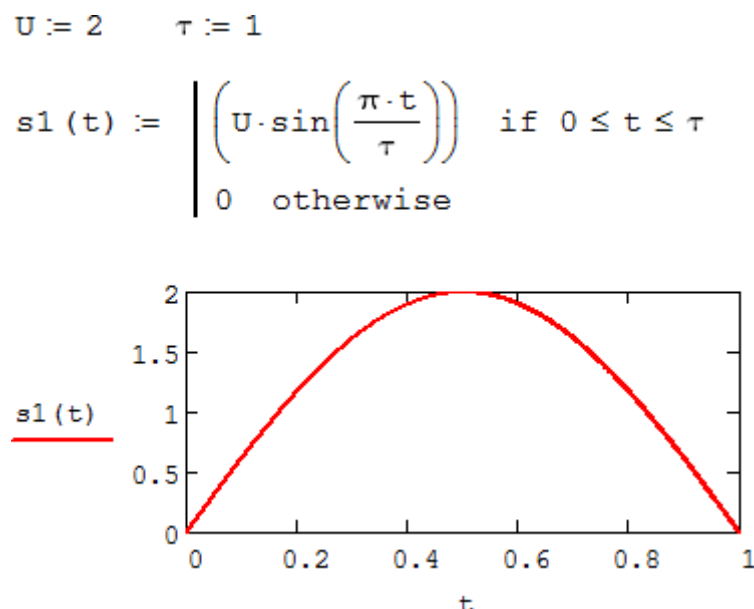


Figure 1.1 – Definition and graph of the $s1(t)$ signal The

duration of the first signal is 1 ms ($TS1 = 1$ ms).

2. According to the specified parameters, we enter the definition of the second signal

$s_2(t) = 1.5, \quad 0 \leq t \leq \tau$ and build its graph (Figure 1.2).

$$A := 1.5$$

$$s_2(t) := \begin{cases} A & \text{if } 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases}$$

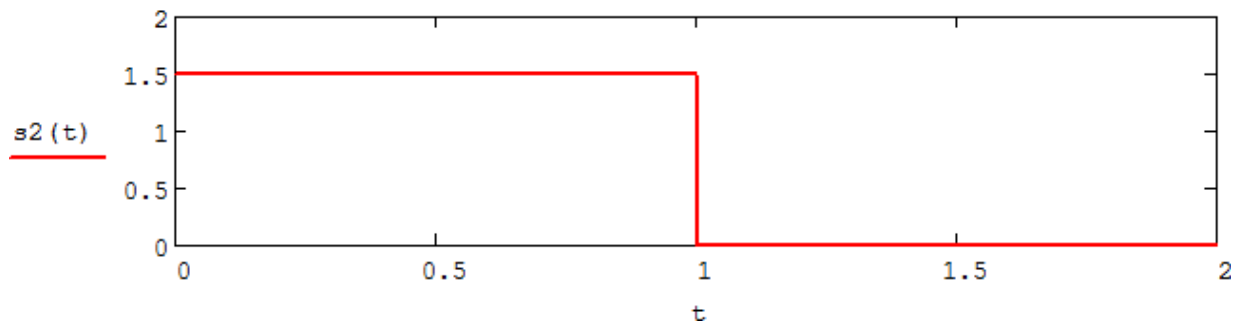


Figure 1.2 – Definition and graph of signal $s_2(t)$ The

duration of the second signal is 1 ms ($TS_2 = 1$ ms).

Note – Since *the* $s_2(t)$ signal has the same duration as *the* $s_1(t)$ signal, you do not need to *redefine* the τ parameter in Mathcad.

3. Using formula (1.4), we determine the norms and squares of the signal norms $s_1(t)$ and

$s_2(t)$ (Figure 1.3).

$$||s_1|| := \sqrt{\int_0^{\tau} s_1(t)^2 dt} = 1.414 \quad ||s_2|| := \sqrt{\int_0^{\tau} s_2(t)^2 dt} = 1.5$$

$$||s_1||^2 = 2$$

$$||s_2||^2 = 2.25$$

Figure 1.3 – Determination of norms and squares of signal norms

4. Using formula (1.7), we determine the distance and the square of the distance between

$s_1(t)$ and $s_2(t)$ (Figure 1.4).

$$ds1_s2 := \sqrt{\int_0^T (s1(t) - s2(t))^2 dt} = 0.656$$

$$ds1_s2^2 = 0.43$$

Figure 1.4 – Determination of the distance and square of the distance between signals

5. Using formula (1.1), we find the dot product of the signals $s1(t)$ and $s2(t)$ (Figure 1.5).

$$s1_s2 := \int_0^T s1(t) \cdot s2(t) dt = 1.91$$

Figure 1.5 – Determination of the dot product of signals

6. Using formula (1.12), we calculate the cosine of the angle and the angle between the signals

$s1(t)$ and $s2(t)$ (Figure 1.6).

$$\cos\psi := \frac{s1_s2}{||s1|| \cdot ||s2||} = 0.9 \quad \psi := \frac{180}{\pi} \arccos(\cos\psi) = 25.8$$

Figure 1.6 – Determination of the distance and square of the distance between signals

Note – Coefficient $180/\pi$ in the formula when calculating the angle between signals $s1(t)$ and $s2(t)$ Figure 1.6 allows you to go from result in radians to degrees.

7. Using formulas (1.9) and (1.10), we determine the energy of the sum and the mutual

The energy of the two signals $s1(t)$ and $s2(t)$ (Figure 1.7).

$$E12 := \int_0^T (s1(t) + s2(t))^2 dt = 8.07$$

$$Es1s2 := 2 \cdot \int_0^T s1(t) \cdot s2(t) dt = 3.82$$

Figure 1.7 – Determination of the energy of the sum and mutual energy of the two signals

Procedure for performing work

1. Run Mathcad.
2. It is required for the signals $s_1(t)$ and $s_2(t)$ to determine:
 - norms and squares of norms;
 - distance and square of the distance between signals;
 - dot product;
 - the cosine of the angle and the angle between the signals;
 - the energy of the sum and mutual energy;

Investigate the effect of changes in the parameters of the $s_1(t)$ and $s_2(t)$ on the

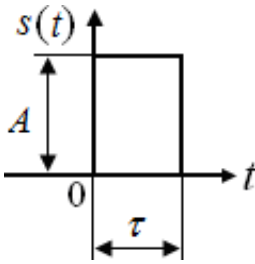
characteristics used in their vector representation.

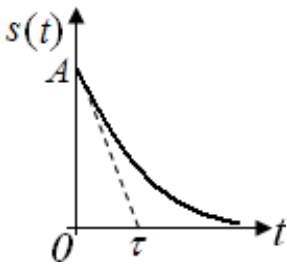
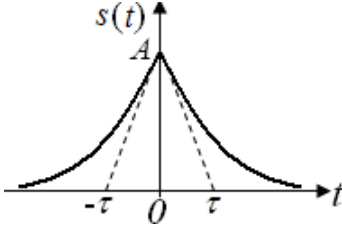
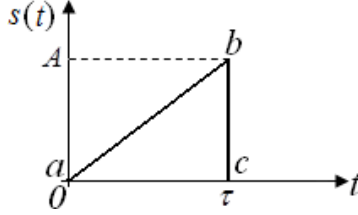
3. Save the results of an individual task.
4. Generate a report on the laboratory work.
5. Protect the work.

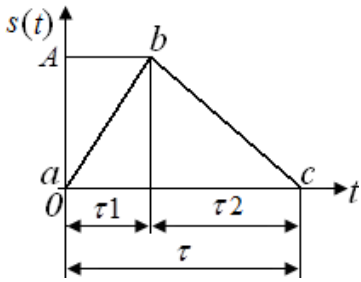
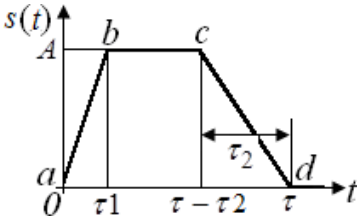
Variants of tasks

The number of the variant corresponds to the number of the student's surname in the list of the study group (subgroup).

Table 1.1 – Name, graphical and analytical definition of signals studied in laboratory work

Name	Graphic Definition	Mathematical model in Mathcad and Parameters
Square Pulse		$s(t) := \begin{cases} A & \text{if } 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases}$ Pulse parameters: A is the height of the rectangular impulse; τ is the duration of the square pulse. <i>Note</i> – Keyword "otherwise" is used in the context to specify the value of the signal "in all other cases"

One-way Exponential Momentum		$s(t) := \begin{cases} A \cdot e^{\frac{-t}{\tau}} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$ Parameters of imuls: A is the height of the one-sided exponential momentum; τ is the time constant one-sided exponential momentum
Bilateral Exponential Momentum		$s(t) := A \cdot e^{\frac{- t }{\tau}}$ Pulse parameters: A is the height of the two-sided exponential momentum; τ is the time constant Bilateral Exponential momentum
Sawtooth Pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau} \right) & \text{if } 0 \leq t \leq \tau \\ 0 & \text{if } t \geq \tau \end{cases}$ Pulse parameters: A is the height of the sawtooth pulse; τ is the duration of the sawtooth pulse. <i>Note</i> – The segment ab is called the forward flow of the sawtooth momentum, the segment bc is called the reverse movement of the Sawtooth Impulse House

Triangular Pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau_1} \right) & \text{if } 0 \leq t \leq \tau_1 \\ \left[A \cdot \left(1 - \frac{t - \tau_1}{\tau_2} \right) \right] & \text{if } \tau_1 \leq t \leq \tau \\ 0 & \text{if } t > \tau \end{cases}$ Pulse parameters: A is the height of the triangular pulse; τ_1 and τ_2 are the duration of the edge and the duration of the triangular pulse cut; $\tau = \tau_1 + \tau_2$ is the duration of the triangular pulse. <i>Note</i> – The segment ab is called the front, and the segment bc is called the treb- Carbon Pulse
Trapezoidal pulse		$s(t) := \begin{cases} 0 & \text{if } t < 0 \\ \left(A \cdot \frac{t}{\tau_1} \right) & \text{if } 0 \leq t \leq \tau_1 \\ A & \text{if } \tau_1 \leq t \leq \tau - \tau_2 \\ \left[A \cdot \left(1 - \frac{t - \tau + \tau_2}{\tau_2} \right) \right] & \text{if } \tau - \tau_2 \leq t \leq \tau \\ 0 & \text{if } t > \tau \end{cases}$ Pulse parameters: A is the height of the trapezoidal pulse; τ is the duration of the trapezoidal pulse; τ_1 is the duration of the trapezoidal pulse edge; τ_2 is the duration of the trapezoidal pulse cutoff. <i>Note</i> – The segment ab is called the front of the trapezoidal pulse, the segment bc is the apex of the trapezoidal pulse, the segment cd is the slice of the trapezoidal pulse. pulse

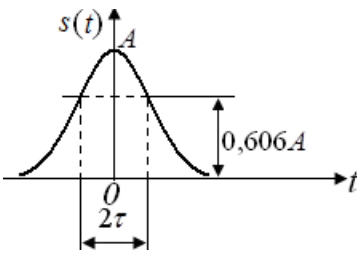
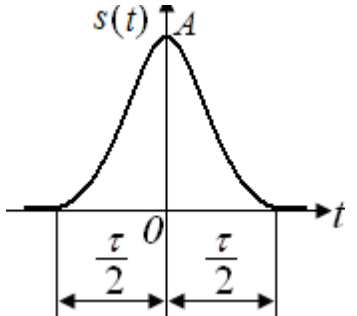
Bell Pulse		$s(t) = A \cdot e^{\frac{-1}{2} \cdot \left(\frac{t}{\tau}\right)^2}$ Pulse parameters: A is the height of the bell-shaped pulse; 2τ is the time interval between the inflection points of the bell-shaped pulse. Note – The value of the parameter 2τK is also determined by the level of 0.606AK.
Cosine-square momentum		$s(t) = \begin{cases} A \cdot \cos^2\left(\frac{\pi}{\tau} \cdot t\right) & \text{if } -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0 & \text{if } t > \frac{\tau}{2} \end{cases}$ Pulse parameters: A is the height of the cosine-square impulse; τ is the duration of the cosine-square pulse. Note – Value of the τ/2 parameter is also determined by the level of 0.5A

Table 1.2 – Variants of individual tasks

Var	Designation signal	Signal type	Pulse Parameters signals
1	$s1(t)$	Trapezoidal pulse	A = 2 V, τ = 3 ms, τ1 = 1 ms, τ2 = 1 ms
	$s2(t)$	Cosine-square momentum	A = 2 V, τ = 6 ms
2	$s1(t)$	Two-way exponential Momentum	A = 2 V, τ = 2 ms
	$s2(t)$	Sawtooth Pulse	A = 2 V, τ = 2 ms
3	$s1(t)$	Triangular Pulse	A = 2 V, τ1 = 1 ms, τ2 = 1 ms
	$s2(t)$	Bell Pulse	A = 2 V, τ = 2 ms
4	$s1(t)$	One-way exponential Momentum	A = 2 V, τ = 2 ms
	$s2(t)$	Cosine-square momentum	A = 2 V, τ = 6 ms
5	$s1(t)$	Sawtooth Pulse	A = 2 V, τ = 5 ms

	$s2(t)$	Triangular Pulse	$A = 2 \text{ V}, \tau1 = 2 \text{ ms}, \tau2 = 3 \text{ ms}$
6	$s1(t)$	Two-way exponential Momentum	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
	$s2(t)$	Cosine-square momentum	$A = 2 \text{ V}, \tau = 8 \text{ ms}$
7	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}, \tau1 = 1 \text{ ms}, \tau2 = 1 \text{ ms}$
	$s2(t)$	Triangular Pulse	$A = 4 \text{ V}, \tau1 = 1 \text{ ms}, \tau2 = 3 \text{ ms}$
8	$s1(t)$	One-way exponential Momentum	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
	$s2(t)$	Sawtooth Pulse	$A = 3 \text{ V}, \tau = 6 \text{ ms}$
9	$s1(t)$	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
10	$s1(t)$	One-way exponential Momentum	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
11	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 5 \text{ ms}, \tau1 = 1 \text{ ms}, \tau2 = 1 \text{ ms}$
	$s2(t)$	Bell Pulse	$A = 3 \text{ V}, \tau = 2 \text{ ms}$
12	$s1(t)$	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
	$s2(t)$	Triangular Pulse	$A = 2 \text{ V}, \tau1 = 2 \text{ ms}, \tau2 = 2 \text{ ms}$
13	$s1(t)$	Trapezoidal pulse	$A = 4 \text{ V}, \tau = 5 \text{ ms}, \tau1 = 2 \text{ ms}, \tau2 = 2 \text{ ms}$
	$s2(t)$	Two-way exponential Momentum	$A = 4 \text{ V}, \tau = 3 \text{ ms}$
14	$s1(t)$	Triangular Pulse	$A = 4 \text{ V}, \tau1 = 1 \text{ ms}, \tau2 = 3 \text{ ms}$
	$s2(t)$	Cosine-square momentum	$A = 4 \text{ V}, \tau = 8 \text{ ms}$
15	$s1(t)$	Trapezoidal pulse	$A = 2 \text{ V}, \tau = 5 \text{ ms}, \tau1 = 2 \text{ ms}, \tau2 = 2 \text{ ms}$
	$s2(t)$	One-way exponential Momentum	$A = 2 \text{ V}, \tau = 2 \text{ ms}$

Note – Graphical and analytical determination of the parameters of the signals under study is given in Table 1.1.

Contents of the report

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1. Name of the laboratory work.
2. The purpose of laboratory work.
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4. Brief conclusions based on the results of laboratory work.

Control questions

1. How to determine the norm of a real signal?
2. How to determine the norm of a complex signal?
3. What does the square of the signal norm coincide with?
4. How to determine the distance between signals?
5. How to determine the energy of the sum of two signals?
6. How to determine the mutual energy of two signals?
7. How to check the correctness of the determination of the energy of the sum of the two signals in Figure 1.7?
8. How do you determine the dot product of two signals?
9. How to determine the cosine of the angle between signals?
10. What is the energy and pulse rate of a cosine shape:

$$s(t) = \begin{cases} 0, & \omega_0 t < -\pi/2; \\ U \cos \omega_0 t, & -\pi/2 \leq \omega_0 t \leq \pi/2; \\ 0, & \omega_0 t > \pi/2. \end{cases}$$

11. How should the height of a square pulse be related?
 A and the amplitude U of the sine wave segment (example 2) so that between the signals $s_1(t)$ and $s_2(t)$ the minimum distance been reached?

12. The $u(t)$ signal is a symmetrical triangular pulse with a height of U and a duration of τ . The signal $v(t)$ is a square-shaped pulse with a height A inscribed in

the $u(t)$ signal (Figure 1.8).

be the amplitude of the square pulse so that the distance between these two signals is minimal?

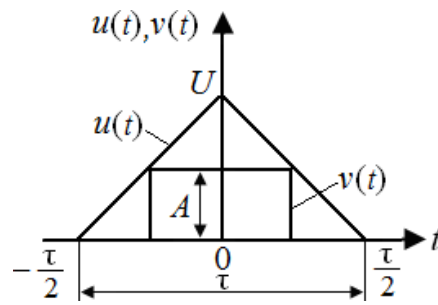


Figure 1.8 – Time diagrams of $u(t)$ and $v(t)$ signals

Practical work 3

Continuous Signal Sampling Study

The aim of the work is to study the basics of discretization and recovery of continuous signals according to V. A. Kotelnikov's theorem.

Theoretical part

At present, all radio-electronic systems, including telephony, broadcasting and television systems, are switching to a digital mode of operation. The conversion of various continuous (analog) signals for their processing in digital form is based on the procedure

discretization. In this regard, the following concepts are introduced:

- A continuous (analog) signal is a signal $x(t)$ Which one is continuous both in time and in meaning;

- discrete signal $x(k\Delta t_d)$, where $k = 0, \pm 1, \pm 2, \pm 3, \dots$ is the signal

Which one It turns out in Result sampling values analog $x(t)$;

- Sampling is the determination of analog signal values at discrete points in time.

An example of an analog signal sampled is shown in Figure 2.1. Note that after the sampling shown in Figure 2.1, the analog signal is represented only at discrete times, and the signal values at these times are equal to the corresponding values of the original analog signal.

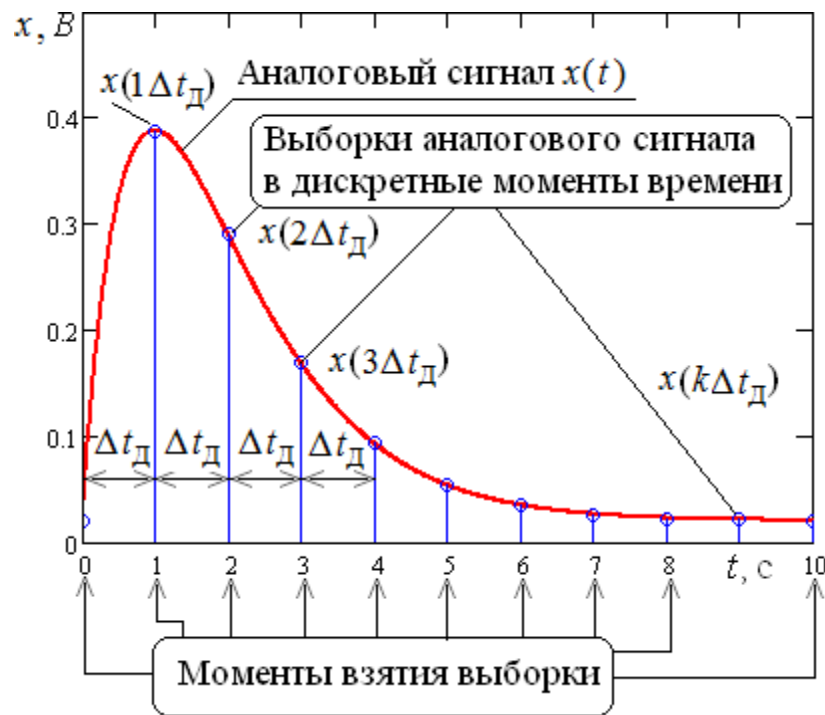


Figure 2.1 – Example of an analog signal sampled

As a rule, when sampling, samples of the analog signal are taken

at regular intervals ΔT (Figure 2.1), called the period D

sampling time (or by interval, sampling step - sample time).

Magnitude, Reverse period sampling, is called frequency sampling (sampling frequency): $f_D = 1/\Delta t_D$. Relevant she circular frequency is defined as $\omega_L = 2\pi/\Delta t_D$.

Thus, a discrete signal is continuous in amplitude, but is only detected at discrete points in time.

However, there is a class of analog signals for which such a loss of information does not occur and which can be accurately reconstructed according to the values of their discrete samples. This class includes signals that obey the theorem formulated in 1933 by the outstanding Russian scientist Vladimir Alexandrovich

Kotelnikov: "Any Signal $s(t)$, Spectrum which not Contains

components with frequencies above some upper cut-off frequency fB (

$\omega_B = 2\pi fB$) can be represented without loss of information by its discrete samples $\{s(k\Delta t_d)\}$ taken at the interval Δt_d satisfying the following inequality:

$$\Delta t_d \leq \frac{1}{2fB} = \frac{\pi}{\omega_B}. \quad (2.1)$$

Note – In foreign sources, this theorem is called Nyquist theorem, as well as Shannon theorem or sampling theorem.

Thus, if fB is the highest frequency component of the signal, then in order for the sampling elements to fully describe the signal, the signal must be sampled at a frequency of at least $2fB$, i.e.

$$fD \geq 2 \quad (2.2)$$

Obviously, an increase in the sample rate of the signal leads to an increase in the speed of message creation, which causes an increase in the required bandwidth of the communication channel. Therefore, the problem of choosing a sample rate (sample period) that provides a minimum of speed of message generation given recovery error

of the original continuous signal $x(t)$ from its set of discrete samples

$\{x(k\Delta t_d)\}$. Restoration of the original continuous signal $x(t)$ by dialing it

discrete readings $\{x(k\Delta t_d)\}$ is made according to the formula:

$$x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta t_d) \frac{\sin 2\pi F_B(t - k\Delta t_d)}{2\pi F_B(t - k\Delta t_d)} \quad (2.3)$$

Formula (2.2) is a series decomposition of the $s(t)$ signal by a system of functions $\{\psi_k(t)\}$ called the Kotelnikov basis:

$$\psi_k(t) = \frac{\sin 2\pi FB(t - k\Delta t_d)}{2\pi FB(t - k\Delta t_d)}. \quad (2.4)$$

The formation of a continuous signal from its discrete samples is explained in Figure 2.2. The dotted line shows the graphs of the individual components of formula (2.3), the solid line shows the restored analog signal.

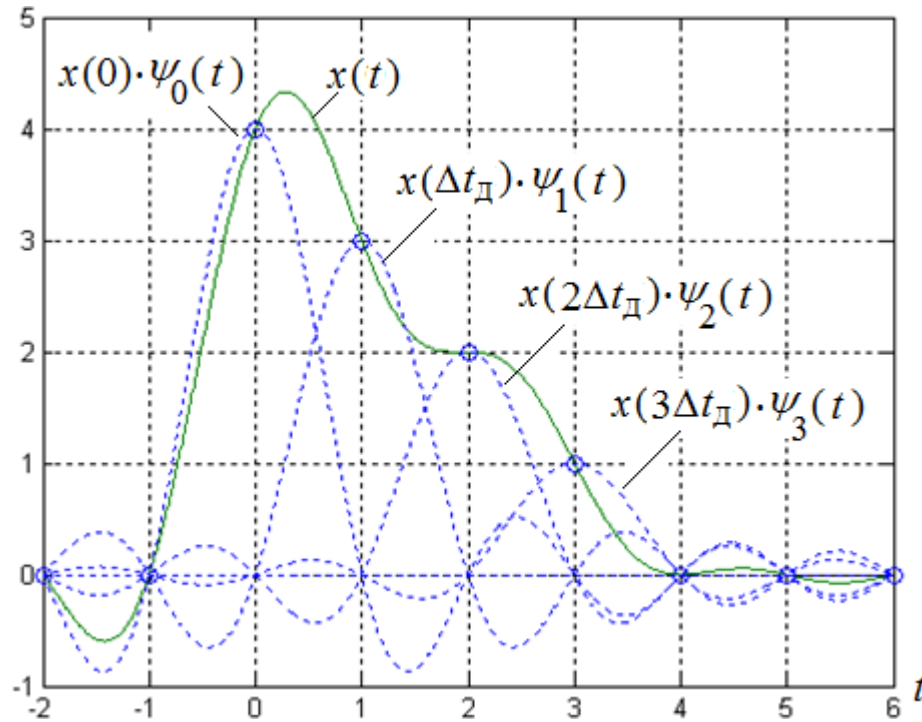


Figure 2.2 – Reconstruction of an analog signal from its discrete samples

Figure 2.2 clearly demonstrates the main property of a signal with a limited spectrum – its infinity in time. Although only a few samples of the signal shown are non-zero, the analog signal turns out to be infinitely oscillating between zero samples (in the figure these are counts with the numbers -2, -1, 4, 5, 6) its values are different from zero.

These oscillations do not end anywhere, although their amplitude tends to zero.

Thus, in order to fully restore the continuous signal $x(t)$

From the set of its discrete references, $\{x(k\Delta t_D) \mid k \in \mathbb{Z}\}$ to be summed up

Infinite A lot of Members range (2.3). However, if Function
with a limited spectrum $x(t)$ Under consideration
on final Interval

Time T (Figure 2.3), the exact expansion (2.3) can be replaced by the following approximate expansion:

$$x_{\varepsilon}(t) = \sum_{k=0}^{n-1} x(k\Delta t_d) \cdot \frac{\sin 2\pi FB(t - k\Delta t_d)}{2\pi FB(t - k\Delta t_d)} = \sum_{k=0}^{n-1} x(k\Delta t_d) \cdot \Psi_k(t), \quad (2.5)$$

where n – finite number of discrete samples, units, continuous signal

$x(t)$, which identifies the recovered signal $x_{\varepsilon}(t)$ with $\Delta t_d = \frac{1}{2fB}$.

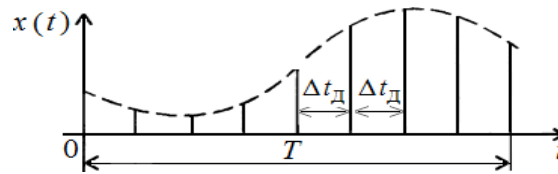


Figure 2.3 – A function with a finite spectrum $x(t)$ is considered at a finite time interval T

Ultimate Number discrete counts n , units, Stackable at a finite time interval T is calculated using the formula

$$n = \frac{T}{\Delta t_d} + 1 = 2fBT + 1. \quad (2.6)$$

The parameter $B = 2fBT$ in formula (2.6) is called the signal base. Obviously, that the error of signal recovery when the number of its samples n is limited will be the greater, the smaller the number of terms is taken into account when summing up in formula (2.5).

Let's evaluate the error qualitatively $|\varepsilon(t)| = |x_{\varepsilon}(t) - x(t)|$. Since all The terms of the series (2.5) are inverted when $t = i\Delta t_d$ to zero at all points, beyond with the exception of the term with the number $k = i$, then in these sections the values of coincide c $x(t)$, t. e. Accuracy $\varepsilon(k\Delta t_d)$ is equal to zero; obviously,

The error will reach the highest value within the interval between samples. In addition, the error value increases towards the edges of the considered time interval T (Figure 2.4). Thus,

We conclude that the first reason for the error in the recovery of a function with a limited spectrum $x(t)$ from the aggregate of its discrete samples) determined at the finite time interval T (formula (2.5)) is that the number of such discrete samples is finite.

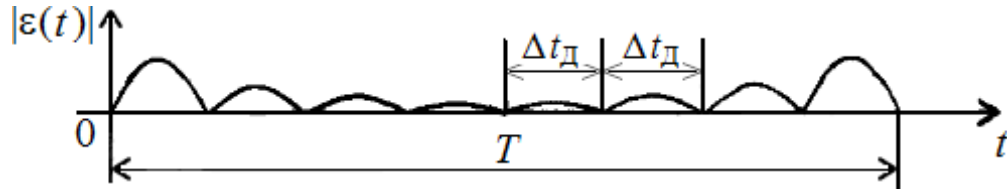


Figure 2.4 – Change in error $\varepsilon(t) = |x_{\varepsilon}(t) - x(t)|$ within time interval T

Another reason for the errors is due to the fact that the spectra of real continuous (analog) signals do not go to zero beyond the upper limit frequency of the analog signal f_B , Hz. Although the main

signal energy is located at frequencies from $-f_B$ to f_B , some part of the energy falls on frequencies above the boundary. Taking this into account, the relative standard error of the restoration of the initial

Continuous signal $x(t)$ by a set of its discrete samples $\{x(k\Delta t_d)\}$

is determined by the ratio

$$\sigma_{\varepsilon}^2 = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df - \int_{-f_B}^{f_B} |S(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df} \cdot 100 = \frac{\Delta E}{E} \cdot 100 = \frac{\Delta E}{\|x\|^2} \cdot 100, \quad (2.7)$$

where ΔE is the part of the energy that is outside the band

frequencies from $-f_B$ to f_B and is not taken into account when restoring the signal, B2c;

E is the total signal energy $x(t)$, B2c;

$\|x\|^2$ is the square of the signal norm $x(t)$, B2s.

Thus, given an error of (2.7), it is possible to determine the required cut-off frequency f_B , and hence the intervals between

$$\text{countdowns (period discretization)} = \frac{\Delta T D}{2fB}. \quad \text{Detailed Research}$$

shows that the errors due to the unaccounted part of the signal spectrum will be greater the slower the signal spectrum decreases beyond the fB boundary frequency.

The third reason for the errors in the reconstruction of the function with a limited spectrum $x(t)$ from the aggregate of its discrete samples) determined at the finite time interval T are the non-ideal characteristics of the low-pass filter (LPF) that forms the functions

$\{\psi_k(t)\}$, which form the Kotelnikov basis on the receiving end of the communication channel.

As it is known from the lecture course, oscillations in the form of the Kotelnikov basic function of the form (2.4) can be obtained at the output of an ideal LPF with a boundary frequency fB under the action of a delta pulse $\delta(t)$ on its input. The amplitude-frequency response (AF) of an ideal LPF is uniform (Figure 2.5):

$$H(\omega) = \begin{cases} \underline{H}(\omega) & |1, 0 \leq f \leq fB; \\ 0, & f > fB \end{cases}$$

where $\underline{H}(\omega)$ – complex transfer function of the ideal LPF.

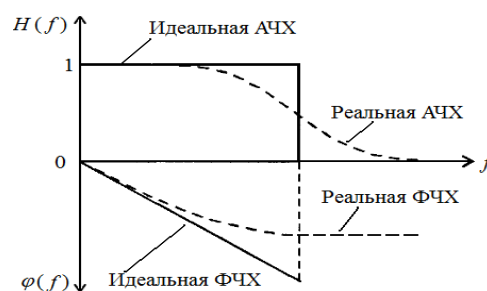


Figure 2.5 – Ideal and real frequency characteristics of the LPF forming the

$\phi\upsilon\nu\chi\tau\iota\omicron\nu\sigma \{\psi_k(t)\}$ $\tau\eta\alpha\tau$ form the Kotelnikov basis on the receiving side of the communication channel

In your queue, phase-frequency Characteristics (CFH) – Linear (Figure 2.5):

$$\varphi(\omega) = \text{Arg}\{H(\omega)\} = -\omega\Delta t,$$

where $\Delta t = \frac{1}{2fB}$ is the time (delay) constant of the ideal LPF.

As is known from the course on the theory of electrical and radio circuits, the impulse response of a filter $h\delta(t)$ can be determined on the basis of Reverse integral Conversions Fourier from Comprehensive Transfer function $H(\omega)$ of the filter:

$$h_{\delta}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega.$$

For considered Case ideal LPF Filter impulse response $h\delta(t)$ is determined by the ratio of the type

$$h_{\delta}(t) = \frac{1}{2\pi} \int_{-fB}^{fB} e^{j\omega t} d\omega = \frac{1}{2\pi} \cdot 2fB \frac{\sin 2\pi fB(t - \Delta t)}{2\pi f(t - \Delta t)} = \frac{fB}{\pi} \frac{\sin 2\pi fB(t - \Delta t)}{2\pi f(t - \Delta t)}.$$

Characteristics of real filters $H(\omega)$ and $\varphi(\omega)$ (dashed curves in Figure 2.5) used on the receiving side of the communication channel differ from the ideal ones (solid curves in Figure 2.5). This results in deviations of real functions $\{\psi_k(t)\}$ (dotted curve in Figure 2.6), forming the Kotelnikov basis on the receiving side of the communication channel, from ideal (solid curve in Figure 2.6) and, as a result, to the appearance of additional errors in the recovery of the $x(t)$ signal by dialing it discrete samples $\{x(k\Delta t_d)\}$, $k = 0, 1, 2, \dots, n-1$ defined on finite time interval T .



Figure 2.6 – Ideal and real basic functions of Kotelnikov

Thus, in order to apply V. A. Kotelnikov's theorem in practice, it is necessary to determine the upper limit frequency of the analog signal fB , Hz. The method for determining the upper limit frequency of the fB signal using two approaches (information and energy) was considered at the first laboratory session "Studies of Numerical Parameters of Signals".

Let us use the results of the example considered at the first laboratory lesson related to the analysis of the numerical parameters of a signal representing a square pulse with a height A and duration τ and consider the method of its sampling and reconstruction according to the theorem of V. A. Kotelnikov.

Equipment and materials

Equipment. To perform laboratory work, you need a personal computer with the following characteristics: a Pentium/Celeron processor with a clock speed of 300 MHz and higher, RAM of at least 128 MB or more, free disk space of at least 100 MB, a CD reader, a monitor of the Super VGA type (number of colors – 256).

Software. To perform the laboratory work, you need the operating system WINDOWS 2000/ XP Professional, the Microsoft .NET Framework library version 1.1 or higher, the program MathCAD 10 and higher.

Safety Instructions

When performing laboratory work, **it is prohibited:**

- independently repair a personal computer, as well as install and remove existing software;
- violate generally accepted safety rules when working with electrical equipment, in particular, touch electrical outlets with metal objects, etc.;
- eat and drink and litter at the workplace of a personal computer user.

In the event of a malfunction of a personal computer, it is necessary to **immediately** inform the laboratory service personnel (system administrator, operator).

Instructions on the procedure for performing work

Let's consider the method of performing the work on the following example.

A continuous signal $s(t)$ is given, which is a rectangular pulse with a height $A = 1$ V and a duration of $\tau = 1$ ms = 10^{-3} s.

$$s(t) = \begin{cases} A, & 0 \leq t \leq \tau; \\ 0, & t < 0 \text{ and } t > \tau. \end{cases} \quad (2.8)$$

It is necessary to determine:

– the upper limit frequency of the spectrum f_V , Hz, and the signal $s(t)$, base using the energy approach;

– Period Δt_d , s, and frequency f_D , Hz, Continuous Sampling $s(t)$, satisfying the theorem of V. A. Kotelnikov;

– n is the number of samples of the discrete signal ($k = 0, 1, 2, \dots, n - 1$);

– Discrete Signal $s(k\Delta t_d)$, Obtained in Result

Continuous S $\square$ T $\square$ Signal Sampling

– Features $\{\psi_k(t)\}$, Components Basis Kotelnikova with Recovery original Continuous signal $s(t)$ by Set his discrete readings $\{s(k\Delta t_d)\}$ (plot for the first few

basic functions, for example, for $k = 0, 1, 2, \dots$);

– The first few terms of the sum sign on the right side of the formula (2.3) are the product of *the k th discrete*

countdowns $s(k\Delta t_d)$ on k Basic function $\psi_k(t)$ (build graphs, for example, for $k = 0, 1, 2, \dots$) and the result of their summation;

– function $s\varepsilon(t)$, representing Result Recovery of the original continuous signal $s(t)$ from its set of discrete samples

$\{s(k\Delta t_d)\}$ (plot and plot in the same reference grid

Plot of the original continuous signal $S(t)$;

– function $\varepsilon(t) = s\varepsilon(t) - s(t)$, representing Module Differences between the signals $s\varepsilon(t)$ and $s(t)$ (plot the graph);

– Square of the distance between the signals $S\varepsilon(t)$ and $S(t)$;

– The square of the distance between the signals $S\varepsilon(t)$ and $S(t)$ expressed as

A percentage of the square of the signal norm $s(t)$;

In addition, it is necessary to investigate how the change in the upper the boundary frequency of the spectrum fB , Hz, the signal $s(t)$ per square of the distance between

signals $s\varepsilon(t)$ and $s(t)$ expressed as a percentage relative to square the signal norm $s(t)$.

Let's consider the solution to the problem in the Mathcad mathematical package:

1. Run the math package Mathcad. According to (2.8) and the specified parameters we introduce the mathematical definition of the continuous signal $s(t)$ and plot it (Figure 2.7).

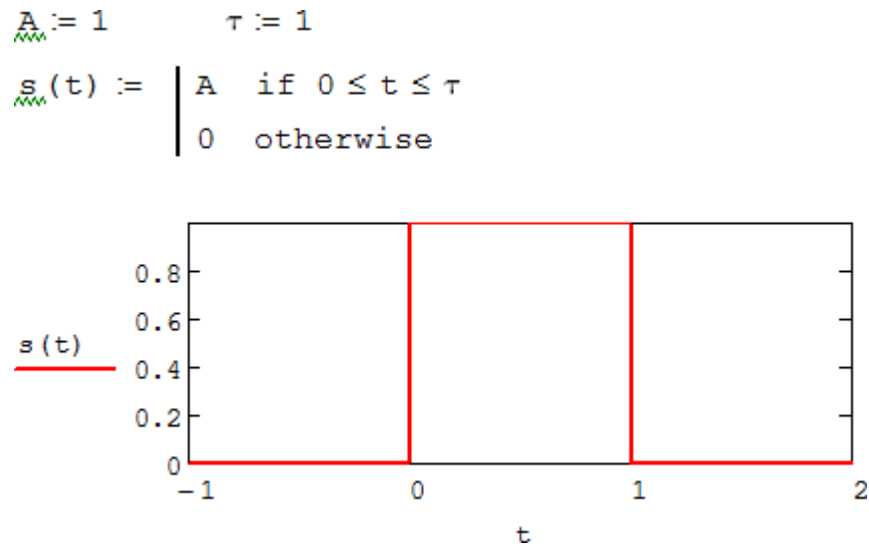


Figure 2.7 – Definition and graph of a continuous signal $s(t)$

2. Using the method considered in the laboratory work, we determine upper boundary frequency spectrum fB signal $s(t)$ on basis energy approach, assuming the signal energy reduction factor due to the artificial limitation of its spectrum width KE to 0.903:

2.1 Using formula (1.30), we find the spectral function of the signal $s(t)$ (Figure 2.8);

$$j := \sqrt{-1}$$

$$S(f) := \int_0^{\tau} s(t) \cdot e^{-j \cdot 2 \cdot \pi \cdot f \cdot t} dt$$

Figure 2.8 – Determination of the spectral function of the signal $s(t)$

2.2 Using the results of determining the upper effective boundary frequency of the signal spectrum obtained in the first laboratory work, we will perform the following steps:

- let us give the initial approximation of the parameter $fB = 1$ kHz ($fb := 1$);
- write down in the Given block the equation relative to the desired of the spectrum cut-off frequency fb , assuming the coefficient of signal energy reduction due to the artificial limitation of its spectrum width to the range from $-fb$ to fb equal to 0.903 (Figure 2.9).

$$fb := 1$$

Given

$$\frac{\int_{-fb}^{fb} (|s(f)|)^2 df}{\int_0^T (|s(t)|)^2 dt} = 0.903$$

$fb := \text{Find}(fb) \quad fb = 1$

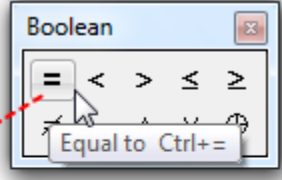
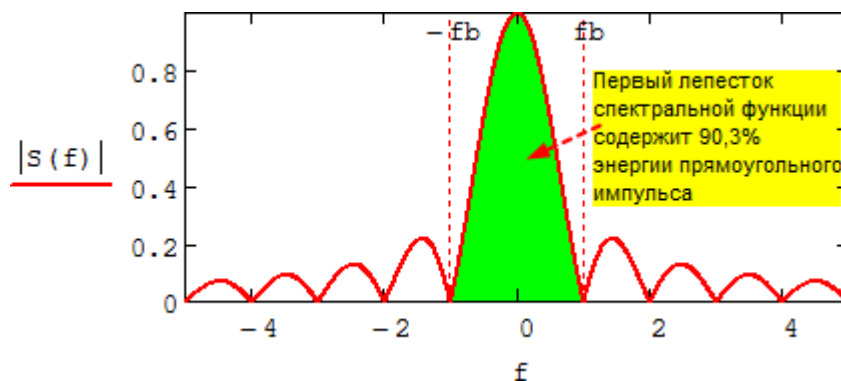


Figure 2.9 – Determination of the boundary frequency of the signal $s(t)$ spectrum

Coming K conclusion, that First Petal spectral function contains 90.3% of the energy of a square pulse (Figure



2.10).

Figure 2.10 – Boundary frequencies of the spectrum of a square pulse

Notes:

- A system of linear and nonlinear equations and inequalities allows you to solve the Given block in combination with the Find function in Mathcad. Attention! The Given block contains a system of equations and/or inequalities to be solved.
- The system of equations and/or inequalities should be written after or to the right of the word Given.
- When writing equations, instead of the = sign, you should type Ctrl+=
- The word Given must be preceded by initial approximations for all variables.
- The Given block is not suitable for searching for indexed variables.
- A sign of the end of a system of equations is the Find function if you want to find an exact solution to the system, or the Minerr function if the system cannot be solved exactly, and you want to find the best approximation that provides the minimum error.
- The Minerr and Find functions must have as many or fewer arguments as the equations and inequalities contained in the given block. If it turns out that the block contains too few equations or inequalities, then it can be supplemented with identities or repeated expressions.
- In the event that a solution cannot be found with the initial approximation selected, a message will appear in the red box Did not find solution.

3. In accordance with Kotelnikov's theorem, we calculate using the formula (2.2)

Sample rate f_D , kHz, and sample period Δt_d , ms, signal $s(t)$ (Figure 2.11).

$$f_D := 2 \cdot f_b = 2 \quad \Delta t_D := \frac{1}{f_D} = 0.5$$

Figure 2.11 – Calculating the sample rate and sample period

Signal $S(t)$

How to See of settlements, of the above on Figure 2.11, Frequency

sampling f_D signal $s(t)$, satisfying Kotelnikov's theorem, is 2 kHz and the sampling period Δt_d is 0.5 ms, respectively.

4. Using formula (2.6), we determine the number of samples of a discrete signal (Figure 2.12).

$$n := \text{floor}(2 \cdot f_b \cdot \tau) + 1 = 3$$

Figure 2.12 – Determination of the number of samples of a discrete signal

Note – Mathcad's built-in floor(z) function returns an integer less than or equal to z. For example, floor(3.364) = 2.

5. Form a discrete reference vector $\{s(k\Delta t_d)\}$ ($k = 0, 1, 2$) continuous signal $s(t)$ and build a time diagram of the discrete signal (Figure 2.13).

$$k := 0 \dots n-1 \quad t_k := k \cdot \Delta t_d \quad s_{dis_k} := s(t_k) \quad t = \begin{pmatrix} 0 \\ 0.5 \\ 1 \end{pmatrix} \quad s_{dis} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

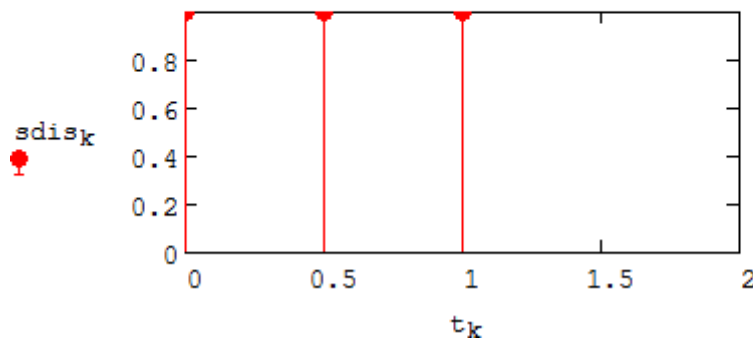


Figure 2.13 – Discrete reference vector and discrete signal time diagram

6. Using formula (2.4), we determine the functions $\{\psi_k(t)\}$ that make up the basis

Kotelnikov in the restoration of the original continuous signal $s(t)$

set of its discrete samples $\{s(k\Delta t_d)\}$ (Figure 2.14).

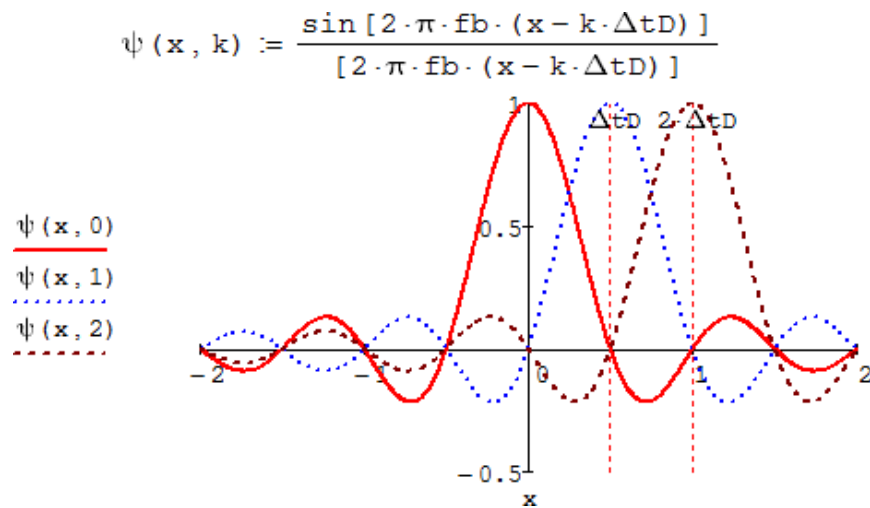


Figure 2.14 – Definition of the functions $\{\psi_k(t)\}$ that make up the basis Kotelnikov when reconstructing the original continuous signal $s(t)$ from a set of its discrete samples $\{s(k\Delta t_d)\}$

7. Define the three terms that are under the sign of the sum in the right Parts of Formula (2.5): Product k -th discrete reference $s(k\Delta t_d)$ on k -yu the basis function $\psi_k(t)$ ($k = 0, 1, 2$) and the result of their summation (Figure 2.15).

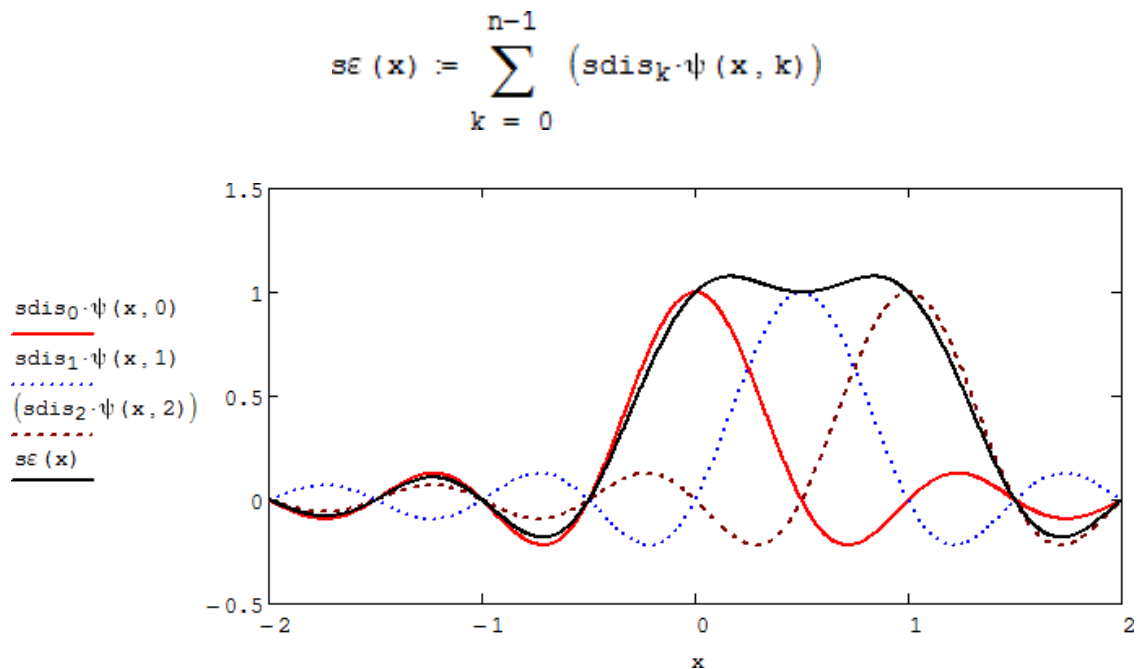


Figure 2.15 – Definition of the first few terms under the sign of the sum in the right part of the formula (2.5) and the result of their summation

8. Using formula (2.5), we determine the function $s\varepsilon(t)$, representing the result of restoring the original continuous signal $s(t)$ over its three discrete readings $\{s(k\Delta t_d)\}$, plot it and plot the original A continuous signal $s(t)$ (Figure 2.16).

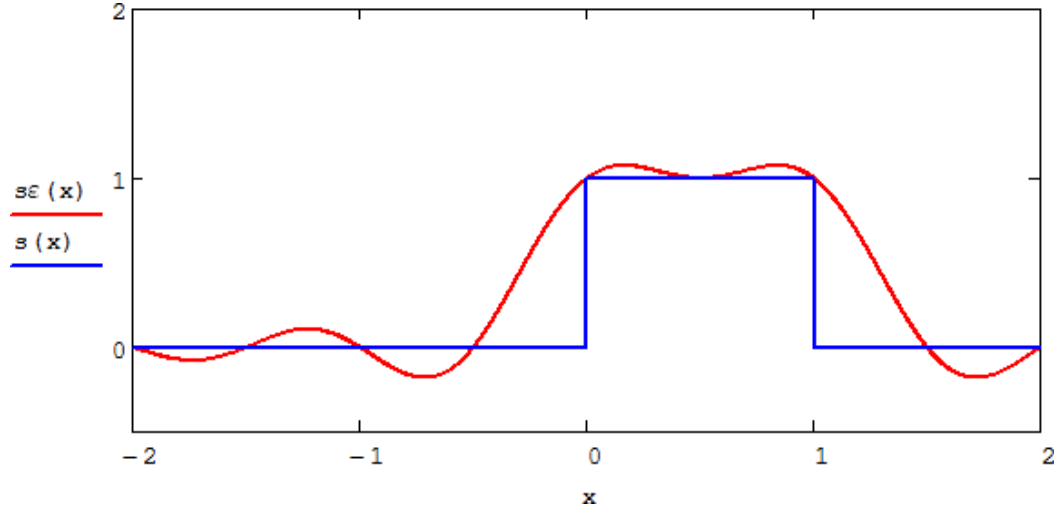


Figure 2.16 – Graphs of the function $s\varepsilon(t)$ representing the result recovery of the original continuous signal $s(t)$ by its three discrete samples $\{s(k\Delta t_d)\}$, and the original continuous signal $s(t)$

6. We build Timeline Features $|s\varepsilon(t) - s(t)|$, t. e. Errors Recovery original Continuous signal $s(t)$ by three his discrete readings $\{s(k\Delta t_d)\}$ (Figure 2.17).

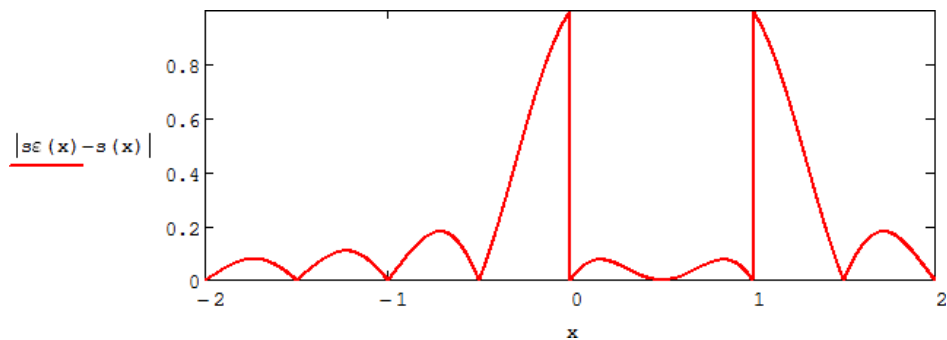


Figure 2.17 – Graph of the function $|s\varepsilon(t) - s(t)|$

7. Using formulas (2.7) and (2.8), determine the distance and the square Distances between $S_{\varepsilon}(T)$ and $S(T)$ σ_{ε}^2 (Figure 2.18).

$$ds_{\varepsilon_s} := \sqrt{\int_{-\infty}^{\infty} (s_{\varepsilon}(x) - s(x))^2 dx} = 0.647 \quad ds_{\varepsilon_s}^2 = 0.418$$

Figure 2.18 – Determination of the distance and the square of the distance between Signals $S_{\varepsilon} \square \tau \square \alpha \nu \delta S \square \tau \square$

8. Using formulas (2.4) and (2.6), we determine the norm and the square of the signal norm

$s(t)$ (Figure 2.19).

$$||s|| := \sqrt{\int_0^T (s(t))^2 dt} = 1 \quad ||s||^2 = 1$$

Figure 2.19 – Determination of the norm and square of the signal norm $s(t)$

9. Expressed as a percentage relative to the square of the signal norm $s(t)$ the square of the distance between the signals $s_{\varepsilon}(t)$ and $s(t)$ (figure 2.20).

$$\sigma_{2\varepsilon} := \left(\frac{ds_{\varepsilon_s}^2}{||s||^2} \cdot 100 \right) = 41.824$$

Figure 2.20 – Expressed as a percentage relative to the square of the norm $s(t)$ square of the distance between $s_{\varepsilon}(t)$ and $s(t)$

We conclude that if the upper effective limit frequency Signal Spectrum $s(t)$ $fB = 1$ kHz, and the sampling period of this signal, found according to Kotelnikov's theorem is $\Delta t d = 0.5$ ms, then the square of the distance between *the signals* $S_{\varepsilon}(t)$ and $S(t)$ is 41.824 per cent relative to the square of the signal norm $s s(t)$.

Investigating how to change the upper effective cut-off frequency of the spectrum fB , Hz, signal $s(t)$ per square of the distance between signals $s_{\varepsilon}(t)$ and

$s(t)$ expressed as a percentage of the square of the signal norm

To do this:

10. Again, let's determine the upper effective cut-off frequency of the spectrum f_B signal $s(t)$ on the basis of the energy approach, now believing that

The coefficient of signal energy reduction due to the artificial limitation of its spectrum width k is equal to 0.95:

- let us give the initial approximation of the parameter $f_B = 2 \text{ kHz}$ ($f_b := 2$);
- write down in the Given block the equation relative to the desired boundary frequency of the spectrum f_b , assuming the coefficient of signal energy reduction due to the artificial limitation of its spectral width to the range from $-f_b$ to f_b equal to 0.95 (Figure 2.21). We conclude that the first two lobes of the spectral function contain 95% of the energy of the square pulse (Figure 2.22);

11. In accordance with Kotelnikov's theorem, we calculate using the formula (2.2) a new sampling rate of f_L , kHz, and a sampling period of Δt_L , ms, $s(t)$ (Figure 2.23).

$$\begin{aligned}
 & f_b := 2 \\
 & \text{Given} \\
 & \frac{\int_{-f_b}^{f_b} (|s(f)|)^2 df}{\int_0^T (|s(t)|)^2 dt} = 0.95 \\
 & \underline{f_b} := \text{Find}(f_b) \quad f_b = 2.073
 \end{aligned}$$

Figure 2.21 – Redetermination of the Signal Spectrum Cutoff Frequency

$s(t)$

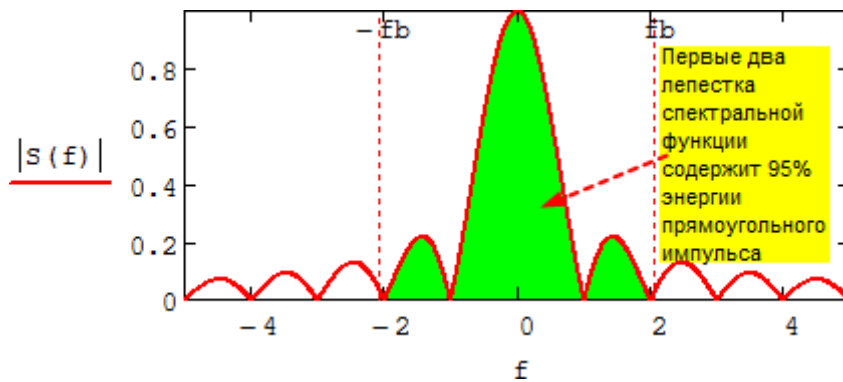


Figure 2.22 – Newly found boundary frequencies of the spectrum of a square pulse

$$f_D := 2 \cdot f_b = 4.146 \quad \Delta t_D := \frac{1}{f_D} = 0.241$$

Figure 2.23 – Calculation of the new sample rate and period

$$S \square t)$$

As can be seen from the calculations shown in Figure 2.23, the new frequency sampling f_D signal $s(t)$, satisfying Kotelnikov's theorem, is 4.146 kHz and the sampling period Δt_d is 0.241 ms, respectively;

12. By formula (2.6) Determine New Quantity samples of the discrete signal (Figure 2.24).

$$n := \text{floor}(2 \cdot f_b \cdot \tau) + 1 = 5$$

Figure 2.24 – Definition of a new number of samples of a discrete signal

13. Form a discrete reference vector $\{s(k\Delta t_d)\}$ ($k = 0, 1, 2, \dots, 4$) continuous signal $s(t)$ and build a time diagram of the discrete signal (Figure 2.25).

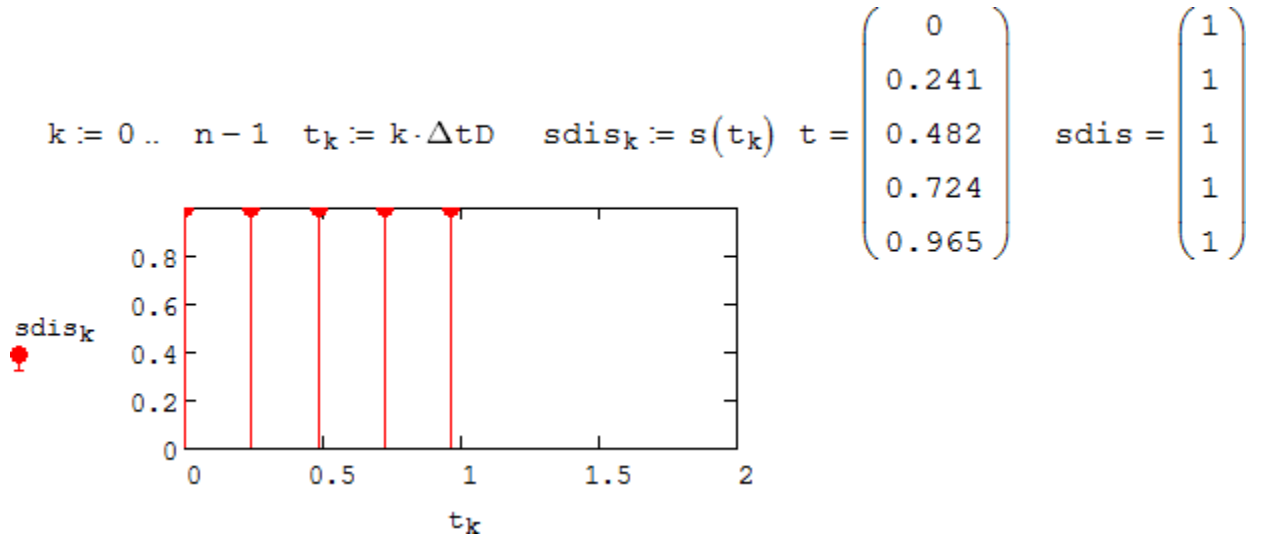


Figure 2.25 – Discrete reference vector and discrete signal time diagram

14. Using formula (2.4), we determine the functions $\{\psi_k(t)\}$ that make up the basis

Kotelnikov in the restoration of the original continuous signal $s(t)$

set of its five discrete samples $\{s(k\Delta t_d)\}$ (Figure 2.26).

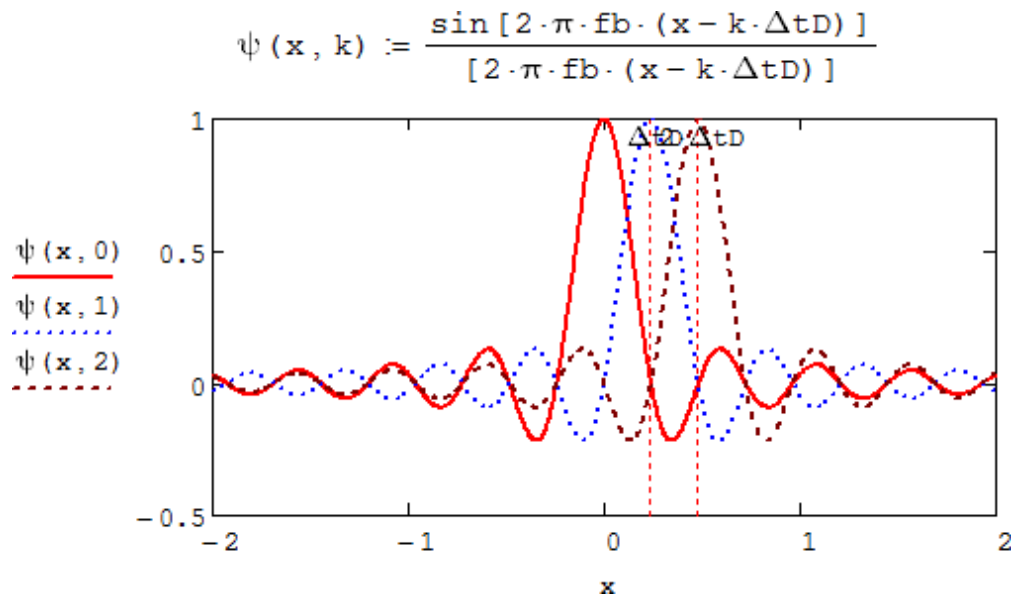


Figure 2.26 – Definition of the functions $\{\psi_k(t)\}$ that make up the basis

Kotelnikov when reconstructing the original continuous signal $s(t)$ from a set of its

five discrete samples $\{s(k\Delta t_d)\}$

15. Determine the five terms that are under the sign of the sum in the right

Parts of Formula (2.5): Product k -th discrete reference

$s(k\Delta t_d)$ on k -yu

The basis function $\psi_k(t)$ and the result of their summation (Figure 2.27).

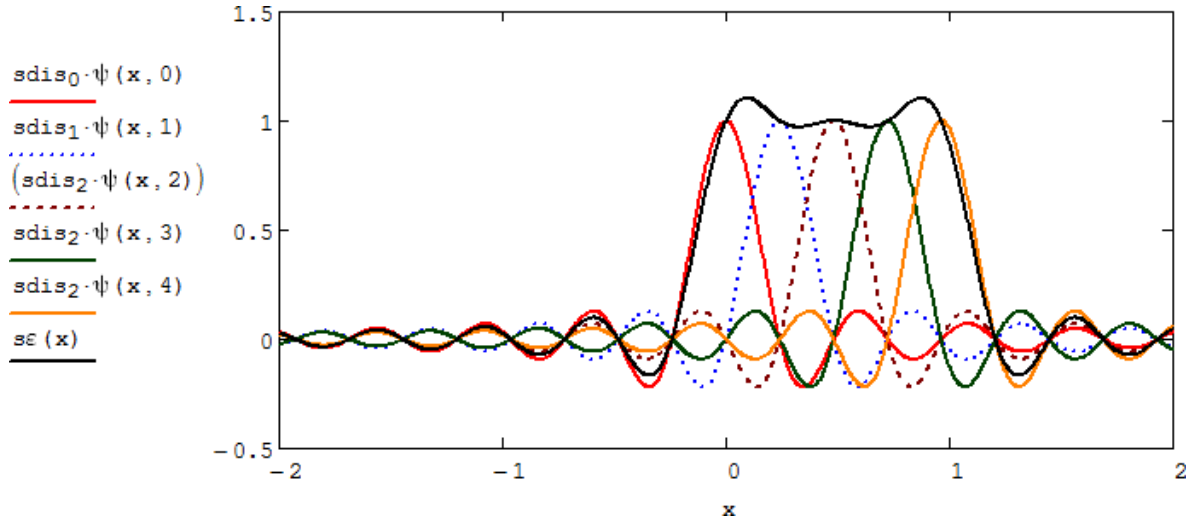


Figure 2.27 – Definition of the five terms under the sign of the sum in the right part of the formula (2.5) and the result of their summation

16. Using formula (2.5), we determine the function $sE(t)$, representing

Result of restoring the original continuous signal $S(t)$ by dialing

of its five discrete references $\{s(k\Delta t_{\text{and}})\}$, we plot it and plot it

The initial continuous signal $s(t)$ (Figure 2.28).

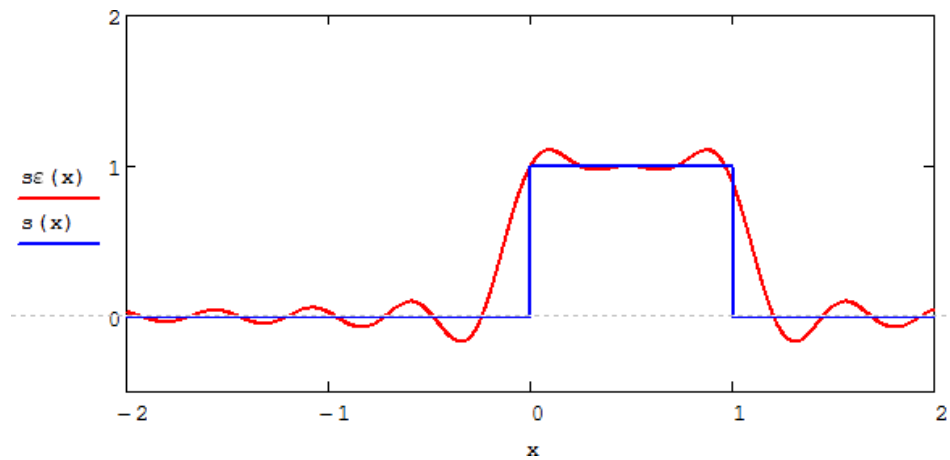


Figure 2.28 – Plots of the $sE(t)$ function and the original continuous $s(t)$

17. We build Timeline Features $|s_{\varepsilon}(t) - s(t)|$, t. e. Errors recovery of the original continuous signal $s(t)$ by dialing five of its discrete samples $\{s(k\Delta t_d)\}$ (Figure 2.29).

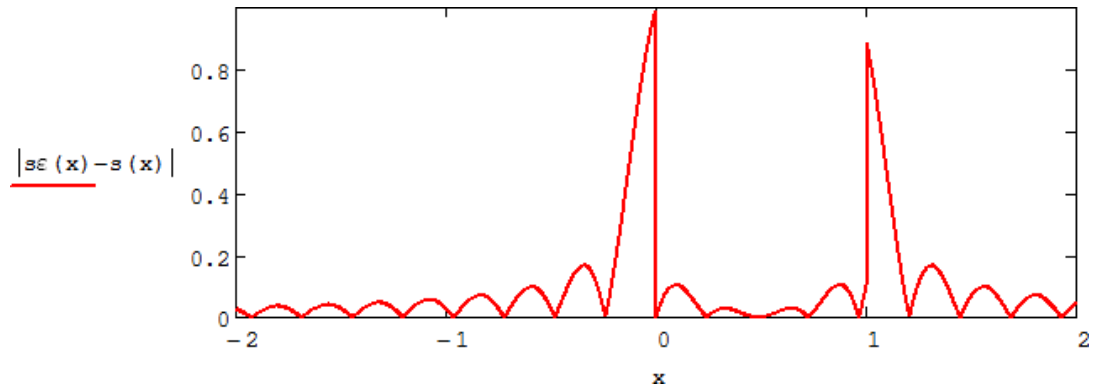


Figure 2.29 – Graph of the function $|s_{\varepsilon}(t) - s(t)|$

18. By formulas (2.7) and (2.8) Determine Distance and Square Distances between $S_{\varepsilon}(T)$ and $S(T)$ $\sigma_{\varepsilon} \gamma \nu \alpha \lambda \sigma$ (Figure 2.30).

$$ds_{\varepsilon_s} := \sqrt{\int_{-\infty}^{\infty} (s_{\varepsilon}(x) - s(x))^2 dx} = 0.403 \quad ds_{\varepsilon_s}^2 = 0.162$$

Figure 2.30 – Determining the distance and the square of the distance between Signals $S_{\varepsilon} \square \tau \square \alpha \nu \delta S \square \tau \square$

19. Expressed as a percentage relative to the square of the signal norm $s(t)$ the square of the distance between the signals $s_{\varepsilon}(t)$ and $s(t)$ (figure 2.31).

$$\sigma_{2\varepsilon} := \left(\frac{ds_{\varepsilon_s}^2}{||s||^2} \cdot 100 \right) = 16.206$$

Figure 2.31 – Expressed as a percentage relative to the square of the norm $s(t)$ square of the distance between $s_{\varepsilon}(t)$ and $s(t)$

We conclude that if the upper effective limit frequency $s(t) f_B = 2.073$ kHz and the sampling period of this signal found according to Kotelnikov's theorem, is $\Delta t_d = 0.241$

ms, the square of the distance between the signals $S_{\varepsilon}(t)$ and $S(t)$ is narrower than 16.206% relative to the square of the signal norm $s(t)$. Thus, according to comparison of the previous result (Figure 2.20), square of distance Between the signals $S_{\varepsilon}(T)$ and $S(T)$ decreased from 41.824% to 16.206%, i.e. 2.581 i.e. the quality of signal recovery $s(t)$ from a set of its discrete $\{s(k\Delta t_d)\}$ has improved.

In a similar manner, it was found that:

- Upper effective cut-off frequency of the fV spectrum $\phi \tau$ found on the basis of the energy approach, given the value of the signal energy reduction factor due to artificial limitation its spectrum width KE is 0.99 is 10.286 kHz;
- according to Kotelnikov's theorem $s(t) fD = 20.572$ kHz and sample period $\Delta t_d = 0.049$ ms;
- number of samples of a discrete signal $n = 21$ units;
- Graph of the function $S_{\varepsilon}(t)$ representing the result of recovery of the original continuous signal $s(t)$ by dialing its 21 discrete samples $\{s(k\Delta t_d)\}$ and plot of the original continuous signal $s(t)$ (Figure 2.32);
- Feature Graph $|s_{\varepsilon}(t) - s(t)|$, i.e., recovery errors of the original continuous signal $s(t)$ from its set of discrete samples $\{s(k\Delta t_d)\}$ (Figure 2.33);
- Distance and square of the distance between the signals $S_{\varepsilon}(t)$ and $S(t)$ (Figure 2.34);
- expressed as a percentage relative to the square of the norm $s(t)$ the square of the distance between the signals $s_{\varepsilon}(t)$ and $s(t)$ (figure 2.35).

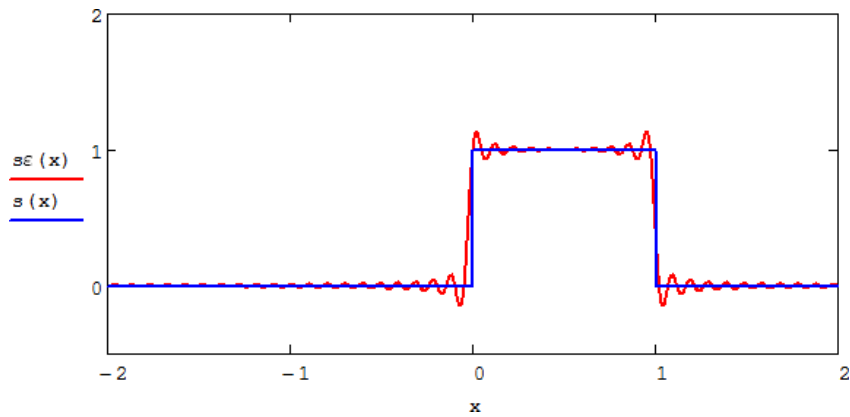


Figure 2.32 – Graphs of the restored $s_\epsilon(t)$ by set his 21st

Discrete Reference and Initial Continuous Signal $s(t)$

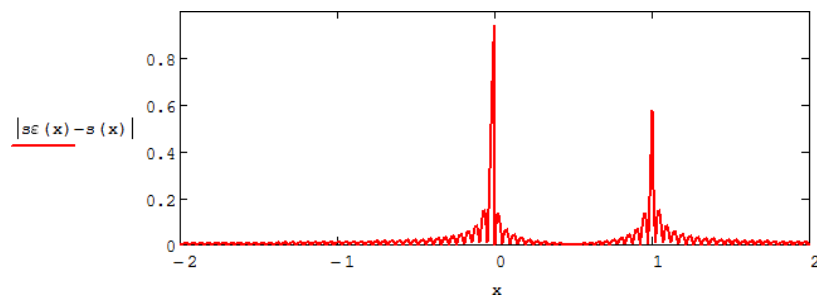


Figure 2.33 – Graph of the function $|s_\epsilon(t) - s(t)|$

$$ds_{\epsilon_s} := \sqrt{\int_{-\infty}^{\infty} (s_\epsilon(x) - s(x))^2 dx} = 0.156 \quad ds_{\epsilon_s}^2 = 0.024$$

Figure 2.34 – Determination of the distance and square of the distance between signals $s_\epsilon(t)$ and $s(t)$

$$\sigma_{2\epsilon} := \left(\frac{ds_{\epsilon_s}^2}{||s||^2} \cdot 100 \right) = 2.428$$

Figure 2.35 – Square of the distance between signals $s_\epsilon(t)$ and $s(t)$, and

expressed as a percentage of the square of the signal norm

$$s(t)$$

We conclude that if the upper limit frequency of the spectrum
 signal $s(t)$ $fB = 10.286$ kHz, and the sampling period of this signal,
)

found in accordance with Kotelnikov's theorem, is

$$\Delta TD = 0,049$$

ms, the square of the distance between $S_\varepsilon(T)$ and $S(T)$ is 2.428 per cent

relative to the square of the signal norm $s(t)$.

The results of the studies will be presented in the form of Table 2.1.

Table 2.1 – Results of the studies carried out

KE	fV , kHz	Δt_d , ms	n , units	σ_ε^2 , %
0,903	1	0,5	3	41,824
0,95	2,073	0,241	5	16,206
0,97	3,444	0,145	7	9,608
0,99	10, 286	0,049	21	2,428

From the data presented in Table 2.1, we conclude that expressed as a percentage of the square of the signal norm $s(t)$

The square of the distance between $S_\varepsilon(t)$ and $S(t)$ is less than 10% if the upper Effective Analog Signal Spectrum Boundary Frequency $s(t) fB = 10.286$ kHz, and its sampling period, found in accordance with the theorem Kotelnikov is $\Delta t_d = 0.049$ ms, while the number of discrete Signal samples $s(t)$ is equal to 21 units.

Work procedure:

13. Run Mathcad.

14. Define in Mathcad for a given continuous

$S(t)$ according to the variant of the individual task (Table 2.2):

- upper Effective boundary frequency spectrum fB , Hz, $s(t)$ the energy approach;
- period Δt_1 , s, and frequency fL , Hz, continuous sampling $s(t)$ satisfying Kotelnikov's theorem;
- n is the number of samples of the discrete signal ($k = 0, 1, 2, \dots, n - 1$);

- Discrete Signal $s(k\Delta t_d)$, Obtained in Result

Continuous Signal Sampling $s(t)$;

- Features $\{\psi_k(t)\}$, Components Basis Kotelnikova with restoration of the original continuous signal $s(t)$ by set it

Discrete Samples $\{s(k\Delta t_d)\}$ (build Charts for multiple

the first basic functions, for example, for $k = 0, 1, 2, \dots$);

- The first few terms of the sum in the right-hand side of formula (2.5) are the product of k -

Discrete Reference $s(k\Delta t_d)$ on k Basic function $\psi_k(t)$

(build graphs, for example, for $k = 0, 1, 2, \dots$);

- function $s_\varepsilon(t)$, representing Result Recovery of the original continuous signal $s(t)$ by the set of its discrete

Counts $\{s(k\Delta t_D)\}$ (build graph);

- a function representing the modulus of the difference $s_\varepsilon(t) - s(t)$ between

$: \varepsilon(t) = |s_\varepsilon(t) - s(t)|$ (build graph);

- Square of the distance between signals $s_\varepsilon(t) - s(t)$ and

- Square of the distance between signals $s(t)$ and $s(t)$, expressed as a percentage of the square of the signal norm $s(t)$.

15. Investigate the effect of changing the upper cut-off frequency

Spectrum fV, Hz, signal per square of the distance between signals $s(t)$ and $s(t)$, expressed as a percentage of the square of the signal norm $s(t)$

expressed as a percentage of the square of the signal norm $s(t)$

and build an appropriate table, for example, by analogy with Table 2.1.

16. Determine under what conditions (upper effective boundary

fV signal spectrum ΔT and etc.) Square of the distance between signals frequency, D

$s\varepsilon(t)$ and $s(t)$ expressed as a percentage relative to the square of the norm signal $s(t)$ not exceeds 10%.

17. Save the results of an individual task.
18. Generate a report on the laboratory work.
19. Protect the work.

Variants of tasks

The number of the variant corresponds to the number of the student's surname in the list of the study group (subgroup).

Table 2.1 – Variants of individual tasks

Variant	Signal type	Pulse Signal Parameters
1	One-way exponential Momentum	$A = 1 \text{ V}, \tau = 1 \text{ ms}$
2	Trapezoidal pulse	$A = 2 \text{ V},$ $\tau = 3 \text{ ms}, \tau_1 = 1 \text{ ms}, \tau_2 = 1 \text{ ms}$
3	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 2 \text{ ms}$
4	Triangular Pulse	$A = 2 \text{ V},$ $\tau_1 = 2 \text{ ms}, \tau_2 = 3 \text{ ms}$
5	One-way exponential Momentum	$A = 2 \text{ V}, \tau = 2 \text{ ms}$
6	Sawtooth Pulse	$A = 3 \text{ V}, \tau = 5 \text{ ms}$
7	Trapezoidal pulse	$A = 2 \text{ V},$ $\tau = 5 \text{ ms}, \tau_1 = 1 \text{ ms}, \tau_2 = 2 \text{ ms}$
8	Triangular Pulse	$A = 4 \text{ V},$ $\tau_1 = 1 \text{ ms}, \tau_2 = 3 \text{ ms}$
9	One-way exponential Momentum	$A = 1 \text{ V}, \tau = 2 \text{ ms}$
10	Trapezoidal pulse	$A = 2 \text{ V},$ $\tau = 5 \text{ ms}, \tau_1 = 1 \text{ ms}, \tau_2 = 1 \text{ ms}$
11	Sawtooth Pulse	$A = 2 \text{ V}, \tau = 4 \text{ ms}$
12	One-way exponential Momentum	$A = 2 \text{ V}, \tau = 3 \text{ ms}$
13	Triangular Pulse	$A = 5 \text{ V},$ $\tau_1 = 3 \text{ ms}, \tau_2 = 2 \text{ ms}$
14	Trapezoidal pulse	$A = 4 \text{ V},$ $\tau = 5 \text{ ms}, \tau_1 = 2 \text{ ms}, \tau_2 = 2 \text{ ms}$
15	Sawtooth Pulse	$A = 4 \text{ V}, \tau = 4 \text{ ms}$

The parameters of the pulses under study are given in Table 1.1.

Contents of the report

The report on the laboratory work is drawn up in the form of a Word document and must include:

20. Name of the laboratory work.
21. The purpose of laboratory work.
22. Wording individual Quests and Result its implementation.
23. Brief conclusions based on the results of laboratory work.

Control questions

24. What signal is called analog?
25. What is the purpose of sampling analog signals?
26. What signal is called discrete?
27. How is V. A. Kotelnikov's theorem formulated?
28. How to restore the original analog signal from a set of its discrete samples?
29. How is the Kotelnikov basis formed?
30. Write down the expression for *the kth basis function of Kotelnikov*.
31. What is the applied significance of V. A. Kotelnikov's theorem for electrical communication?
32. What is the Kotelnikov Analog Sample Rate?
33. How to determine the upper limit frequency of the signal spectrum?
34. How is the Kotelnikov series written if the function with a limited spectrum $x(t)$ to be sampled is given on a finite time interval t ?
35. How to determine the finite number of discrete samples if the $x(t)$ spectrum-bounded function to be sampled is set to

final time interval T ?

36. What is called base signal? What Was the base of the signal examined during the performance of the individual task equal?

37. What is the reason for the error of signal recovery based on the set of its discrete samples?

38. How to reduce the error of signal recovery from a set of its discrete samples?

39. How to Define relative RMS Accuracy σ_{ε}^2 restoring the original continuous signal $x(t)$ by set of its discrete samples?

40. Calculate the sample period of the signals:

$$s1(t) = \frac{\sin \omega_0 t}{\omega_0 t}, \quad s2(t) = \cos \omega_0 t.$$

What Quantity counts Necessary for sampling of these signals?

41. What can the hardware implementation of the synthesis of the signal presented by Kotelnikov's row look like?

42. What is the methodology for performing an individual task?

43. What Conclusions Yes, you can do by Results of an individual task?

Practical work 4

Investigation of spectral analysis and synthesis of periodic signals

The purpose of the work is to acquire the skills of spectral analysis and synthesis of periodic signals.

Theoretical part

In general, the periodic signal $s(t)$ is described by the ratio

$$s(t) = s(t + nT), \quad (2.1)$$

where T – a constant quantity called period, s;

n is any integer (positive or negative).

The periodic signal $s(t)$ can be decomposed into a Fourier series.

Such a decomposition existed, a fragment of the $S(t)$ signal lasting in one period must satisfy the Dirichlet conditions:

- there should be no breaks of the second kind (with branches of the function extending to infinity);
- the number of breaks of the first kind (jumps) must be finite;
- The number of extremes must be finite (as an example of a function that has an infinite number of extremes on a finite interval, we can cite $\sin(1/x)$ in the vicinity of zero).

There are three forms of notation of the Fourier series: sinus-cosine form, real form and complex form.

The sinus-cosine form of the Fourier series has the following form:

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left[a_k \cos(k\omega_1 t) + b_k \sin(k\omega_1 t) \right], \quad (2.2)$$

where $k = 1, 2, \dots, \infty$ is the harmonic number of the Fourier spectrum of the periodic signal;

$\omega_1 = 2\pi / T$ is the basic circular frequency of the periodic signal (frequency first harmonic), rad/s;

$k\omega_1$ – circular frequency of the k th harmonic, rad/s;

a_k – coefficient k "cosine" component of the Fourier spectrum
periodic signal, B;
 b_k – Coefficient k -th "sinus" component spectrum Fourier
periodic signal, B;
 $a_0 / 2$ – Constant Component spectrum Fourier periodic
signal, which is the average value of the signal over the period, B.

The coefficients used in (2.2) are a_k , b_k ($k = 1, 2, \dots, \infty$) and a_0 are determined by the formulas:

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos(k\omega_1 t) dt, \quad (2.3)$$

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \sin(k\omega_1 t) dt, \quad (2.4)$$

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} s(t) dt, \quad (2.5)$$

Note – The integration limits in formulas (2.3) to (2.5) need not be $-T/2$ to $T/2$. Integration can be performed at any interval of length T , the result will not change. Specific limits are chosen for reasons of computational convenience; for example, it may be more convenient to perform an integration from 0 to T or from $-T$ to 0.

On the basis of analysis (2.2), it can be seen that the basic functions of the sinus-cosine form of the Fourier series can be represented as:

$$\left\{ \begin{array}{l} 1, \cos \omega_1 t, \cos 2\omega_1 t, \dots, \cos k\omega_1 t, \dots \\ \sin \omega_1 t, \sin 2\omega_1 t, \dots, \sin k\omega_1 t, \dots \end{array} \right\}. \quad (2.6)$$

Note – All functions of the system (2.6) are orthogonal in pairs on the interval $(-T/2; T/2)$ or $(0; T)$, and the frequencies of the cosine and sinusoidal components form an arithmetic progression.

Thus, in general, the periodic signal spectrum $s(t) \propto \tau$ use sinus-cosine Forms range Fourier consists of of constant component of $a_0/2$ and an infinite set of coefficients $\{ak\}, \{bk\}$ ($k = 1, 2, \dots, \infty$) with the "cosine" and "sinus" components of the spectrum.

In the special case, if the signal $s(t)$ is an even function, then all Odds b_k will be equal to zero and, in the formula of the Fourier series will be only "cosine" terms are present.

If the signal $s(t)$ is an odd function, will be equal to zero, on the contrary, cosine coefficients ak , and only "sinus" terms.

A certain inconvenience of the sinus-cosine form of the Fourier series is that for each value of the summation index k (that is, for each harmonics with a frequency of $k\omega_1$) in formula (2.2) there are two terms – sine and cosine. Since the ratio is valid:

$$a \cos(\omega t) + b \sin(\omega t) = A \cos(\omega t - \varphi),$$

where $A = \sqrt{a^2 + b^2}$ – amplitude of the harmonic signal, B;

$\varphi = \arctg(b/a)$ – the initial phase of the harmonic signal, radians or degrees, then the series (2.2) can also be represented in a compact form:

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_1 t - \varphi_k). \quad (2.7)$$

Formula (2.7) represents the real form of the notation of the Fourier series. If the signal $s(t)$ is an even function, phase φ_k can take Only the values of 0 and π , and if $s(t)$ is an odd function, then the possible values for the phase are $\pm\pi/2$.

Reverse Transition from real K sinus-cosine The form of notation of the Fourier series is carried out on the basis of the following relations:

$$ak = Ak \cos \varphi_k, \quad bk = Ak \sin \varphi_k. \quad (2.8)$$

Comparing the real to the sinus-cosine form of the Fourier series, we note that:

– with use sinus-cosine Forms Decomposition periodic signal to the trigonometric series (2.2) coefficients Ak and bk depend on the choice of the reference point, and when using the real Forms of decomposition (2.7) of harmonic amplitude Ak do not depend on the choice and are defined only by the signal type $s(t)$, and the arguments harmonics φ_k depend on the reference when choosing the limits of integration in formulas (2.3) – (2.5);

– when the reference point is shifted along the time axis t by $\pm t_0$ amplitude Ak is conserved and the phases φ_k are incremented $\pm k\omega_1 t_0$ (the shift theorem).

The complex form of representation of the Fourier series is the most general and is obtained from the real form by the representation of the cosine as a half-sum of complex exponents (such a representation follows from the Euler's formulas $e^{jx} = \cos x + j \sin x$):

$$\cos x = \frac{1}{2} (e^{jx} + e^{-jx}).$$

Applying this transformation to the real form of the Fourier series, we get the sums of complex exponents with positive and negative exponents:

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{Ak}{2} [\exp(jk\omega_1 t + j\varphi_k) + \exp(-jk\omega_1 t - j\varphi_k)]. \quad (2.9)$$

We will treat exponents with a minus sign in the indicator as members of a series with negative numbers. Within the framework of the same general approach, the constant

As a result, the term $a_0 / 2$ will become a series term with a zero number. As a result, a complex form of notation of the Fourier series will be obtained:

$$s(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_1 t}. \quad (2.10)$$

Complex coefficients of the series C_k are related to amplitudes A_k and phases φ_k appearing in the real form of the notation of the Fourier series (2.7) with the following relations:

$$C_k = \frac{1}{2} A_k e^{j\varphi_k} \quad A_k = 2 |C_k| \quad \text{and} \quad \varphi_k = \arg(C_k)$$

The formulas for the relationship C_k with the coefficients a_k and b_k also look simple

sinus-cosine form of the Fourier series (2.2):

$$C_k = \frac{a_k}{2} - j \frac{b_k}{2}, \quad a_k = 2\operatorname{Re}(C_k) \quad \text{and} \quad b_k = -2\operatorname{Im}(C_k)$$

Hence Immediately Same should be and Formula direct Calculation

coefficients C_k of the Fourier series in complex form:

$$C_k = \frac{1}{T} \int_{-T/2}^{T/2} s(t) \exp(-jk\omega_1 t) dt, \quad (2.11)$$

where k – any integer.

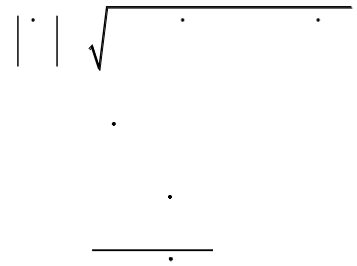
Formula (2.11) Determines Comprehensive Spectrum periodic signal.

If $s(t)$ is an even function, the coefficients of the series C_k will be purely real, and if $s(t)$ is an odd function, the coefficients of the series will turn out to be purely imaginary.

The set of harmonic amplitudes of the Fourier series is often called the amplitude spectrum of a periodic signal (spectrum), and the set of their phases is called the phase spectrum of a periodic signal (Table 2.1).

Table 2.1 – Characteristics of periodic signals

The term	Definition	Mathematical formula and Value Designation
Periodic Signal Period (Period)	A parameter equal to the smallest time interval after which instantaneous Periodic Signal Values	T
Frequency of periodic signal (frequency)	A parameter that is the inverse of the period Periodic Signal	$Fl = \frac{1}{T}, \quad \omega_1 = 2\pi Fl = \frac{2\pi}{T}$
Comprehensive Periodic Spectrum	A complex function of a discrete argument equal to an integer number of periodic signal frequency values, which is the values of the Fourier complex series coefficients for a periodic signal	$C_k = \frac{1}{T} \int_{-T/2}^{T/2} s(t) \exp(-jk\omega_1 t) dt$ where k is any integer
Amplitude spectrum of periodic signal (spectrum)	A discrete argument function that is a complex spectrum module Periodic Signal	$A_k = C_k = \operatorname{Re} \{ C_k \} + \operatorname{Im} \{ C_k \}$
Phase spectrum	A discrete argument function that is	$\varphi_k = \arg \{ C_k \}$



If the analyzed signal $s(t)$ is real, then its amplitude and phase spectra have even and odd symmetry, respectively:

$$A_{-k} = A_k, \quad \varphi_{-k} = -\varphi_k,$$

a Odds complex range Fourier – complex-conjugate symmetry:

$$\dot{C}_{-k} = \dot{C}_k^*,$$

where $*$ is the symbol of the mathematical operation of complex conjugation.

A spectral diagram of a periodic signal is a graphical representation of the Fourier series coefficients for a given signal. A distinction is made between amplitude and phase spectral diagrams (Figures 2.1 and 2.2).

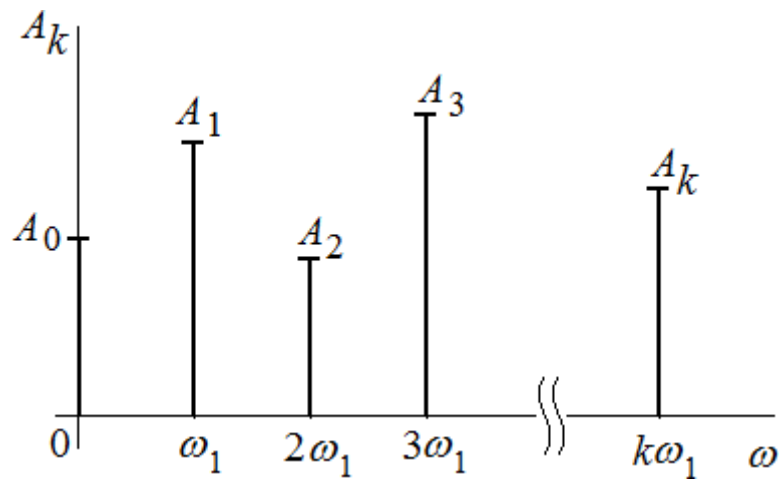


Figure 2.1 – Amplitude spectral diagram of the periodic signal

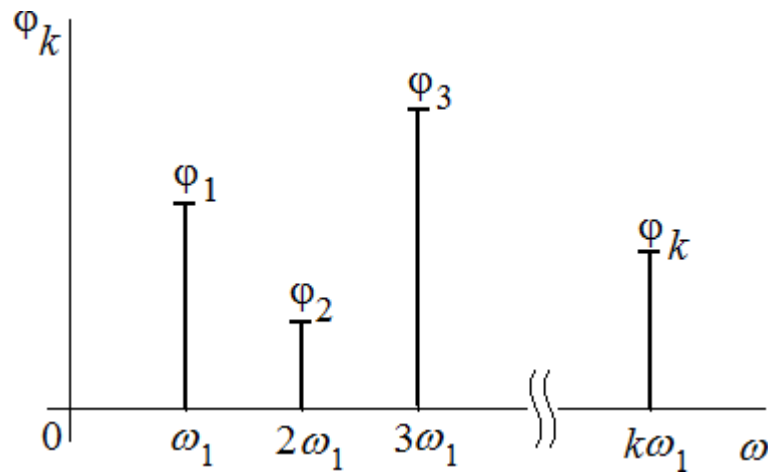


Figure 2.2 – Phase spectral diagram of a periodic signal

Let us formulate the main properties of spectral diagrams of periodic signals:

- The first property of the spectra depicted in Figures 2.1 and 2.2 is that the spectral diagrams of a periodic signal are discrete functions of frequency, so they are also called barred.

- The second property of the spectra shown in Figures 2.1 and 2.2 is that these spectra are harmonic. This means that they consist of equidistant spectral lines, since the harmonic frequencies are in simple multiples of the

Frequencies $k\omega_1$ first harmonic frequency $\omega_1 = 2\pi / T$).

Note – Of course, individual harmonics, sometimes even the first one, may be absent, i.e., their amplitudes may be equal to zero, but this, however, does not violate the harmony of the spectrum.

Thus, the Fourier series represents a given periodic signal $s(t)$ as a sum harmonic oscillations (sine wave and cosine wave) of multiple frequencies with appropriately selected amplitudes and phases. If we limit ourselves to a finite number of terms of the series (2.2) and (2.7), we get

Periodic Alarm $s_N(t)$ which is an approximate image $s(t)$:

$$s_N(t) = \frac{a_0}{2} + \sum_{k=1}^N \left[a_k \cos(k\omega t) + b_k \sin(k\omega t) \right]; \quad (2.12)$$

$$s_N(t) = \frac{a_0}{2} + \sum_{k=1}^N A_k \cos(k\omega t - \varphi_k). \quad (2.13)$$

As the number of terms N increases, the approximation naturally improves, and in the limit, at $N \rightarrow \infty$, $\tau \eta \varepsilon$ approximate equations (2.12) and (2.13) become exact equations (2.2) and (2.7), respectively.

At the same time, it is essential that the approximations (2.12) and (2.13) are always the best approximations for any N , i.e., when it becomes necessary to approximate a periodic function by a trigonometric polynomial, the smallest square deviation will be obtained if the coefficients of the polynomial are determined by the formulas (2.3) – (2.5).

at each N takes the lowest value when the

odds ak, bk polynomial $PN(t)$ are taken Relevant

Fourier odds (2.3) – (2.5) ak and bk Features $s(t)$, i.e. when

trigonometric polynomial $PN(t)$ is a partial sum

$$s_N(t) \equiv \frac{a_0}{2} + \sum_{k=1}^N \left[a_k \cos(k\omega t) + b_k \sin(k\omega t) \right] \quad (2.14)$$

Fourier series of functions $s(t)$.

It should be borne in mind that when Fourier series are truncated, a certain distortion of functions always exists. But with a small fraction of the energy of the cut off part of the signal (with a rapid attenuation of the spectra of functions with an increase in the harmonic number), this effect, called the Gibbs effect, may be hardly noticeable. At jumps and breaks of functions, the Gibbs effect is most pronounced, which can have a very significant effect on the quality and accuracy of signal processing:

- the steepness of the differences is "blurred", since it cannot be greater than the steepness (at the zero point) of the last preserved harmonic of the series (2.12) or (2.13);

- On both sides of the "blurred" differences, outliers and damped oscillations appear with a frequency equal to the frequency of the last stored or first discarded term of the series (2.12) or (2.13).

The manifestation of the Gibbs effect will be considered in more detail later in the example of spectral analysis and synthesis of the periodic sequence of square pulses.

Parseval's theorem. By determining the average power of the signal $s(t)$, represented as a Fourier series (2.2), for the oscillation period T , we get:

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt = \left(\frac{a_0}{2} \right)^2 + 2 \sum_{k=1}^{\infty} (a_k^2 + b_k^2). \quad (2.15)$$

When deriving the formula (2.15), the ratio

$$\frac{1}{T} \int_0^T \cos\left(\frac{2\pi t}{T}\right) dt = \frac{1}{2}.$$

The relation (2.15) is a mathematical notation of Parseval's theorem, from which it follows that if the periodic signal $s(t)$ decomposes into the Fourier series, then the average power of this signal over the period is the sum of the powers of the constant component and the harmonics of the Fourier spectrum.

When using the real form of notation of the Fourier series (2.7), the Parseval equation (2.15) can also be rewritten in the form

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt = \left(\frac{a_0}{2}\right)^2 + \sum_{k=1}^{\infty} A_k^2 \quad (2.16)$$

or

$$P_{cp} = P_0 + \sum_{k=1}^{\infty} P_k, \quad (2.17)$$

where $P_0 = \left(\frac{a_0}{2}\right)^2$ – power, constant component of the Fourier spectrum;

$P_k = \frac{1}{2} A_k^2$ is the power of the k th harmonic of the Fourier spectrum.

And, finally, for complex range Fourier Equality Parseval takes the form:

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt = \sum_{k=-\infty}^{\infty} |C_k|^2, \quad (2.18)$$

Set of magnitudes $|C_k|^2$ included in (2.18) is called a spectrum the power of the periodic signal.

It should be taken into account that with truncation rows Fourier N Parseval equations take the form of inequalities:

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt \geq \left(\frac{a_0}{2}\right)^2 + \sum_{k=1}^N (a_k^2 + b_k^2); \quad (2.19)$$

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt \geq \left(\frac{a_0}{2} \right)^2 + \frac{1}{2} \sum_{k=1}^N A_k^2 = P_0 + \sum_{k=1}^N P_k. \quad (2.20)$$

$$P_{cp} = \frac{1}{T} \int_0^T s^2(t) dt \geq \sum_{k=-N/2}^{N/2} |\dot{C}_k|^2. \quad (2.21)$$

Equipment and materials

To perform the laboratory work, you need a personal computer with the following characteristics: a Pentium/Celeron processor with a clock speed of 300 MHz or higher, RAM of at least 128 MB or more, free disk space of at least 100 MB, a CD reader, a Super VGA monitor (number of colors – 256).

To perform the laboratory work, you need the operating system WINDOWS 2000/ XP Professional, the Microsoft .NET Framework library version 1.1 or higher, MathCAD 13 and higher.

Safety Instructions

When performing laboratory work, **it is prohibited:**

- independently repair a personal computer, as well as install and remove existing software;
- violate generally accepted safety rules when working with electrical equipment, in particular, touch electrical outlets with metal objects, etc.;
- eat and drink and litter at the workplace of a personal computer user.

In the event of a malfunction of a personal computer, it is necessary to **immediately** inform the laboratory service personnel (system administrator, operator).

Instructions on the procedure for performing work

The method of performing the work will be considered on the example of spectral analysis and synthesis in a truncated Fourier basis (limit ourselves to considering ten harmonics, i.e. $N = 10$) of a periodic sequence of square pulses with a height $A = 1$ V, duration $\tau = 0.2$ s and period $T = 1$ s (Figure 2.3).

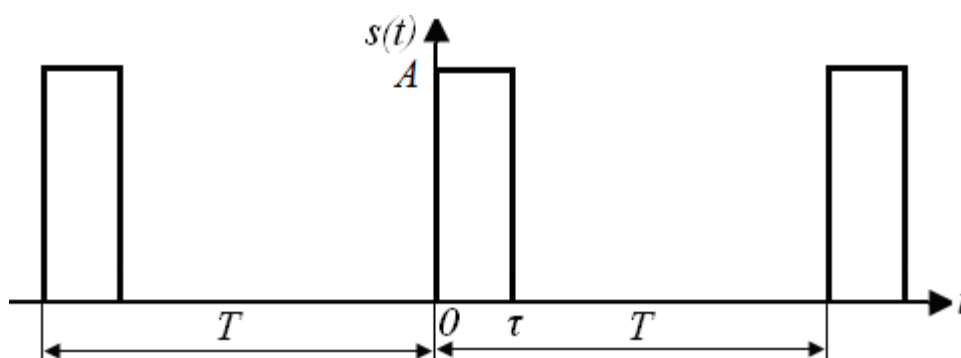


Figure 2.3 – Periodic sequence of square pulses

It is required for the $s(t)$ signal to determine:

- ω_1 is the basic circular frequency of the periodic signal (frequency first harmonic), rad/s;
- a_k is the coefficient of the k th cosine component of the Fourier spectrum periodic signal, B;
- b_k is the coefficient of the k th sine component of the Fourier spectrum periodic signal, B;
- $a_0 / 2$ is the constant component of the periodic Fourier spectrum signal, B;
- A_k is the amplitude of the k -th harmonic of the periodic Fourier spectrum signal, B;
- φ_k – Initial phase k -th harmonics spectrum Fourier periodic signal, radians, or degrees;
- amplitude and phase spectral diagrams of a periodic signal (build bar spectra);

- Refurbished by truncated row Fourier Signal $s_N(t)$ which is an approximate image of the $s(t)$ signal;
- distance and square of the distance between the signals $s_N(t)$ and $s(t)$ per fluctuation period T expressed as a percentage of the square Signal standards $s(t)$;
- the average signal power $s(t)$ for the oscillation period T ;
- the sum of the powers of the constant component and harmonics of the Fourier spectrum, B2, and the fulfillment of the Parseval inequality;
- t_m is the point in time when the truncated series Fourier Signal $s_N(t)$ Accepts Maximum Significance (research Gibbs effect), c;
- Δ is the maximum value of the absolute error of signal recovery due to the presence of the Gibbs effect, B;
- $\Delta\%$ is the maximum value of the relative error of signal recovery, expressed as a percentage, due to the presence of the Gibbs effect;
- to investigate how the change in the number of harmonics of the truncated Fourier series affects the change in the parameters of the Gibbs effect - t_m , Δ and $\Delta\%$.

Let's consider the solution of the problem in the Mathcad mathematical package.

In the following Mathcad document, the following notations are adopted:

- $s(t)$ is a function that describes the dependence of a single pulse that makes up a periodic sequence of pulses on the current time within one period T ;
- $s_N(t)$ is a periodic function reduced (spectral synthesis) based on the results of spectral analysis (Fourier series decomposition) of the $s(t)$ function over N harmonics;
- T – period, s;

- τ is the pulse duration, s;
- A is the height of the pulse, B;
- N – the number of calculated harmonics of the periodic pulse sequence spectrum (the problem of spectral analysis);
- a_k, b_k is the coefficient of the k th cosine and sine component Fourier spectrum of a periodic signal (formula (2.2)), $k = 1, 2, 3, \dots, N$;
- A_k, φ_k – Amplitude and initial phase k -harmonic spectrum Fourier of a periodic signal (formula (2.7)), $k = 1, 2, 3, \dots, N$;
- a_0, A_0 – Double the constant component of the spectrum Fourier periodic signal (formulas (2.2) and (2.7));
- t_m is the moment in time when the signal $s_N(t)$ reconstructed from the truncated Fourier series takes its maximum value (study of the Gibbs effect);
- Δ is the maximum value of the absolute error of signal recovery due to the presence of the Gibbs effect;
- $\Delta\%$ is the maximum value of the relative error of signal recovery due to the presence of the Gibbs effect;

Solution:

1. Run the math package Mathcad. According to the waveform $s(t)$ (Table 2.3) and the specified parameters, enter the signal definition and plot it within one period (Figure 2.4).

$$A := 1 \quad \tau := 0.2 \quad T := 1$$

$$s(t) := \begin{cases} A & \text{if } 0 \leq t \leq \tau \\ 0 & \text{if } \tau \leq t \leq T \end{cases}$$

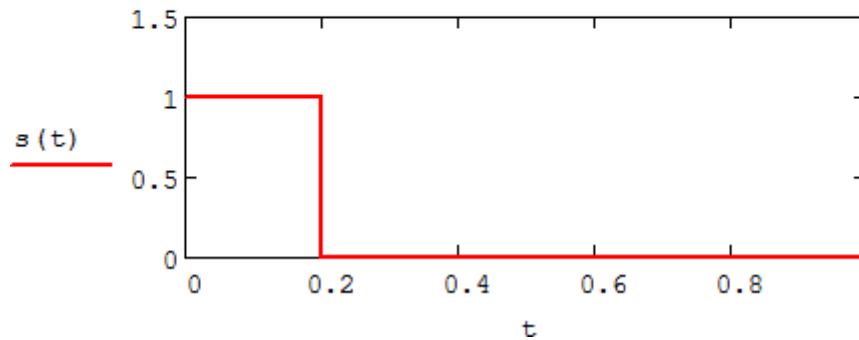


Figure 2.4 – Definition and graph of the $s(t)$ signal within a single period

2. Calculate the fundamental circular frequency of the periodic signal (first harmonic frequency), rad/s, (Figure 2.5).

$$\omega_1 := \frac{2 \cdot \pi}{T} = 6.283$$

Figure 2.5 – Calculating the basic circular frequency of a periodic signal (first harmonic frequency)

As can be seen from Figure 2.5, the frequency of the first harmonic is 6.283 rad/s.

3. Set the number of harmonics of the truncated Fourier series ($N = 10$), form the ranked variable k ($k = 0, 1, 2, \dots, 10$) and calculate

Coefficient a_k k -the cosine component of the Fourier spectrum

periodic signal (Figure 2.6).

$$a_k := \frac{2}{T} \cdot \int_0^T s(t) \cdot \cos(k \cdot \omega_1 \cdot t) dt$$

Figure 2.6 – Calculating the coefficient of the k th cosine component of the Fourier spectrum of a periodic signal

4. Calculating the coefficient b_k of the k -th sine component of the spectrum Fourier of the periodic signal (Figure 2.7).

$$b_k := \frac{2}{T} \cdot \int_0^T s(t) \cdot \sin(k \cdot \omega_1 \cdot t) dt$$

Figure 2.7 – Calculating the coefficient of the kth sine component of the Fourier spectrum of a periodic signal

5. Display the calculated values of the coefficients in the form of vectors a_k and b_k (Figure 2.8).

a =		0	b =		0
	0	0.4		0	0
	1	0.303		1	0.22
	2	0.094		2	0.288
	3	-0.062		3	0.192
	4	-0.076		4	0.055
	5	-1.411·10 ⁻⁷		5	-1.128·10 ⁻⁶
	6	0.05		6	0.037
	7	0.027		7	0.082
	8	-0.023		8	0.072
	9	-0.034		9	0.024
	10	-5.581·10 ⁻⁷		10	-2.257·10 ⁻⁶

Figure 2.8 – Calculated values of the a_k and b_k coefficients

6. Calculate permanent component $a_0 / 2$ spectrum Fourier of the periodic signal (Figure 2.9).

$$a_0 := \frac{2}{T} \cdot \int_0^T s(t) dt = 0.4 \quad \frac{a_0}{2} = 0.2$$

Figure 2.9 – Calculating the constant component of the Fourier spectrum of a periodic signal

Note – Strictly speaking, it was possible not to recalculate the constant component of the Fourier spectrum of the periodic signal, since the double value of this component coincides with the value of the element of the vector a (figure) with the value of the index $k = 0$.

7. We calculate and display the amplitudes A_k , B , and the initial phases φ_k , rads, harmonics of the Fourier spectrum of the periodic signal (Figure 2.10).

Notes

– In Figure 2.10, the function $\text{atan2}(x, y)$ returns the angle measured from the x-axis counterclockwise to the direction of the point with the x and y coordinates $\pi \pi \pi$.

– In calculating the initial phases of φ_k harmonics,

Limitation on the absolute value of *the kth sinus component of the spectrum*

Fourier periodic signal $|b_k| \leq 10^{-3}$ t. k. without this restriction,

For example, for the fifth harmonic, we have $B_5 = -1.128 \cdot 10^{-6}$, $A_5 = -1.411 \cdot 10^{-7}$ and $\text{atan2}(a_5, b_5) = -1.659$ rad or -97.127 degrees, which is an error.

$$A_k := \sqrt{(a_k)^2 + (b_k)^2}$$

$$\varphi_k := \begin{cases} \text{atan2}(a_k, b_k) & \text{if } |b_k| \geq 10^{-3} \\ 0 & \text{if } |b_k| < 10^{-3} \end{cases}$$

$A =$

	0
0	0.4
1	0.374
2	0.303
3	0.202
4	0.094
5	$1.137 \cdot 10^{-6}$
6	0.062
7	0.086
8	0.076
9	0.042
10	$2.325 \cdot 10^{-6}$

$\varphi =$

	0
0	0
1	0.628
2	1.256
3	1.885
4	2.513
5	0
6	0.629
7	1.256
8	1.884
9	2.512
10	0

Figure 2.10 – Amplitudes and initial phases of the harmonic of the Fourier spectrum of a periodic signal

8. We build amplitude and phase Spectral periodic signal diagrams (Figures 2.11 and 2.12).

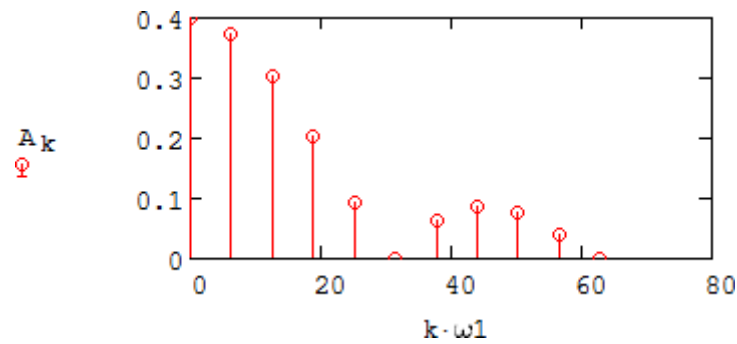


Figure 2.11 – Amplitude spectral diagram of the periodic signal

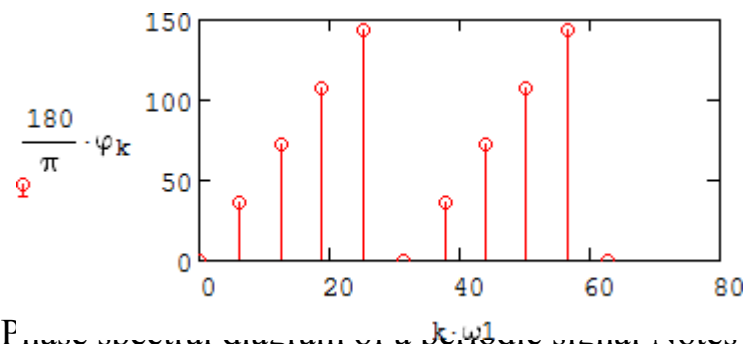


Figure 2.12 – Phase spectral diagram of a periodic signal

- In Figures 2.11 and 2.12, the circular frequency of *the kth* harmonic, rad/s, is plotted along the abscissa axis;
- In Figure 2.11, the amplitude of the *kth* harmonic is plotted along the ordinate axis, including twice the value of the constant component (*A0*) of the Fourier spectrum of the periodic signal in volts.
- In Figure 2.12, the initial phase of *the k-toy* is plotted along the ordinate axis harmonics of the Fourier spectrum of a periodic signal in degrees.

9. We find the signal restored from the truncated Fourier series $s_N(t)$,

which is an approximate image of the signal $s(t)$, and build it

a time chart within two periods, and a time chart

initial signal (Figure $s(t)$ in limits set within one period
2.13).

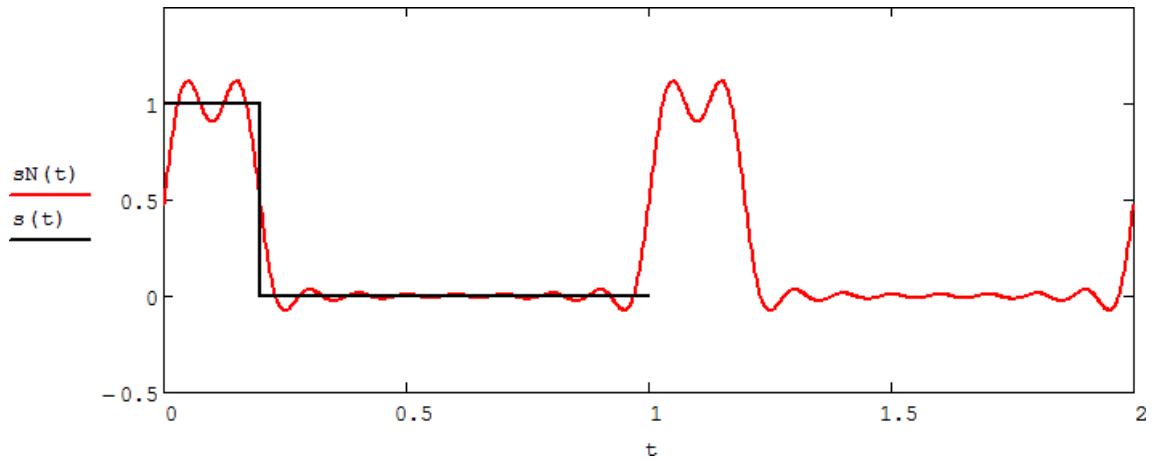


Figure 2.13 – Truncated Fourier series $s_N(t) \propto \nu \delta$

The original signal $s(t)$ defined within the same period

10. Distance and square of the distance between the signals $s_N(t)$ and $s(t)$ over the oscillation period T (Figure 2.14).

$$ds_{sN} := \sqrt{\int_0^T (s_N(t) - s(t))^2 dt} = 0.1 \quad ds_{sN}^2 = 0.01$$

Figure 2.14 – Determining the distance and the square of the distance between $s_N(t)$ and $s(t)$

11. Square of the distance between signals $s_N(t)$ and $s(t)$ for the period T (Figure 2.15), expressed as a percentage relative to square the signal norm $s(t)$.

$$Es := \int_0^T s(t)^2 dt = 0.2 \quad \varepsilon := \frac{ds_{sN}^2}{Es} \cdot 100 = 5.009$$

Figure 2.15 – Square of the distance between the signals $s_N(t)$ and $s(t)$, expressed as a percentage of the square of the signal norm $s(t)$

We conclude that if we limit ourselves to the consideration of ten harmonics of the Fourier spectrum, then the square of the distance between the signals $s_N(t) \propto \nu \delta$ $s(t)$ is approximately 5 per cent relative to the square of the signal norm $s(t)$.

12. Find the average signal strength $s(t)$ for the oscillation period T (Figure 2.16).

$$P_{cp} := \frac{1}{T} \cdot \int_0^T s(t)^2 dt = 0.2$$

Figure 2.16 – Average signal power $s(t)$ for the oscillation period T

13. We calculate the sum of the powers of the constant component and harmonics of the Fourier spectrum and check the Parseval inequality (Figure 2.17).

Figure 2.17 – Sum of the powers of the constant component and ten

$$P_0 := \left(\frac{a_0}{2} \right)^2 = 0.04$$

$$P_k := \frac{1}{2} \cdot (A_k)^2$$

$P =$

$$P_{\Sigma} := P_0 + \sum_{k=1}^N P_k = 0.19$$

	0
0	0.08
1	0.07
2	0.046
3	0.02
4	$4.372 \cdot 10^{-3}$
5	$6.466 \cdot 10^{-3}$
6	$1.943 \cdot 10^{-3}$
7	$3.738 \cdot 10^{-3}$
8	$2.866 \cdot 10^{-3}$
9	$8.628 \cdot 10^{-4}$
10	$2.702 \cdot 10^{-12}$

harmonics of the Fourier spectrum

Comparing P_{cp} (Figure 2.16) and P_{Σ} (Figure 2.16), we are convinced that $P_{cp} > P_{\Sigma}$, i.e. the Parseval inequality, holds.

14. As can be seen from Figure 2.13, the sequence of rectangular momenta is not well suited for representation by a Fourier row – it contains jumps, and the sum of any number of harmonic components with any amplitudes will always be a continuous (and, moreover, infinitely differentiable at all points) function. Therefore, the behavior of the Fourier series in the vicinity of the discontinuities is of particular interest. The time diagram shown in Figure 2.13 clearly shows that in the vicinity of the break point, the summation of ten terms of the Fourier series gives

inclined region. At the very point of discontinuity (at time $t = 0$), the Fourier series converges to the half-sum of the right and left limits:

$$x(t_0) = \frac{1}{2} \left[\lim_{t \rightarrow t_0^-} s(t) + \lim_{t \rightarrow t_0^+} s(t) \right] = \frac{1}{2}(0 + 1) = 0,5 .$$

Here $s(t)$ is the original signal, $sN(t)$ is the sum of the truncated Fourier series for it.

In the regions adjacent to the gap, the sum of the truncated Fourier series gives noticeable pulsations. This phenomenon, inherent in Fourier series for any signals with discontinuities of the first kind (jumps), as noted earlier, is called the Gibbs effect. In a number of papers, for example, it has been shown that the amplitude of the first (largest) ejection is approximately 9% of the jump value

$sN(t)$ takes the first maximum value after time $t = 0$
(Figure 2.18).

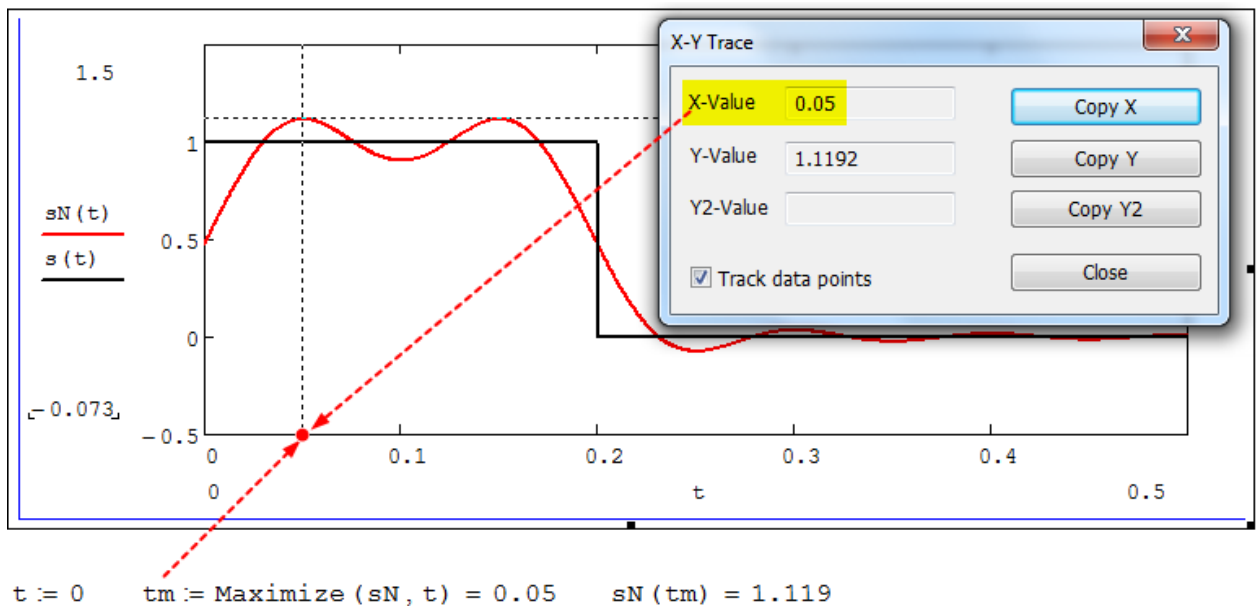


Figure 2.18 – The point in time when the restored from the truncated series
Fourier Transform signal $sN(t)$ maximizes

Note – Figure 2.18 implements two ways to determine the time t_m :

- the first method is based on the use of the X-Y Trace graph trace window;
- the second method is based on using the built-in Mathcad function $\text{Maximize}(f, \text{var1})$, which returns the value of the variable var1 in which the function $f(\text{var1})$ takes the maximum value. In order to eliminate the ambiguity of the definition of the t_m parameter (in the figure

2.18 The $sN(t)$ Available several maximums) before addressing the
)

of the $\text{Maximize}(f, \text{var1})$ function is set to the initial approximation of this parameter $t := 0$, i.e., the t_m parameter is searched at the beginning of the coordinate system with t equal to zero.

15. We calculate the maximum value of the absolute error Δ of signal recovery due to the presence of the Gibbs effect (Figure 2.19).

$$\Delta := |sN(t_m) - s(t_m)| = 0.119$$

Figure 2.19 – Maximum value of the absolute error of signal recovery due to the presence of the Gibbs effect

16. We determine the maximum value of the relative error of signal recovery $\Delta\%$ due to the presence of the Gibbs effect, expressed as a percentage (Figure 2.20).

$$\Delta\% := \frac{\Delta}{s(t_m)} \cdot 100 = 11.917$$

Figure 2.20 – Maximum value of the relative error of signal recovery due to the presence of the Gibbs effect, expressed as a percentage

As can be seen from Figure 2.20, in the example under consideration, due to the Gibbs effect, the amplitude of the first (largest) ejection is approximately 12% of the jump value.

17. Let's study how a change in the number of harmonics of the signal spectrum will affect the parameters of the Gibbs effect. Let's increase the number of harmonics by two

and three times ($N = 20$ and $N = 30$). The result of signal recovery and the Gibbs effect parameters are shown in Figures 2.21 and 2.22.

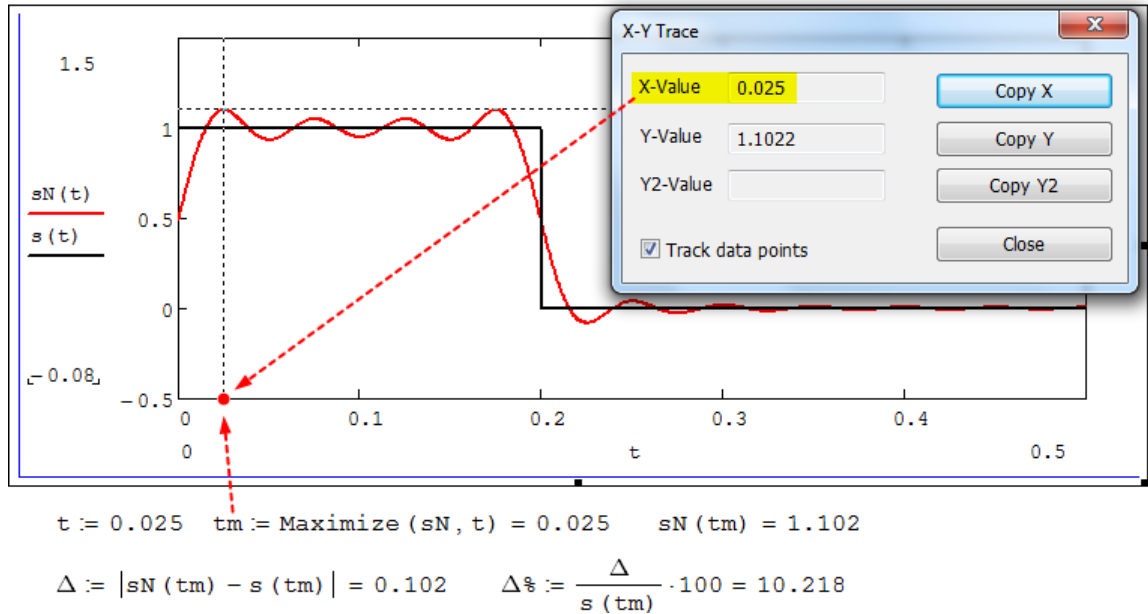


Figure 2.21 – Parameters of the Gibbs effect at the number of harmonics $N = 20$

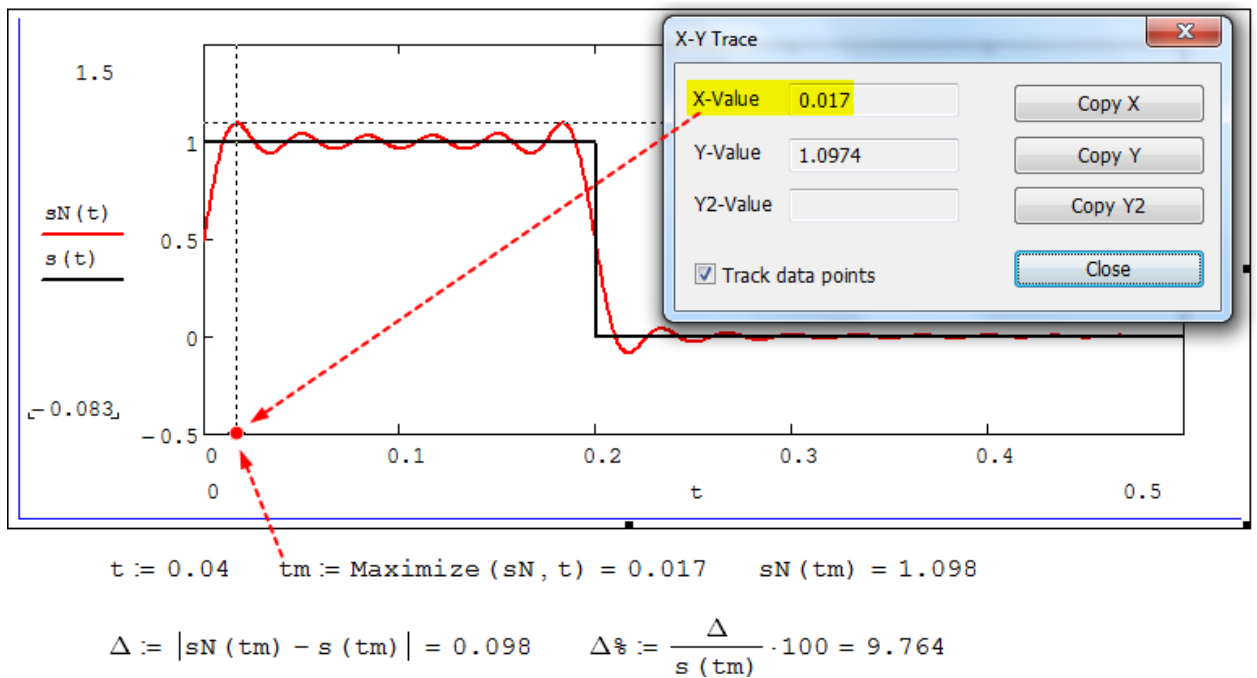


Figure 2.22 – Parameters of the Gibbs effect at the number of harmonics $N = 30$

Analysis of the graph and data presented in Figures 2.21 and 2.22 shows that with a two- and three-fold increase in the number of harmonics (i.e., from 10 to 20 and 30), the number of pulsations of the restored signal also doubled (tripled), and the amplitude of the first (largest) outburst

is about 10% of the jump value, that is, in the laboratory, it has not changed. Thus, the studied deterministic, periodic signal is a periodic sequence of rectangular pulses (Figure 2.3) can be specified by the $s(t)$ (temporary approach) or is represented as a linear spectrum of harmonic amplitudes and phases in the Fourier basis (Figures 2.11 and 2.12) (spectral approach).

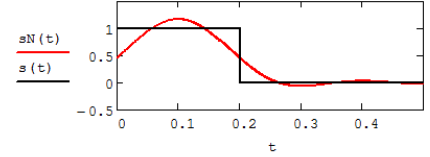
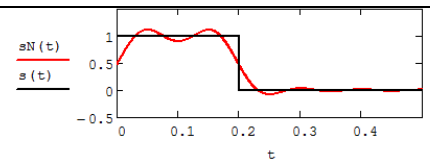
Note – In the course of the study of the effect of changing the number of harmonics of the Fourier spectrum of the signal on the parameters of the Gibbs effect, other parameters were simultaneously studied (Table 2.2).

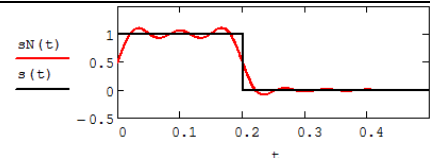
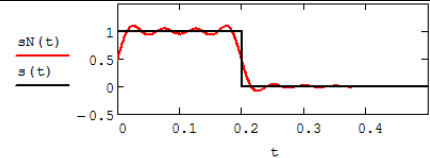
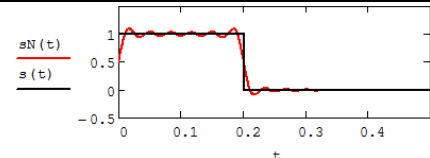
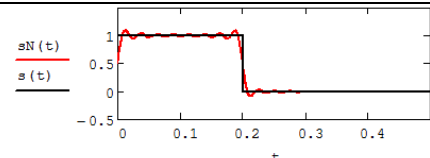
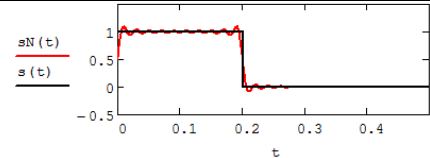
From the analysis of the data presented in Table 2.2, it is possible to draw conclusions about

degree of influence by changing the number of harmonics of the Fourier spectrum of the signal on the

Quality Recovery by truncated row Fourier signal $s_N(t)$,
Execution of Parseval inequality and parameters of the Gibbs effect of the signal $s_N(t)$.

Table 2.2 – Results of the study of spectral analysis and synthesis
of the periodic signal

N, unit.	Graphs of the truncated Fourier series signal $s_N(t)$ and the original signal $s(t)$ given in within one period	Square of the distance between signals $s_N(t)$ and $s(t)$ for fluctuation period T , %	Sum of the powers of the constant component and harmonics of the Fourier spectrum, B^2	Parameters Gibbs effect	
					$\Delta\%$
5		9,720	0,181		17,223
10		5,009	0,190	0,05	11,917

15		3,389	0,193		10,73
20		2,526	0,195		10,218
30		1,688	0,197		9,764
40		1,265	0,197	0,01 2	9,557
50		1,013	0,198	0,00 9	9,441

Procedure for performing work

1. Run Mathcad.
2. Determine in Mathcad for a given periodic $S(t)$ according to the individual task (Table 2.3):

– ω_1 – The main circular frequency periodic signal (frequency of the first harmonic), rad/s;

– a_k – coefficient k "cosine" component of the spectrum Fourier of periodic signal, B;

– b_k is the coefficient k th of the "sinus" component of the Fourier spectrum periodic signal, B;

– $a_0 / 2$ – permanent component spectrum Fourier periodic signal, B;

– A_k is the amplitude of the k th harmonic of the periodic Fourier spectrum signal, B;

- φ_k – Initial phase k -th harmonics spectrum Fourier periodic signal, radians, or degrees;

- amplitude and phase Spectral periodic signal diagrams (build bar spectra);

- the signal reconstructed from the truncated Fourier series $s_N(t)$ which is an approximate image of the $s(t)$ $\sigma_{\gamma N \alpha \lambda}$;

- distance and square of the distance between *the signals* $s_N(t)$ and $s(t)$ for the fluctuation period T , expressed as a percentage relative to square of the signal norm $s(t)$;

- the average signal power $s(t)$ for the oscillation period T ;

- amount capacities permanent component and harmonics of the Fourier spectrum and check the fulfillment of the Parseval inequality;

- t_m is the point in time when the truncated row Fourier Signal $s_N(t)$ Accepts Maximum Significance (study of the Gibbs effect);

- Δ – Maximum Significance absolute signal recovery errors due to the presence of the Gibbs effect;

- $\Delta\%$ – Maximum Significance relative signal recovery errors due to the presence of the Gibbs effect;

3. Explore Impact Changes Quantities harmonics spectrum Fourier signal for the quality of recovery on the truncated Fourier series

$s_N(t)$ signal, the Parseval inequality and effect parameters

Gibbs signal $s_N(t)$. The results of the study are presented in the form of Table 2.2.

4. Save the results of spectral analysis and synthesis of a given periodic signal.

5. Generate a report on the laboratory work.

6. Protect the work.

Variants of tasks

The number of the variant corresponds to the number of the student's surname in the list of the study group (subgroup).

Table 2.3 – Variants of the signals under study

Variant	Signal type	T, s
1		$20 \cdot 10^{-3}$
2		$10 \cdot 10^{-3}$
3		$15 \cdot 10^{-3}$
4		$4 \cdot 10^{-3}$
5		$2 \cdot 10^{-3}$

6		$20 \cdot 10^{-3}$
7		$10 \cdot 10^{-3}$
8		$8 \cdot 10^{-3}$
9		$16 \cdot 10^{-3}$
10		$8 \cdot 10^{-3}$
11		$24 \cdot 10^{-3}$
12		$32 \cdot 10^{-3}$

13		$40 \cdot 10^{-3}$
14		$8 \cdot 10^{-3}$
15		$16 \cdot 10^{-3}$

Contents of the report

The report on the laboratory work is drawn up in the form of a Word document and must include:

- the name of the laboratory work;
- the purpose and program of laboratory work;
- Wording individual Quets and Result its implementation;
- brief conclusions based on the results of laboratory work.

Control questions

1. What signal is called a periodic signal?
2. What is the main idea of spectral analysis of periodic signals?
3. What is the Periodic Signal Spectrum?
4. What requirements must a periodic time function $s(t)$ meet in order to be subjected to a Fourier spectral analysis?

5. What is the Periodic Signal Period?
6. What is the fundamental frequency of a periodic signal?
7. What does the notation of the Fourier series look like for periodic *signals* $s(t)$ on

Interval complex $-\infty \leq t \leq \infty$ (sinus-cosine, Material and
form)?

8. What are the basic functions of the sinus-cosine form of the Fourier series?
9. How to Define permanent component
spectrum Fourier of a periodic signal? What is the meaning of
this parameter?

10. How to Define Coefficient k -th "cosine"
("sine") component of the Fourier spectrum of a periodic signal?

11. What Conclusions Yes, you can do of
Comparisons between real to the sinus-cosine form of notation of
the Fourier series?

12. What is the amplitude spectrum of a periodic signal?
13. What is the amplitude spectral diagram of a periodic signal?
14. What is the phase spectrum of a periodic signal?

15. What Such a thing phase Spectral Diagram
periodic signal?

16. What is harmonica?

17. Continue the phrase: "If the analyzed signal $s(t)$ is real, then ...".

18. What are the main properties of periodic signal spectral diagrams?

19. Continue the phrase: "When there is a need to approximate a periodic function by a trigonometric polynomial, then the smallest square deviation will be obtained if...".

20. How is Parseval's equality formulated?

21. What is called the power spectrum of a periodic signal?