

**Ministry of Education and Science of the Russian Federation
Federal state autonomous educational
institution of higher education
"The North Caucasian federal university"
Institute of information technologies and telecommunications**

**Methodological Recommendations
TO INDEPENDENT WORK OF STUDENTS**

on discipline

Digital Signal Processing / Цифровая обработка сигналов

for students majoring in 02.04.01 Mathematics and Computer Science
profile "Training Program for Electronic Information"

Stavropol, 20 26

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INTRODUCTION

The practical work consists of four laboratory works. The first three works are performed in system of mathematical modeling MathCAD. The computer meeting the following requirements is necessary for their performance:

- the volume of random access memory is not less than 32 MB;
- Windows operating system of version 95 or more late;
- system of mathematical modeling MathCAD of version 6.0 or more late;
- the text editor of Word of version 8 (97) or later is necessary for viewing of electronic version of the description of laboratory works.

The fourth laboratory work (sections 1, 2, 3) are carried out independently by means of the available BAT and EXE files in the corresponding catalogs. Before performing this work, it is necessary to read attentively all text files which are available in the corresponding catalogs.

Each laboratory work has the theoretical part describing the basic concepts. However before its reading it is necessary to read additional literature on the subject of the work performed. Knowledge gained thus will need to be applied both when performing the practical part of work, and by drawing up conclusions.

Except the theoretical each part work has the practical part tasks describing, and also the order and ways of their performance. Before performance of this part follows attentively and completely to read it and to clear for itself all unclear moments.

The list of the available laboratory works is included below:

Work no. 1. “ANALYSIS of CHARACTERISTICS of DIGITAL FILTERS FOR PROCESSING of ONE-DIMENSIONAL SIGNALS».

Work no. 2. “SYNTHESIS OF DIGITAL FILTERS FOR PROCESSING OF ONE-DIMENSIONAL SIGNALS».

Work no. 3. “ DIGITAL PROCESSING OF TWO-DIMENSIONAL SIGNALS ”

Work no. 4 (section 1). "DIGITAL LINEAR FILTERING OF IMAGES".

Work no. 4 (section 2). "DIGITAL NONLINEAR PROCESSING OF IMAGES."

Work no. 4 (section 3). "DIGITAL METHODS OF CORRECTION OF IMAGES".

Program modules are developed for laboratory works together with the staff of TU department: AG Kostevich, VN Ulyanov and SV Popov.

WORK NO. 1

ANALYSIS OF CHARACTERISTICS OF DIGITAL FILTERS FOR PROCESSING OF ONE-DIMENSIONAL SIGNALS

Work purpose: studying the bases of the analysis of characteristics of digital filters.

Prologue

The linear digital filter (DF) is the device in which the current counting of the output signal is presented in the form of linear combination of the current counting of the input signal and the previous entrance and output counting. Processing of input data linear TsF (without effects of quantization of data) is described by difference equation

$$y(nT) = \sum_{i=0}^M a_i x(nT - iT) + \sum_{j=1}^N b_j y(nT - jT),$$

where $x(nT)$ and $y(nT)$ – counting of entrance and output signals of the filter respectively; a_i and b_j – filter coefficients; M and N – the integral numbers defining the filter order; T – sample period of input data.

Treat the main characteristics of linear TsF: transfer (system) function in the Z -form; pulse and transitional characteristics; amplitude-frequency and phase-frequency characteristics; precision characteristics.

Transfer function of $H(z)$ of the filter call the relation of the Z -image of the output signal of $Y(z)$ to the Z -image of the input signal of the filter $X(z)$ under zero initial conditions, ie at $y(-T) = y(-2T) = \dots = y(-NT) = 0$ and, besides, $x(nT) = 0$ at $n < 0$:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^M a_i z^{-i}}{1 - \sum_{j=1}^N b_j z^{-j}}.$$

1. PULSE AND TRANSITIONAL CHARACTERISTICS

The Pulse Response (PR) of linear TsF is called reaction (the output response) of the filter under zero initial conditions on entrance influence in the form of the unit impulse

$$\delta(nT) = \begin{cases} 1 & \text{при } n = 0, \\ 0 & \text{при } n \neq 0. \end{cases}$$

Using expression for transfer function of $H(z)$, believing also $(nT) = \delta(nT)$, and considering that $Z\{(nT) \delta\} = 1$, we receive $Z\{h(nT)\} = H(z)$. Thus, transfer function of the filter and its pulse response are connected unambiguously through Z -transformation.

Knowing the pulse response of $h(nT)$ and the input signal $x(nT)$, it is possible to receive the output signal of the $y(nT)$ filter in the form of discrete convolution

$$y(nT) = \sum_{k=0}^n x(kT)h(nT - kT) = \sum_{k=0}^n h(kT)x(nT - kT).$$

The Surge Characteristic (SC) of linear TsF is called reaction (the output response) of the filter under zero initial conditions on entrance influence in the form of single step function

$$U(nT) = \begin{cases} 1 & \text{при } n \geq 0, \\ 0 & \text{при } n < 0. \end{cases}$$

Discrete signals the unit impulse $\delta(nT)$ and single step function of $U(nT)$ are connected by ratios

$$U(nT) = \sum_{k=0}^n \delta(kT),$$

$$\delta(nT) = U(nT) - U(nT - T).$$

2. AMPLITUDE-FREQUENCY AND PHASE-FREQUENCY CHARACTERISTICS

The Complex Frequency Characteristic (CFC) of the filter represents result of substitution $z = e^{j\omega T}$ in transfer function of $H(z)$.

Function $H(e^{j\omega T})$ it makes the following physical sense. If on the entrance of the filter the complex harmonic signal is given $\{e^{jn\omega T}\}$, that the output signal of the filter in the set mode (at $n \rightarrow \infty$) has the appearance $\{H(e^{j\omega T}) e^{jn\omega T}\}$.

The module of the complex frequency characteristic is called the amplitude-frequency characteristic (AFC) of the filter $A(\omega) = |H(e^{j\omega T})|$, and her argument - the phase frequency characteristic (FChH)

$$\varphi(\omega) = \arg[H(e^{j\omega T})] + 2k\pi, \quad \text{где } k = 0, \pm 1, \pm 2, \dots$$

The $\arg(z)$ function is defined as follows (z – the imaginary):

$$\arg(z) = \begin{cases} \arctg \frac{\text{Im}(z)}{\text{Re}(z)} & \text{Re}(z) > 0, \quad \text{Im}(z) - \text{любоe}, \\ \pi + \arctg \frac{\text{Im}(z)}{\text{Re}(z)} & \text{Re}(z) < 0, \quad \text{Im}(z) > 0, \\ -\pi + \arctg \frac{\text{Im}(z)}{\text{Re}(z)} & \text{Re}(z) < 0, \quad \text{Im}(z) < 0. \end{cases}$$

That is the $\arg(z)$ function changes in limits $\arg(z) \in [-\pi, \pi]$.

Thus, if on the entrance of the filter the discrete harmonic signal in the form of the sinusoid is given – $\{\sin n\omega T\}$, that AChH determines its amplitude, and FChH – the phase of the sine signal at the filter exit. As $\{e^{j\omega T}\}$ – periodic function with the period on frequency $\omega_{\text{ш}} = \frac{2\pi}{T}$, that and functions $H(e^{j\omega T})$, $A(\omega)$, $\varphi(\omega)$ have the same period of repetition. The frequency characteristic of the filter is, as a rule, set on the interval $\left[0, \frac{\pi}{T}\right]$.

Let's define one more characteristic of the digital filter – the group delay time $\tau(\omega)$:

$$\tau(\omega) = -\frac{d\varphi(\omega)}{d\omega}.$$

Approximately constant characteristic of the group delay time in all filter pass band is preferable.

3. PRECISION CHARACTERISTICS

Functions of estimates of mistakes in the output signal depending on data processing parameters belong to precision characteristics of the digital filter. It is possible to allocate four intrinsic errors arising at calculations in the digital filter: the errors connected with determination of coefficients of the filter at its synthesis; inexact representation of coefficients of the filter due to their rounding to the finite number of discharges; quantization noises of counting of input signals on level; noise of rounding of results of arithmetic operations at calculations.

ATsP quantization noise

The device intended for transformation of continuous fluctuation to the sequence of reports each of which is temporary selection, quantized on level, of input fluctuation, is called the analog-to-digital converter (ATDC).

Work of ATsP can be presented in the form of two-stage process. At the first stage the sequence forms $S(nT) = S(t)|_{t=nT}$. At the second stage value of each counting $S(nT)$ is represented the number consisting of the finite number of binary places. As a result the new sequence turns out $S_{\text{KB}}(nT)$. Difference $e(nT) = S(nT) - S_{\text{KB}}(nT)$ is called the quantization noise of the input signal on level or analog-to-digital conversion noise.

The mistakes caused by inexact values filter coefficients

At synthesis of digital filters of value of coefficients (filter parameters), turned out as a result of calculation, it is necessary to round with the set degree of accuracy. As a result of it the TsF actual parameters differ from rated a little. When rounding values of coefficients there can be significant misregistration of zero concerning poles or their full combination. At misregistration even at the small size

because zero and poles are close to rather unit circle in the plane Z , there will be the sharp change of characteristics of the filter. Therefore, development of any TsF shall be followed by the research of influence of inaccuracy of the task of coefficients of TsF that is especially important for recursive filters and filters of the high order.

The mistakes caused by quantization of results of calculations

At implementation of the algorithm of linear digital filtering are carried out addition operation and multiplication by constant numbers (filter coefficients – a_i, b_j). Performance of operation of multiplication is connected with round-off errors (truncation): the work of two numbers from the fixed comma according to b_1 and b_2 discharges may contain $(b_1 + b_2)$ discharges, and this work is usually placed in the register with $b_{ym} < (b_1 + b_2)$ discharges.

Round-off error $e_{ok}(nT)$ for this source it can be evaluated by the upper bound

$$\max |e_{ok}(nT)| = \frac{1}{2} 2^{-b_{ym}} = \frac{\Delta}{2},$$

where $\Delta = 2^{-b_{ym}}$ – the step of quantization of the output data of the multiplier, or can be considered as the digital stable process with the uniform spectral power density with the zero average and the dispersion equal

$$\sigma_{BX}^2 = \frac{\Delta^2}{12} = \frac{2^{-2b_{ym}}}{12}.$$

Having accepted such linear model for each node of multiplication, it is possible to consider all structure of TsF as linear and to calculate the mistake in the output signal of the filter as superposition of mistakes, $e_{ok \text{ BIX}}, k = 1, \dots, K$, caused by all K sources of noise of rounding.

For this purpose it is necessary to define pulse responses of parts of structure of the filter from each k -go of the source of noise (the multiplier k -go exit) to the exit of the filter and to calculate parcels

$$e_{ok \text{ BIX}}(nT) = \sum_{l=0}^{\infty} h_k(lT) e_{ok}(nT - lT).$$

The dispersion of noise of rounding at the filter exit caused by all K sources of noise will be equal

$$\sigma_{OK \text{ BIX}}^2 = \sum_{k=0}^K \sigma_{k \text{ BIX}}^2,$$

where $\sigma_{k \text{ BIX}}^2$ – dispersion of the component of output noise from source k -go.

The dispersion of noise in the output signal caused by k m the source

$$\sigma_{k_{\text{ВЫХ}}}^2 = \sigma_{k_{\text{ВХ}}}^2 \sum_{n=0}^{\infty} h_k^2(nT),$$

where $\sigma_{k_{\text{ВХ}}}^2 = \frac{\Delta_k^2}{12}$.

Using Parseval's equality

$$\sum_{n=0}^{\infty} h_k^2(nT) = \frac{T}{\pi} \int_0^{\pi/T} |H_k(e^{j\omega T})|^2 d\omega$$

it is possible to write down equivalent expression for calculation of dispersion from noise source k-go:

$$\sigma_{k_{\text{ВЫХ}}}^2 = \sigma_{k_{\text{ВХ}}}^2 \frac{T}{\pi} \int_0^{\pi/T} |H_k(e^{j\omega T})|^2 d\omega,$$

where $|H_k(e^{j\omega T})|$ – amplitude-frequency characteristic for noise source k-go.

Calculation of dispersion of noise of rounding is possible also with use of transfer functions for noise source k-go – $H_k(z)$

$$\sigma_{k_{\text{ВЫХ}}}^2 = \sigma_{k_{\text{ВХ}}}^2 \frac{1}{2\pi j} \oint H_k(z) H_k(z^{-1}) z^{-1} dz.$$

Limit cycles

Owing to rounding of results of calculation at the exit of TsF there can be specific periodic fluctuations. Let's explain it the example.

On the entrance of TsF of the first order described by difference equation

$$y(nT) = x(nT) - 0.9y(nT-T),$$

Let's give the signal

$$x(nT) = \begin{cases} 10 & \text{при } n = 0, \\ 0 & \text{при } n \neq 0. \end{cases}$$

At the exit of real TsF in which results of calculations are rounded to integer values the signal will have the following appearance: +10, -9, +8, -7, +6, -5, +5, -5 Limit cycles are very undesirable phenomenon as leading to the emergence of spurious oscillations at the exit of TsF which level can exceed quantization noise level considerably. The most exact and universal method of suppression of limit cycles with simultaneous increase in accuracy of filtering of the entrance array is the way of recursive calculations taking into account the remains from performance of arithmetic operations in TsF.

Description of program modules

Laboratory work consists of 7 parts, each of which is realized in the form of the program document for MathCAD. On each of parts the program module which is in the LAB1 group is organized.

1. Start of the program module

For start of the program module it is necessary to point by the mouse manipulator to the program module (to left-click once of the manipulator) and to press the <Enter> key. As a result of this action the MathCAD system will be started and the chosen program module will automatically be loaded.

2. The operating procedure with program modules

2.1. Program $h(nT)$ module

Possibilities of this program module are that you can set coefficients of recursive and nonrecursive parts of the filter interesting you and to obtain information on the pulse response of the filter. At the beginning of the program module there is the description of the variables used in the module and also the following restrictions are introduced:

1. Sample period = 1 page.
2. Number of counting of entrance and output signals of $L = 20$.
3. Maximum order of the digital filter $K = 20$.

Coefficients of the filter are set in the section “Task of Initial Conditions”. Expression 1 (hereinafter in program modules) sets the input signal. Expression 2 defines the response to entrance influence in the form of the unit impulse $\delta(nT)$ which actually and is the pulse response.

2.2. Program $g(nT)$ module

Possibilities of this program module are that you can set coefficients of recursive and nonrecursive parts of the filter interesting you and to obtain information on the surge characteristic of the filter. At the beginning of the program module there is the description of the variables used in the module with the comments and restrictions described at point 2.1. Coefficients of the filter are set in the section “Task of Initial Conditions”. Expression 3 sets the input signal.

Expression 4 creates the response to entrance influence in the form of step function of $U(nT)$ which actually and is the surge characteristic.

2.3. Program $h(j)\omega$ module

Possibilities of this program module are that you can set coefficients of recursive and nonrecursive parts of the filter interesting you and to receive schedules of amplitude-frequency and phase-frequency characteristics of the filter.

At the beginning of the program module there is the description of the variables used in the module with comments and, the restrictions described at point 2.1.

Coefficients of the filter are set in the section “Task of Initial Conditions”. According to 5 the transfer characteristic is calculated. Expression 6 is used for creation of AChH. Expression 7 is used for creation of FChH.

2.4. Program sw module

The program module “Discrete Convolution of the Input Signal with the Pulse Response of the Filter” is constructed as follows.

At the beginning there is the description of the variables used in the program module with comments at the introduced restrictions:

1. Sample period = 1 page.
2. Number of counting of the input signal of $L = 100$.
3. Number of counting of THEIR filter $K = 100$.

Then the most often used input signals are given: unit impulse of $x_1(n)$, step function of $x_2(n)$, harmonic function of $x_3(n)$, rectangular $x_4(n)$ function, triangular $x_5(n)$ function.

As the pulse response of the filter two functions are set: $h_1(n) = e^{-n/5}$ and $h_2(n) = e^{5 \cdot x_3(n)}$. Expression 8 – the formula of discrete convolution. This program module has the following opportunities: it is possible to choose any of five input signals given in the module or to set the; to choose the pulse response or to set the. The chosen functions need to be set in the section - “The task of initial conditions” as follows:

$$x(n) = \{x_1(n) \div x_7(n)\}, h(n) = \{h_1(n) \div h_3(n)\}.$$

2.5. Program display module

This program module is used for finding of dispersion. At the beginning of the program module there is the description of the variables used in the module. Expression 9 – the set transfer function. Expression 10 – the formula used for finding of dispersion.

2.6. Program modules graf1, graf2, graf3

These program modules are used for creation of the two-dimensional field of round-off errors in TsF of the second order.

The possibility of change of level on which the field of mistakes is limited is provided in all three program modules. It is possible to set another maximum value

of functions near the three-dimensional schedule after record: “function restriction $f_{\max} = \dots$ ”. Change of coefficients of b_{1n} and b_{2n} in the section “Permeability Coefficients of B1 and B2” is also possible. It is possible to set section b_{1i}, b_{2j} and to see the section of the two-dimensional field of round-off errors on this section.

3. The exit from the program module

For the exit from the program module it is necessary to press the combination of the <ALT+F4> keys, at the same time the window reporting about the need of preservation of the changed module will appear. Choose option <is NOT PRESENT>, having pressed it the left button of the mouse manipulator. After that you will appear in the LAB1 window and will be able to continue work, carrying out section 1.

4. The list of the program modules used in laboratory work No. 1

- **h(nT)** – the pulse response of TsF.
- **g(nT)** – the surge characteristic of TsF.
- **h(j) ω** – AChH and FChH TsF.
- **sw** – discrete convolution.
- **disp** – finding of dispersion of round-off errors at TsF exit.
- **graf1** – the two-dimensional field of round-off errors in TsF of the second order without the remains depending on values of permeability coefficients b_1 and b_2 .
- **graf2** – the two-dimensional field of round-off errors in TsF of the second order taking into account the remains depending on permeability coefficients.
- **graf3** – the two-dimensional field of the relative variance of mistakes for two options of creation of TsF of the second order.

Work performance order

1. To study the Description of Program Modules block.
2. On the basis of the transfer functions given below find coefficients of digital filters and setting up them in program modules sketch received by IH, PH, AChH and FChH. Provide block diagrams of the studied digital links. To begin the research of the vsepropuskayushchy and smoothing links with AChH.

The digital integrator with limited time of summing ($M = 3; 4; 5;$)	$H(z) = \frac{1 - z^{-M}}{1 - z^{-1}}$
Digital Differentiator (B1P)	$H(z) = 1 - z^{-1}$
Calculator of the 2nd difference (B2P)	$H(z) = 1 - 2 \cdot z^{-1} + z^{-2}$
All-permeable link ($K = -0.8; \dots; 0.8$)	$H(z) = \frac{K - z^{-1}}{1 - K \cdot z^{-1}}$

The smoothing link ($K = 0.3; \dots; 0.9$)	$H(z) = \frac{1}{1 - K \cdot z^{-1}}$
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3. On the basis of the variant of the task given by the teacher substitute filter coefficients in program modules and sketch the received AChH, FChH, IH and PH. Describe features of characteristics of the filter investigated by you. Changing one of coefficients in recursive or nonrecursive parts of the digital filter describe the changes which happened to characteristics of the filter. Explain these changes. Using the direct form of implementation of TsF provide the block diagram of the filter.

4. In the program sw module in the section “Task of Initial Conditions” set two any of these five input signals, receive discrete convolution and sketch it. Then in the section “The task of initial conditions “set one of the pulse responses given in the program module, and receive discrete convolution. Repeatedly sketch discrete convolution. Write down expressions for signals and pulse responses participating in the parcel.

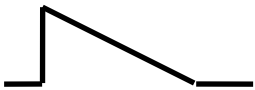
5. Prodiskretiziruyte the continuous signal set to you also receive its analytical description of $x_6(n)$ through unit impulses $\delta(n-k)$ (number of **significant** counting not less than five). Enter the received description of the pro-sampled signal in the section “Task of Initial Conditions”. Sketch discrete convolution of this signal with the symmetric signal of $x_7(n)$. In the section “Task of Initial Conditions” set other formula of the pulse response and sketch discrete convolution again. Write down expressions for signals and pulse responses participating in the parcel.

6. See the program disp module. Sketch dependence of dispersion of the quantization noise of ATsP at the exit of the digital filter from coefficient of smoothing of K . Write down expression for the system $H(z)$ function and sketch the block diagram of TsF. Calculate theoretically dependence of dispersion of the quantization noise of ATsP at the exit of the digital filter from coefficient of smoothing of K . Compare calculation to the obtained data.

7. See the fields of mistakes calculated by program modules graf1, graf2, graf3. Sketch this field of mistakes and explain functional connections from values of coefficients b_1 и b_2 . Setting sections on b_{1i}, b_{2j} , sketch cuts of the two-dimensional field of mistakes on these sections. On each of coefficients set not less than three sections. In total not less than 18 sections will turn out.

Variants of tasks

1 option	Butterworth's filter $a_0=0.01761$ $a_1=0.07044$ $b_1=1.60874$ $a_2=0.10566$ $b_2=-1.31517$	
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	$a_3=0.07044$ $b_3=0.50437$ $a_4=0.01761$ $b_4=-0.07969$	
Option 2	Chebyshev's filter $a_0=0.03493$ $a_1=0.13974$ $b_1=2.55456$ $a_2=0.20961$ $b_2=-3.03822$ $a_3=0.13974$ $b_3=1.85615$ $a_4=0.03493$ $b_4=-0.49433$	
3 option	Bessel's filter $a_0=4.0352810 \cdot 10^{-4}$ $a_1=0.00161$ $b_1=3.05674$ $a_2=0.00242$ $b_2=-3.55347$ $a_3=0.00161$ $b_3=1.85923$ $a_4=4.0352810 \cdot 10^{-4}$ $b_4=-0.36896$	
Option 4	Q=5 resonator $a_0=0.251$ $b_1=0.556$ $a_1=-0.049$ $b_2=-0.778$	
5 option	Rejection Q=5 filter $a_0=0.749$ $b_1=0.556$ $a_1=-0.507$ $b_2=-0.778$ $a_2=0.778$	

Contents of the report

1. The schedules of IH, PH, AChH and FChH received in points 2, 3.
2. Block diagrams of filters on points 2, 3.
3. The schedules of discrete convolutions received in points 4, 5.
4. Schedule of dependence of dispersion on coefficient of K and theoretical calculation of dispersion for point 6.
5. Schedules of characteristic “cuts” from two-dimensional fields of mistakes for recursive TsF of the second order on point 7.
6. Analytical conclusions on work.

List of references

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4. Citizens of Kazan GD, Kuryachy MI, Pustynsky IN Measuring television. - M.: The higher school, 1994. - 288 pages.
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WORK NO. 2

SYNTHESIS OF DIGITAL FILTERS FOR PROCESSING OF ONE-DIMENSIONAL SIGNALS

Work purpose: acquisition by students of skills of design of digital filters and optimization of their parameters.

Prologue

1. SYNTHESIS OF DIGITAL DEVICES FOR PROCESSING OF ONE-DIMENSIONAL DATA

Design of digital filters includes five main stages.

1. The solution of the problem of approximation for the purpose of determination of coefficients of the digital filter at which the filter meets requirements to time or frequency characteristics.

2. Choice of structure (implementation form) of the digital filter.

3. Task of realization of coefficients of the filter, entrance and output signals and arithmetic devices.

4. Check by means of mathematical or simulation modeling of compliance of characteristics of the developed TsF set.

5. Hardware or program sale of the digital filter.

Like calculation of analog filters, calculation of digital filters includes the process of finding of suitable transfer function which properly meets the imposed requirements.

Calculation of digital TsF consists of the following two stages.

1. Receiving suitable transfer function of the analog $H_a(p)$ filter prototype.

2. Creation of the procedure of transition which will transform the $H_a(p)$ function to the corresponding transfer function of $H(z)$.

Let's call the main methods of transformation of the analog filter in digital:

- invariant transformation of the pulse response;
- displays of differentials;
- bilinear transformation;
- Z-forms.

Let's submit the short description of some of these methods.

Method of bilinear Z-transformation

Bilinear transformation represents conformal representation of points *of the p-plane* in points on *the Z-plane* and uses replacements of the look:

$$p \Rightarrow \frac{2}{T} \cdot \frac{z-1}{z+1}. \quad (1)$$

Use of substitution (1) provides unambiguous transformation of transfer function **H(p)** of the analog filter prototype to transfer function **of H(z)** of the digital filter:

$$H(z) = H(p) \Big|_{p = \frac{2}{T} \cdot \frac{z-1}{z+1}}.$$

Let's consider transformation (1) .

To each point of the complex **p-plane** ($p = \sigma + j \omega$) puts in compliance the certain end **of the z-plane** ($z = \exp (\sigma + j \omega) T$).

The conjugate axis **of the p-plane** ($p = j \omega$ for $-\infty < \omega < \infty$) is displayed in the unit circle in **the z-plane** ($z = \exp (jT) \omega$).

The left half **of the p-plane** is displayed in the part **of the z-plane** in the unit disk ($|z| < 1$).

Two circumstances are very important:

First, as all poles of the stable analog filter are located in the left half **of the p-plane** , when transforming the analog filter to digital also stable filter will turn out.

Secondly, as the conjugate axis **of the p-plane** is displayed on the unit circle **of the z-plane** , all maxima and minima **of AChH |H (j) ω|** the analog filter will remain also in **AChH |H (ejT) ω|** the digital filter. Remains as well irregularity **of AChH** for the corresponding frequency bands.

Thus, selective analog filters will be transformed to the corresponding digital filters.

So, the ratio between “analog” frequencies **Ω** and “digital” frequencies **ω** which can be received from (1) is nonlinear:

$$\Omega = \frac{2}{T} \cdot \operatorname{tg} \left(\frac{\omega \cdot T}{2} \right) = \frac{2}{T} \cdot \operatorname{tg} (\pi \cdot w),$$

where $w = \frac{\omega_d}{\omega_d}$ – normalized digital frequency.

Thus, *deformation of the frequency diagram* upon transition from the analog filter to digital takes place.

Deformation of the frequency diagram for frequency-selective filters (**FNCh**, **FVCh**, **PF**, **the Russian Federation**) which approximated **AChH** has the step function appearance, does not lead to violation of selective properties of the filter at bilinear transformation. Deformation of the frequency diagram can be

compensated by means of specification of values of separate frequencies in the analog filter.

Method of Z-forms

By **Z-transformation** definition - $z = e^{p \cdot T}$, and the left half plane **of the p-plane** is displayed in the unit disk **of the z-plane** ($|z| = 1$), and the right halfplane – out of it, at the same time the circle corresponds to the conjugate axis **of j p-plane** ω . Transformation $z = e^{p \cdot T}$ has nonlinear character. For approximation of such transformation for the purpose of receiving the linearized ratios between operators z and p the approximate analytical ratios called **by Z-forms** were received.

The essence of the method of receiving **Z-forms** consists in the following. Taking the logarithm the ratio $z = e^{p \cdot T}$, we have

$$p^{-1} = \frac{T}{\ln z}. \quad (2)$$

Presenting the $\ln z$ function in the row form, we will receive

$$\ln z = 2 \cdot \left[\frac{z-1}{z+1} + \frac{(z-1)^3}{3 \cdot (z+1)^3} + \frac{(z-1)^5}{5 \cdot (z+1)^5} + \dots + \frac{(z-1)^{2n+1}}{(2n+1) \cdot (z+1)^{2n+1}} + \dots \right]. \quad (3)$$

Taking into account (3) ratio (2) takes the form

$$\begin{aligned} p^{-1} &= \frac{T}{2} \cdot \frac{1}{u + \frac{1}{3} \cdot u^3 + \frac{1}{5} \cdot u^5 + \dots + \frac{1}{(2n+1)} \cdot u^{2n+1} + \dots} = \\ &= \frac{T}{2} \cdot \left[\frac{1}{u} - \frac{1}{3} \cdot u - \frac{4}{45} \cdot u^3 - \frac{44}{945} \cdot u^5 + \dots \right], \end{aligned} \quad (4)$$

$$\text{where } u = \frac{(z-1)}{(z+1)}.$$

Considering that the row (4) quickly meets, we will be limited to its body in the look

$$p^{-1} = \frac{T}{2} \cdot \frac{1}{u} = \frac{T}{2} \cdot \frac{z+1}{z-1}. \quad (5)$$

Expression (5) is also **the Z-form** corresponding to p^{-1} operator.

Building both parts of expression (4) in the corresponding degree, we can receive **the Z-form** of p^k operator of higher order:

$$p^{-k} = \frac{N_k(z)}{(z-1)^k}, \quad (6)$$

where $N_k(z)$ – the polynomial of z .

The further technique of transformation is as follows. Transfer function of the known analog filter registers on negative degrees **of $p-1$** instead of **p** . Then each degree **of $p-1$** is replaced with the corresponding rational z - expression from the table **of Z-forms** .

<i>Operator of the p-plane</i>	<i>Corresponding "Z-form"</i>
$p-1$	$\frac{T}{2} \cdot \frac{z+1}{z-1}$
$p-2$	$\frac{T^2}{12} \cdot \frac{z^2 + 10 \cdot z + 1}{(z-1)^2}$
$p-3$	$\frac{T^3}{2} \cdot \frac{z \cdot (z+1)}{(z-1)^3}$
$p-4$	$\frac{T^4}{6} \cdot \frac{z \cdot (z^2 + 4 \cdot z + 1)}{(z-1)^4} - \frac{T^4}{120}$
$p-5$	$\frac{T^5}{24} \cdot \frac{z \cdot (z^3 + 11 \cdot z^2 + 11 \cdot z + 1)}{(z-1)^5}$
$p-6$	$\frac{T^6}{120} \cdot \frac{z \cdot (z^4 + 26 \cdot z^3 + 66 \cdot z^2 + 26 \cdot z + 1)}{(z-1)^6} + \frac{T^6}{30240}$

As a significant proportion of projectible digital BIH-filters demand understanding of computational methods of filters in continuous time, we will give rated formulas for several standard types of analog filters: Butterworth, Bessel and Chebyshev of type 1.

Butterworth's filters

Square of the amplitude characteristic rated (ie the having frequency of the cut 1 ^{I am glad / c)} Butterworth's filter it is equal:

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}, \quad (7)$$

where **n** is the filter order.

Analytically continuing function (7) on all **p** - the plane, we will receive:

$$H^2(p) = \frac{1}{1 + (-p^2)^n} \quad (8)$$

All poles (8) are on the unit circle at identical distance from each other in p - the plane.

Let's express transfer function of $H(p)$ through the poles which are located in left half plane **of** p :

$$H(p) = \frac{k_0}{\prod_{k=1}^n (p - p_k)},$$

$$\text{where } p_k = \exp\left[j\pi \cdot \left(\frac{1}{2} + \frac{(2 \cdot k - 1)}{2 \cdot n}\right)\right], \quad k = 1, 2, \dots, n,$$

k_0 – the rationing constant.

Let's formulate the main properties of the rated filter of Butterworth of the lower frequencies.

1. At any n such ratios are fair:

$$|H(j0)|^2 = 1, \quad |H(j1)|^2 = 0.5 \quad \text{and} \quad |H(j)\omega|^2 \rightarrow 0.$$

From here follows that cut frequency on the level of 3 dB is equal 1 I am glad / page ·

2. Function of the module of transfer of filters of Butterworth monotonously decreases at $\omega \geq 0$. Therefore, $|H(j)\omega|$ has the maximum value at $\omega = 0$.

3. Butterworth's filters are characterized by the fact that have the most flat amplitude-frequency characteristic in origin of coordinates in p - the plane.

4. The steepness of AChH of the filter of Butterworth **of** n -go of the order at high frequencies makes $20n$ ·dB/decade.

Chebyshev's filters

The square of the module of function of transfer of the filter of Chebyshev can be written down in the look

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 T_n^2(\omega)},$$

where ε represents the parameter which establishes the size of irregularity of transfer, and $T_n(\omega)$ – Chebyshev's polynom determined by expression:

$$T(\omega) = \cos(n \cdot \cos^{-1} \omega).$$

The rated low-pass filter of Chebyshev **of** n -go of the order has the following main properties.

1. For $|\omega| \leq 1$ values of the $|H(j\omega)|^2$ function **fluctuate** between two limits $\frac{1}{1+\varepsilon^2}$ and 1. In total on the interval $0 \leq \omega \leq 1$ there is **n** of critical points in which the $|H(j\omega)|^2$ function **reaches** the maximum value equal 1, or the minimum value equal $\frac{1}{1+\varepsilon^2}$.

2. At $\omega \geq 1$ $|H(j\omega)|^2$ function **monotonously** decreases and tends to zero. The steepness of recession at high frequencies makes **20n** ·dB/decade.

3. The $|H(j\omega)|^2$ function **meets** the following conditions:

$$|H(j1)|^2 = \frac{1}{1+\varepsilon^2}, \text{ and}$$

$$|H(j0)|^2 = 1 \text{ if } n \text{ is odd, or}$$

$$|H(j0)|^2 = \frac{1}{1+\varepsilon^2}, \text{ if } n \text{ is even.}$$

Function of the filter of Chebyshev has only poles – the numerator represents it the stationary value. Poles of the filter of Chebyshev are located on the ellipse. The big axis of this ellipse passes on the conjugate axis **of p** - the plane whereas the small axis – along the real axis.

Transfer function of the filter of Chebyshev is defined as follows:

$$H(p) = \frac{k_0}{\prod_{k=1}^n (p - p_k)},$$

where **k₀** – the rationing constant,

pk – poles: $p_k = \sigma_k + j\xi_k, \quad k = 1, 2, \dots, n.$

Here

$$\sigma_k = -\text{sh} \left[\left(\frac{1}{n} \right) \cdot \text{arcsh} \left(\frac{1}{\varepsilon} \right) \right] \cdot \sin \frac{2 \cdot k - 1}{2 \cdot n} \cdot \pi,$$

$$\xi_k = \text{ch} \left[\left(\frac{1}{n} \right) \cdot \text{arcsh} \left(\frac{1}{\varepsilon} \right) \right] \cdot \cos \frac{2 \cdot k - 1}{2 \cdot n} \cdot \pi.$$

The main difference of filters of Chebyshev is that they have property of the optimality. In other words, if any n-go filter of the order containing only poles has in the transmission band the best characteristics in comparison with Chebyshev's filter of the order **of n**, then in the strip of not transmission of the characteristic of this filter will be for certain worse.

Bessel's filters

Bessel's filters are characterized by the most smooth characteristic of the group delay time in origin of coordinates in **the p-plane** . The surge characteristic of filters of Bessel has very small emission (normal less than 1%), and both pulse and amplitude-frequency characteristics aim at the Gaussian curve in process of increase in the order of the filter.

Transfer function of filters of Bessel registers in the look:

$$H(p) = \frac{d_0}{B_n(p)},$$

where **B_n(p)** – order **n-go** Bessel function,

and $d_0 = \frac{(2 \cdot n)!}{2^n \cdot n!}$, **d0** – the rationing constant.

Bessel function can be presented in the form:

$$B_n(p) = \sum_{k=0}^n d_k \cdot p^k,$$

where $d_k = \frac{(2 \cdot n - k)!}{2^{n-k} \cdot k! \cdot (n - k)!}$, $k=0, 1, \dots, n$.

Bessel's filters have only poles which are located on the circle with the center on the transverse positive half shaft **of the p-plane** . Unlike Butterworth's filters the frequency of the cut of filters of Bessel depends on the filter n order.

2. SYNTHESIS OF KIH-FILTERS BY METHOD OF TEMPORARY WINDOWS

As the frequency characteristic **of $H(ej\omega)$** of any TsF represents periodic function of frequency ω , it can be spread out in a row by Fourier:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-j\omega n}. \quad (9)$$

One of possible ways of receiving digital KIH-filters is truncation of the infinite series (9) to the finite number of members.

However follows from the well-known Gibbs phenomenon that truncation of the row (9) causes emissions and fluctuations in the required frequency characteristic before and after any discontinuity point. The size of emission makes about 9% of amplitude in discontinuity point.

The method of weighing is used for obtaining the finite weight sequences **of $W(n)$** called by windows which modify Fourier's coefficients in the equation (9) for receiving the required pulse response **of $h_0(n)$** of final duration.

At this **$h_0(n) = h(n)W(n)$** ,

where $W(n)$ – the sequence of final duration, ie

$$W(n) = 0 \text{ for } n < 0 \text{ and } n > N - 1,$$

and $h(n)$ – the Fourier series coefficients representing the pulse response of TsF

$$h(n) = \frac{1}{\pi} \int_0^{\pi} H(e^{j\omega}) \cdot e^{j\omega \cdot n} d\omega.$$

As multiplication of two sequences in the time domain is equivalent to convolution of their ranges in frequency domain, the method of weighing provides significant smoothing of emissions of the initial frequency characteristic of TsF.

In conclusion we will give some functions of temporary windows which are often used in practice.

Dirikhle's window (rectangular window)

$$W(n) = \begin{cases} 1 & , \quad 0 \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1. \end{cases}$$

Bartlett's window (triangular window)

$$W(n) = \begin{cases} \frac{2 \cdot n}{N-1} & , \quad 0 \leq n < \frac{N-1}{2}, \\ 2 - \frac{2 \cdot n}{N-1} & , \quad \frac{N-1}{2} \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1. \end{cases}$$

where $N - 1$ is an even number.

Window Hanna

$$W(n) = \begin{cases} \frac{1}{2} \cdot \left(1 - \cos\left(\frac{2\pi \cdot n}{N-1}\right)\right) & , \quad 0 \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1. \end{cases}$$

Hamming's window

$$W(n) = \begin{cases} 0,54 - 0,46 \cdot \cos\left(\frac{2\pi \cdot n}{N-1}\right) & , \quad 0 \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1. \end{cases}$$

Blackman's window

$$W(n) = \begin{cases} 0,42 - 0,5 \cdot \cos\left(\frac{2\pi \cdot n}{N-1}\right) + 0,08 \cdot \cos\left(\frac{4\pi \cdot n}{N-1}\right) & , \quad 0 \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1. \end{cases}$$

Kaiser's window

$$W(n) = \begin{cases} \frac{I_0 \cdot \left[\omega_a \cdot \sqrt{\left(\frac{N-1}{2}\right)^2 - \left(n - \frac{N-1}{2}\right)^2} \right]}{I_0 \cdot \left[\omega_a \left(\frac{N-1}{2}\right) \right]} , & 0 \leq n \leq N-1, \\ 0 & , \quad n < 0, n > N-1, \end{cases}$$

where $I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{\left(\frac{x}{2}\right)^k}{k!} \right]^2$ – modified Bessel function of the order zero of the first sort, and ω_a – window form parameter. The most typical values

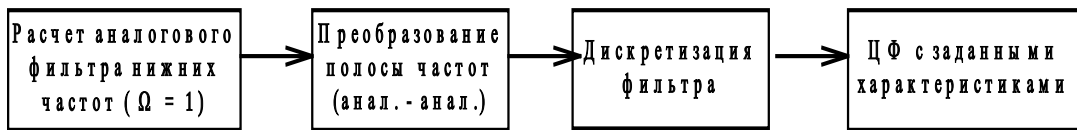
$$\omega_a \cdot \frac{N-1}{2} = 4 \div 9.$$

3. FREQUENCY TRANSFORMATIONS

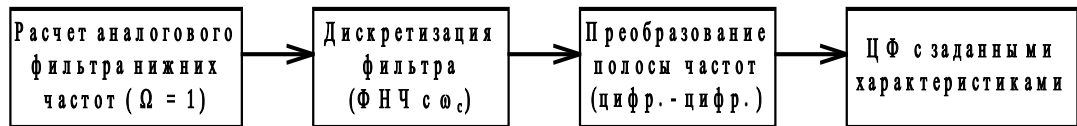
Above we considered only low-pass filters. When calculating digital high-pass filters, strip and rejection two approaches presented in figures 1 and 2 are used.

Frequency transformations of low-pass filters

Method 1



Method 2



Let's consider the method 2. Formulas for the FNChFNCh1 transformations →(with other strip), FNChFVCh, →FNChPF, →FNChRF are included below →.

1. FNCH →FNCH1

$$z^{-1} \rightarrow \frac{z^{-1} - \alpha}{1 - \alpha \cdot z^{-1}},$$

ω_c - FNCh cut
frequency

ω_u - FNCh1 cut
frequency

$$\alpha = \frac{\sin\left\{\left(\left[\omega_c - \omega_u\right] \middle/ 2\right) \cdot T\right\}}{\sin\left\{\left(\left[\omega_c + \omega_u\right] \middle/ 2\right) \cdot T\right\}}.$$

2. FNCH → FVCH

$$z^{-1} \rightarrow -\frac{z^{-1} + \alpha}{1 + \alpha \cdot z^{-1}},$$

ω_c - FNCh cut frequency

ω_u - FVCh cut frequency

$$\alpha = -\frac{\cos\left\{\left(\left[\omega_c - \omega_u\right] \middle/ 2\right) \cdot T\right\}}{\cos\left\{\left(\left[\omega_c + \omega_u\right] \middle/ 2\right) \cdot T\right\}}.$$

3. → PF FNCH

$$z^{-1} \rightarrow -\frac{z^{-2} - \frac{2\alpha k}{k+1}z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1}z^{-2} - \frac{2\alpha k}{k+1}z^{-1} + 1},$$

ω_u , ω_l - upper and lower frequencies of the cut of the PF respectively

$$\alpha = \frac{\cos\left\{\left[\left(\omega_u + \omega_l\right) \middle/ 2\right] \cdot T\right\}}{\cos\left\{\left[\left(\omega_u - \omega_l\right) \middle/ 2\right] \cdot T\right\}},$$

$$k = \operatorname{ctg}\left[\left(\frac{\omega_u - \omega_l}{2}\right) \cdot T\right] \cdot \operatorname{tg}\left(\frac{\omega_c \cdot T}{2}\right).$$

4. FNCHRF →

$$z^{-1} \rightarrow -\frac{z^{-2} - \frac{2\alpha}{1+k}z^{-1} + \frac{1-k}{1+k}}{\frac{1-k}{1+k}z^{-2} - \frac{2\alpha}{1+k}z^{-1} + 1},$$

ω_u , ω_l - upper and lower frequencies of the cut of the Russian Federation respectively

$$\alpha = \frac{\cos\left\{\left[\left(\omega_u - \omega_l\right)/2\right] \cdot T\right\}}{\cos\left\{\left[\left(\omega_u + \omega_l\right)/2\right] \cdot T\right\}},$$

$$k = \operatorname{tg}\left[\left(\frac{\omega_u - \omega_l}{2}\right) \cdot T\right] \cdot \operatorname{tg}\left(\frac{\omega_c \cdot T}{2}\right).$$

4. THE MISTAKES CAUSED BY ROUNDING OF COEFFICIENTS OF THE FILTER

At synthesis of digital filters of value of coefficients (filter parameters), turned out as a result of calculation, it is necessary to round with the set degree of accuracy. As a result of it the TsF actual parameters differ from rated a little. When rounding values of coefficients there can be significant misregistration of zero concerning poles or their full combination. At misregistration even at the small size because zero and poles are close to rather unit circle in the plane Z, there will be the sharp change in characteristics of the filter. Therefore, development of any TsF shall be followed by the research of influence of inaccuracy of the task of coefficients of TsF that is especially important for recursive filters and filters of the high order.

Description of program modules

For start of the program module perform the following operations:

- Launch the MathCAD 5.0 environment having double clicked “mouse” on the corresponding pictogram in the application window.
 - Having entered the MathCAD environment select the Open Document item from the File menu.
 - In the appeared dialog box choose the necessary module and press <OK>.
- Upon termination of work with the module select the Close Document item from the File menu.

Program modules for synthesis of the rejection contour:

Program res_1.mcd module

In this program module comparison of different methods of synthesis of digital filters is implemented. Setting the center frequency of the contour, its good quality and sampling rate it is possible to observe implementation of several methods of approximation of analog circuits on schedules of AChH and FChH.

In the program module are designated:

HB, HI, HO, HZ – transfer functions of TsF;

AB, AI, AO, AZ – TsF amplitude-frequency characteristics;

β_B , β_I , O, Z β_B – phase frequency characteristics of TsF.

The values recommended of parameters:

Q: ((1 5) ÷;

F0:= (0.5 ÷3) ·106 Hz;

F: ((5 10)F0 ÷.

Program bilin.mcd module

This module carries out synthesis of the rejection contour by method of bilinear transformation with correction of frequency and without it. Thus it is possible to observe qualitative character of detuning of frequency at BLP comparing ChH TsF.

In the program module are designated:

HB and N – transfer functions of TsF;

AB and A-TsF amplitude-frequency characteristics;

β_B and β_I – phase frequency characteristics of TsF.

The values recommended of parameters:

Q: ((1 5) ÷;

F0:= (0.5 ÷3) ·106 Hz;

F: ((5 10)F0 ÷.

Program modules for synthesis of TsF LF

Program modules batter.mcd, bessell.mcd, chebysh.mcd

These modules realize Butterworth, Bessel and Chebyshev's rated digital filters of the lower frequencies. Setting the order N analog filters prototypes, we receive its ChH, and then sampling it by method of bilinear transformation, we compare AChH and FChH of the synthesized TsF and the analog filter prototype.

Further it is offered to find coefficients and to construct structure of TsF. For this purpose write down system function of the filter in expanded form by the example given in the module. All subsequent actions are made with use of the symbolical processor. For this purpose select the Load Symbolic Processor item from the Symbolic menu. Having allocated convertible expression with the blue frame make the following actions selecting the corresponding items from the Symbolic menu:

- “Simplify” - simplification of all expression;
- “Expand Expression” - decomposition of expression having allocated numerator;
- “Expand Expression” - decomposition of expression having allocated the denominator.

The received expression will represent fractional and rational expression $H(z) = \frac{M(z)}{N(z)}$, $M(z)$ and $N(z)$ - polynomials from z in positive degrees of z .

Lead the received system function to the look convenient for implementation of TsF.

For this purpose:

- divide numerator and the denominator into z^N where N – the maximum degree in expression;
- normalize the denominator so that the coefficient at z^0 became equal to unity;
- divide number before all expression into value of the normalization;
- allocate all expression with the blue frame and implement clause from the Symbolic - “Evaluate” menu - “Floating Point Evaluation ...” with the accuracy of representation of number – 10^{-6} ;
- lead numerator and the denominator to the convenient look in decreasing order of z^n degrees.

Program `l_f_filt.mcd`, `u_f_filt.mcd`, `b_f_filt.mcd` and `r_f_filt.mcd` modules

In these program modules the FNCh frequency transformations to FNCh1, FVCh, PF and the Russian Federation are carried out, respectively setting the coefficients received earlier and frequencies of the calculated filter, it is possible to calculate AChH of the initial and transformed filters.

Normalized frequencies of transition are recommended to be chosen within $0 \div \pi$.

Program modules of assessment of mistakes at change of coefficients

Program `o10.mcd` module

The module realizes calculation and display of mistakes in representation of recursive coefficients of the digital filter in the percentage ratio. Setting coefficients of the filter and the error of display of coefficients (in %), we will observe as a result changes of AChH owing to the error of the task of one of coefficients and function of the mistake. In the same module coefficients of the filter are quantized on the certain number of binary places after the comma.

Work performance order

1. Study the Description of Program Modules block.
2. Make synthesis of the rejection filter by different methods:
 - invariant transformation of the pulse response;
 - displays of differentials;
 - bilinear transformation;
 - **Z-forms** .

Set the center frequency, good quality of the contour and sampling rate. Draw conclusions on the difference of methods of synthesis of digital filters and area of their applicability.

3. At synthesis of the rejection filter make correction of frequency for the method of bilinear transformation, explain need and physical sense of its shift.

4. Make synthesis of the set TsF LF of the 3rd or 4th order by method of bilinear transformation. Sketch amplitude and phase frequency characteristics and the characteristic of the group delay time.

5. Pass from $H(p)$ to $H(z)$, replacing $p \rightarrow \frac{2}{T} \cdot \frac{z-1}{z+1}$ and substituting numerical values T and denominator roots; execute symbolical processing and calculate coefficients of a_i and b_j , draw structure of the filter.

6. Execute all frequency transformations for the set filter (see section 3) and sketch the received AChH of the initial and transformed filters.

7. Make the analysis of influence of errors of the task of coefficients of digital FNCh for AChH (changing one of b_j coefficients). Describe the nature of change of ChH. Draw the conclusion on influence of change of one of coefficients on the behavior of the filter.

8. Quantize coefficients of the digital filter on such number of binary places that the maximum deviation of AChH from initial made about 10 - 20%. Sketch AChH and describe the nature of its change.

Table of tasks

Option No.	Name of the LF filter	Order No.
1	Butterworth	3
2	Butterworth	4
3	Chebysheva	3
4	Chebysheva	4
5	Bessel	3
6	Bessel	4

Contents of the report

1. Schedules of AChH and FChH and group delay of digital filters.
2. The block diagram of TsF LF received in item 5 .
3. The schedules characterizing the influence of errors of the task of coefficients of the filter on AChH.
4. Conclusions on work.

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DIGITAL PROCESSING OF TWO-DIMENSIONAL SIGNALS

Work purpose : assessment of quality of linear and nonlinear (rank) filtering of images, studying of the main algorithms of processing of images.

1. INTRODUCTION

The two-dimensional discrete signal is the function defined on set of the arranged couples of integral numbers:

$$x = \{ x(n_1, n_2), 0 \leq (n_1, n_2) < \infty \}.$$

Now widely use the two-dimensional filters corresponding to space structure of the image to processing of images. For linear two-dimensional filters, physically implementable and invariant in time, the output signal registers in the form of two-dimensional discrete convolution [1]

$$y(n_1, n_2) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} x(i_1, i_2) \cdot h(n_1 - i_1, n_2 - i_2), \quad (1)$$

where $h(n_1, n_2)$ – the pulse response of the filter.

Function of transfer of the two-dimensional linear filter can be written down in the look:

$$H(z_1, z_2) = \frac{\sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} \cdot z_1^{-i_1} z_2^{-i_2}}{1 - \sum_{j_1=0}^{N_1} \sum_{j_2=0}^{N_2} b_{j_1 j_2} \cdot z_1^{-j_1} \cdot z_2^{-j_2}}, \quad (j_1, j_2) \neq (0, 0)$$

where values $a_{i_1 i_2}$ and $b_{j_1 j_2}$ determine filtering parameters. Let's enter the concept of basic areas of the filter (the set of values of the signals used at calculations) on input and output signals respectively:

$$\begin{aligned} & \{ (i_1, i_2): 0 \leq i_1 < M_1, 0 \leq i_2 < M_2 \}, \\ & \{ (j_1, j_2): 0 \leq j_1 < N_1, 0 \leq j_2 < N_2, (j_1, j_2) \neq (0, 0) \}. \end{aligned}$$

At $b_{j_1 j_2} = 0$ Nonrecursive algorithms of data processing are implemented by 0 two-dimensional filter, and expression for discrete convolution of the input signal and the pulse response of the filter has the appearance

$$y(n_1, n_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} \cdot x(n_1 - i_1, n_2 - i_2)$$

Thus, counting of the output signal of the two-dimensional nonrecursive filter represents the weighed sum of counting of the input signal in basic area. At sale of such filter for formation of each of entrance counting it is necessary to execute

$(M1+1)(M2+1)$ of umnozheniye and $(M1M2+M1+M2)$ of additions over the suspended counting of the input signal. Reading of video signal in raster systems is made “the element behind the element” and “the line after line”, and in this case when processing of the $z1-1$ and $z2-1$ operators Who is this video? in real time for implementation lag lines are used.

Let's give examples the most often used at information processing in television measuring systems of nonrecursive operators, the decomposition 3×3 elements in size. In fig. 1 coefficients of two-dimensional filtering $a_{i_1 i_2}$ are provided in the type of the masks describing pulse responses of the corresponding filters [2].

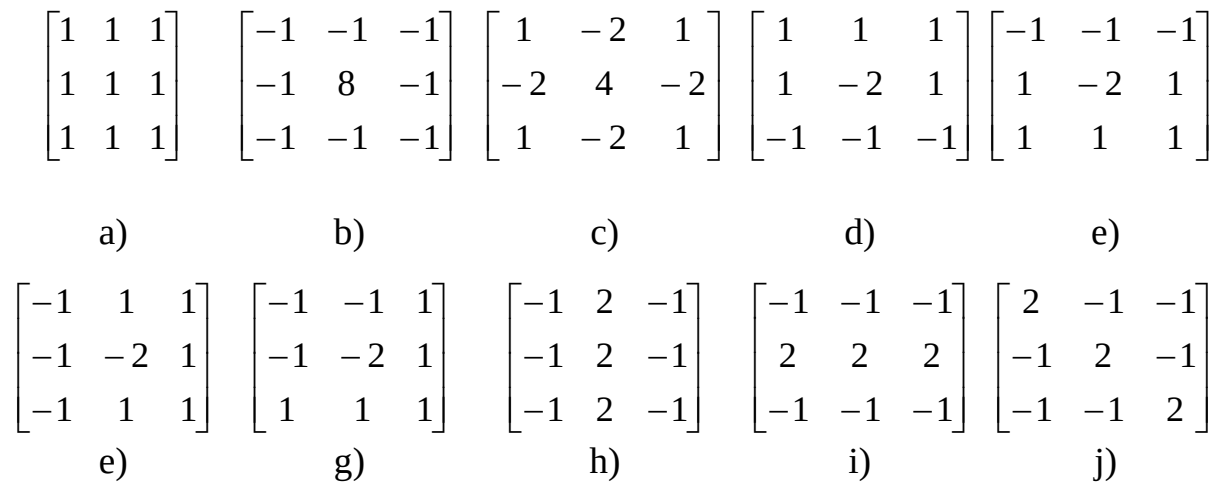


Fig. 1. – Pulse responses of linear two-dimensional filters

"Sliding average" filter (two-dimensional low-pass filter):

The filter which is summing up counting of the input signal with the equal weight realizes the calculation algorithm of “the sliding average” (fig. 1a). When using such filter for processing of the video information it is possible to increase the relation signal/noise in the output signal [2]. The output signal of the “filter the sliding average” is calculated according to (1) at coefficients $a_{i_1 i_2} = 1$ also has the appearance

$$y_1(n_1, n_2) = x(n_1, n_2) + x(n_1-1, n_2) + x(n_1-2, n_2) + x(n_1, n_2-1) + x(n_1-1, n_2-1) + x(n_1-2, n_2-1) + x(n_1, n_2-2) + x(n_1-1, n_2-2) + x(n_1-2, n_2-2).$$

Low-frequency two-dimensional filters like the filter in fig. 1a leave low-frequency components untouched and weaken high-frequency components. Such filters are used for decrease in the visual noise which is contained in the image and also for removal of high-frequency components from the image it was possible to investigate carefully contents low-frequency the component. Let's consider the part of the image which is not containing high-frequency the component. It means that all values of elements of the image are constant or slowly changing. As the low-frequency mask passes through area of the image, new value of convertible elements of the image are calculated according to (1). If all values of elements of the image in the field of the adjunction are identical (are constant), new values of

elements of the image will be same as well as initial. Thus, when processing low-frequency components remain, any fast changes of intensity are averaged with the remained image elements in the field of the adjunction and by that level high-frequency the component goes down. Visual result of low-frequency filtering is weak unsharpness of the image [3].

Laplacian for “eight neighbors” (the two-dimensional filter high frequencies)

The operator of processing of two-dimensional data described by the mask given on fig. 1b realizes function of allocation of signals of small-sized objects from the background component in video signal. The output signal of this filter is connected with the input signal the look equation

$$y_2(n_1, n_2) = -x(n_1, n_2) - x(n_1-1, n_2) - x(n_1+1, n_2) - x(n_1, n_2-1) - x(n_1, n_2+1) - x(n_1-1, n_2-1) - x(n_1+1, n_2-1) - x(n_1-1, n_2+1) - x(n_1+1, n_2+1).$$

High pass filters increase the level of small-sized details in the image. Such filtering is used when it is necessary to investigate high-frequency structure of the object [3].

Similarly it is possible to describe communication of output and entrance signals of two-dimensional nonrecursive filters which masks are given in fig. 1, into.

Allocation of “edge” (difference of brightness in the image)

Allocation of “edge” is used as the preliminary step in the course of extraction of signs of the image. Though allocation of “edge” is generally used in machine sight, it of course, has also other applications. For example, information on “edge” obtained in the course of its allocation can be used in the initial image for strengthening of its clarity.

Allocation of “edge” can be used as the method for production of original images which can then be retouched in programs of drawing for creation of the highly artistic image [3].

Let's consider two methods of strengthening of "edge":

Allocation of “edge” according to Laplace

The method of strengthening of edges according to Laplace differs from other methods in the fact that edges are highlighted irrespective of the direction. Function $f(x, y)$ Laplace registers in the look

$$L(f(x, y)) = d^2f/dx^2 + d^2f/dy^2,$$

where d^2f/dx^2 – the second partial differential coefficient on x , and d^2f/dy^2 – the second partial differential coefficient on y .

For discrete functions second differential coefficients can be approximated as follows:

$$\begin{aligned} d^2f/dx^2 &= f(x+1, y) - 2f(x, y) + f(x-1, y), \\ d^2f/dy^2 &= f(x, y+1) - 2f(x, y) + f(x, y-1). \end{aligned}$$

Thus, the Laplacian can be written down in the following look:

$$L(f(x, y)) = f(x+1, y) + f(x-1, y) + f(x, y-1) + f(x, y+1) - 4f(x, y).$$

This expression can be considered as the pulse response of the filter, written down in the form of function $f(x, y)$. For convenience signs before coefficients change for opposite. The Laplacian mask for “four neighbors” has the appearance:

$$p(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$

The example of the mask of the Laplacian for “eight neighbors” is given in fig. 1b.

Allocation of “edge” by method of the directed gradient

Sometimes it is necessary to highlight also other “edges” of the image, not strictly vertical or horizontal. Diagonal “edges” can be important. Selective highlighting of edges in the different directions is used sometimes in order that the user gained general idea about the image. The method of the directed gradient is just used for this purpose. For highlighting “edges” in the different directions 8 different masks are used, four of them are given in fig. 1g-1zh. These directions are called as part of the world: “North”, “South”, “East”, “southeast” (all there are 8 masks).

Intensity of the output element of the image will depend on the gradient of change of brightness (the more the inclination, the is brighter the element). For example, the gradient “East” will strengthen “edge” which contains transition from black to white [5] “from left to right”.

Allocation of lines

The masks given on fig. 1z – 1k, are called operators of allocation vertically, horizontally and the inclined oriented details, respectively.

Rank filtering

The specific place among algorithms of nonlinear processing of signals is held by procedures of rank data filtering. Rank data processing call transformation of set of counting of the signal to the variational series in which counting of the signal is placed on the importance (increase of values of amplitudes). The first (minimum) rank in the varying series of numbers occupies counting of the signal with the minimum amplitude. The last (maximum) rank in the varying series of numbers occupies counting of the signal with the maximum amplitude. Most often when processing signals algorithms of median filtering which realize procedures of calculation of the center of the variation series [2] are used.

Let's consider the principle of data processing in the median filter on the example.

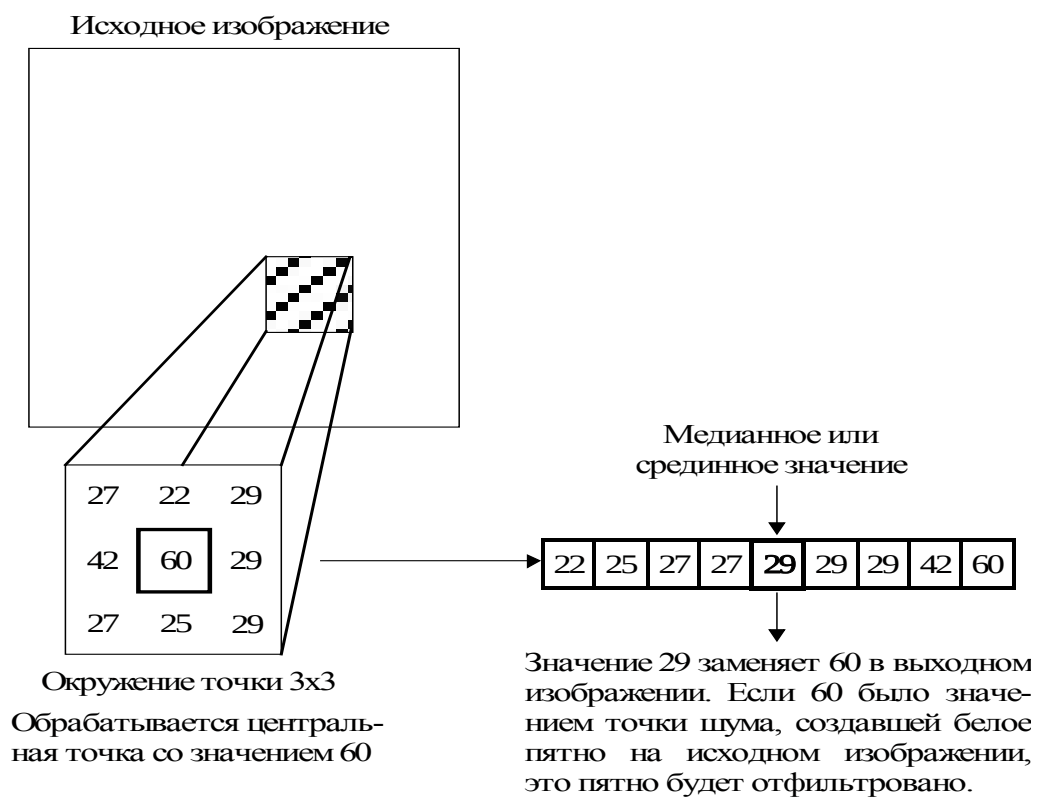


Fig. 2. – Operation of the median filter

2. LINEAR AND RANK FILTERING (PART 1)

2.1. Description of tool programs

Name program s	Assignment	Syntax
view	Carries out viewing of images	view < file name >
med	Carries out median filtering	med <name of the vkh.file> <name of the vykh.file> At implementation of this command on the screen there are names of the files containing weighting coefficients of filters. Choose the necessary file with the *.msk expansion and press <Enter>
fir	Carries out two- dimensional linear convolution	fir <name of the vkh.file> <name of the vykh.file> At implementation of this command on the screen there are names of the files containing weighting coefficients of filters. choose the necessary file with the *.msk expansion and press <Enter>
norm	Carries out the file normalization	norm < file name >
add	Carries out addition of two images	add<slag.1> <slag.2> <sum>
sub	Subtracts two images	sub <deductible> <subtractor> <difference>
stat	Shows distribution of the image (histogram), the mean square deviation, the ensemble average	stat <file name>
cut	Carries out viewing of oscillograms of lines of the site of the image	cut <name of the vkh.fayl> <name of the vykh.fayl> move the white square, the specified sizes, on the respective site of the image and press the F3 button. There will be oscillograms of lines of the allocated site of the image.

The fir program carries out convolution of the image with the set mask $a_{i_1 i_2}$ according to expression

$$y(n_1, n_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} x(n_1 - i_1, n_2 - i_2), \quad n_1 \geq 0, \quad n_2 \geq 0, \quad (2)$$

where $x(n_1, n_2)$, $y(n_1, n_2)$ – counting of entrance and output images, $a_{i_1 i_2}$ – weighting coefficients of the mask $M_1 \times M_2$ size. They say that mask $a_{i_1 i_2}$ "slides" according to the image and for each provision of the mask expression is calculated (2).

Entrance counting has to be located in the file is byte-by-byte, and output counting registers in the output file as two-byte to avoid overflow. To provide the filter coefficient equal 1, it is necessary to normalize all reports of the processed image on the constant equal to the sum of output coefficients of the mask. In the MED program for the image the mask also "slides" and for each provision of the mask one value of the output image is calculated. At each provision of the mask in the separate array reports of the image to which there correspond nonzero elements of the mask are copied. Then these reports are sorted in ascending order of values (the variable series is under construction) and as day off value of the report which reached central (median) position in the variational series is accepted.

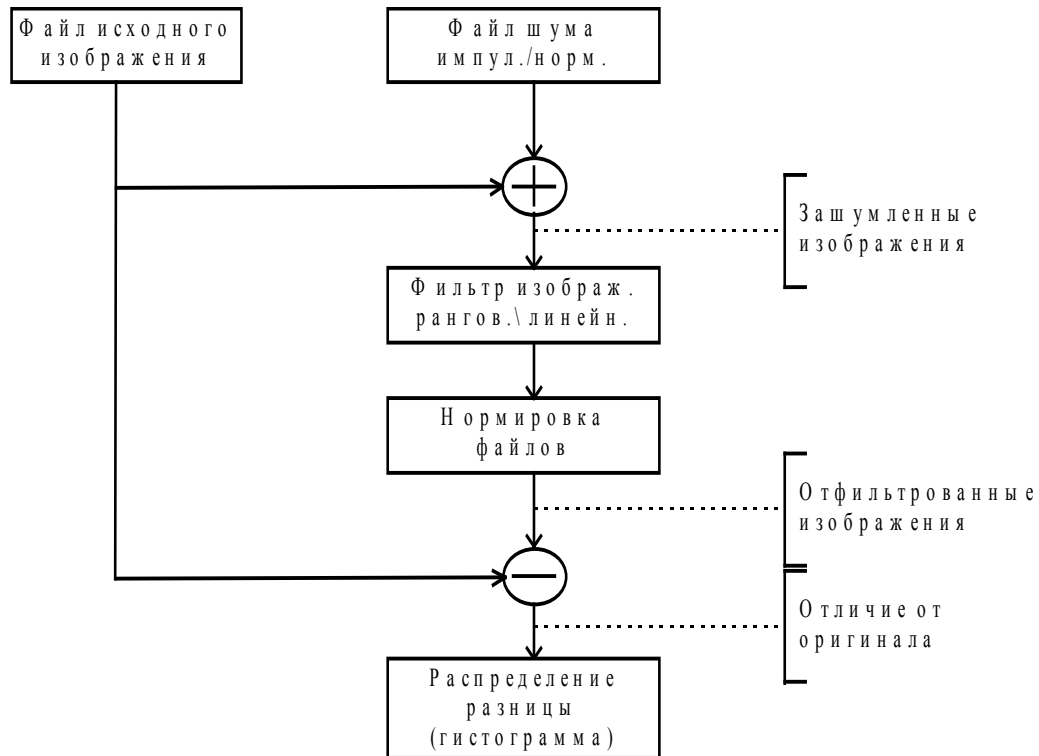
2.2. Input data:

- the initial image – Lena.dat;
- the image containing the impulse noise – in.dat;
- the image containing normal noise – n.dat.

Files with weighting coefficients of filters:

File name	M1.msk	M2.msk	M3.msk	M4.msk	M5.msk
Weight filter coefficients	0 0 0 1 1 1 0 0 0	0 1 0 1 1 1 0 1 0	1 1 1 1 1 1 1 1 1	0 0 1 0 0 0 1 1 1 0 1 1 1 1 1 0 1 1 1 0 0 0 1 0 0	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Number of weighting coefficients	3	5	9	13	16

2.3. Work performance algorithm



2.4. Order of performance of the part of 1 work

In the manager of programs, in the group “Appendices”, “double click” on the pictogram of «LAB3».

2.4.1. Check the initial image, the image of impulse and normal noise. For this purpose gather in the command line:

view Lena im n

2.4.2. Create two files of the image, noisy impulse and normal noise. Gather in the command line:

add Lena im i0

i0.dat – the image, noisy the impulse noise.

Gather in command hundred-fate:

add Lena n n0

2.4.3. See initial, noisy impulse, normal noise of the image. Gather in the command line:

view Lena i0 n0

2.4.4. Execute median filtering of the image, noisy the impulse noise. Gather in the command line:

med i0 ri1

Choose the M1.msk file and press <Enter>.

In the catalog the file of the filtered image – ri1.dat will appear.

Similarly, using masks Sq.m, M3, M4, M5, receive the filtered images ri2.dat, ri3.dat, ri4.dat, ri5.dat. For this purpose gather in the command line:

```
med i0 ri2  
med i0 ri3  
med i0 ri4  
med i0 ri5
```

2.4.5. Look at the results of filtering. For this purpose gather in the command line:

```
view i0 ri? .dat
```

2.4.6. Execute median filtering of the image, noisy normal noise. Carry out the procedure of filtering similar to item 4.4. Gather in the command line:

```
med n0 rn1  
med n0 rn2  
med n0 rn3  
med n0 rn4  
med n0 rn5
```

2.4.7. See results of filtering. For this purpose gather in the command line:

```
view n0 rn? .dat
```

2.4.8. Execute linear filtering of the image with pulse responses of the filter (M1, Sq.m, M3, M4, M5). Gather in the command line:

```
fir i0 Li1
```

Choose the M1.msk file and press <Enter>. In the catalog the file of the filtered image – Li1.dat will appear. Similarly receive the filtered images Li2.dat, Li3.dat, Li4.dat, Li5.dat. For this purpose gather in the command line:

```
fir i0 Li2  
fir i0 Li3  
fir i0 Li4  
fir i0 Li5
```

2.4.9. Look at the results of filtering. For this purpose gather in the command line:

```
view i0 Li? .dat
```

2.4.10. Execute linear filtering of the image, noisy normal noise. Carry out the procedure of filtering similar to item 2.4.6. Gather in the command line:

```
fir n0 Ln1  
fir n0 Ln2  
fir n0 Ln3  
fir n0 Ln4  
fir n0 Ln5
```

2.4.11. Look at the results of filtering. For this purpose gather in the command line:

view n0 Ln? .dat

2.4.12. Make the normalization of the files received after median filtering. Gather in the command line:

norm/vr/?? .dat

At implementation of the norm program the histogram of the image will appear. For implementation of the normalization press <Enter>. In the specified way normalize all files with the filtered images.

2.4.13. Make the normalization of the files received after linear filtering. Gather in the command line:

norm/v L? 1.dat

At implementation of the norm program the histogram of the image will appear. By keys <the arrow up>, <the arrow down> to establish normalization coefficient (1/ratio) equal to number of weighting coefficients in corresponding filter masks. So for files L? 1.dat 1/ratio has to make 3. Keys <Shift>+ <the arrow up>, the coefficient of the normalization is established by <Shift>+ <the arrow down> more precisely. Then press <Enter>.

norm/v L? 2.dat

norm/v L? 3.dat

norm/v L? 4.dat

norm/v L? 5.dat

For files L? 2.dat 1/ratio has to make 5, for L? 3.dat – 9, for L? 4.dat – 13, L? 5.dat – 16.

2.4.14. For assessment of quality of filtering it is necessary to define on how many the filtered image differs from initial. Carry out subtraction of the initial image from filtered. Gather in the command line:

sub ri1 Lena ris1

sub ri2 Lena ris2

sub ri3 Lena ris3

sub ri4 Lena ris4

sub ri5 Lena ris5

sub rn1 Lena rns1

sub rn2 Lena rns2

sub rn3 Lena rns3

sub rn4 Lena rns4

sub rn5 Lena rns5

sub Li1 Lena Lis1

sub Li2 Lena Lis2

sub Li3 Lena Lis3
sub Li4 Lena Lis4
sub Li5 Lena Lis5

sub Ln1 Lena Lns1
sub Ln2 Lena Lns2
sub Ln3 Lena Lns3
sub Ln4 Lena Lns4
sub Ln5 Lena Lns5

As a result of subtraction of the initial image from filtered, you have to receive 20 files provided in tab. 1.:

Table 1

Type of filter and noise	File names
Rank filtering, impulse noise	ris1.dat ris2.dat ris3.dat ris4.dat ris5.dat
Rank filtering, normal noise	rns1.dat rns2.dat rns3.dat rns4.dat rns5.dat
Linear filtering, impulse noise	Lis1.dat Lis2.dat Lis3.dat Lis4.dat Lis5.dat
Linear filtering, normal noise	Lns1.dat Lns2.dat Lns3.dat Lns4.dat Lns5.dat

2.4.15. By means of the STAT program see distributions of differences from the original after linear and median filtering. Enter values of the mean square deviation (SKO) in the table. For this purpose gather in the command line STAT and through the gap names of files. Repeat this operation for all files.

stat ris1 Lis1
stat ris2 Lis2
stat ris3 Lis3
stat ris4 Lis4
stat ris5 Lis5

stat rns1 Lns1

stat rns2 Lns2
stat rns3 Lns3
stat rns4 Lns4
stat rns5 Lns5

Table 2

Type of filter and noise	File names	Number weight coefficient filters	CKO
Rank filtering, impulse noise	ris1.dat ris2.dat ris3.dat ris4.dat ris5.dat		
Rank filtering, normal noise	rns1.dat rns2.dat rns3.dat rns4.dat rns5.dat		
Linear filtering, impulse noise	Lis1.dat Lis2.dat Lis3.dat Lis4.dat Lis5.dat		
Linear filtering, normal noise	Lns1.dat Lns2.dat Lns3.dat Lns4.dat Lns5.dat		

2.4.16. On the basis of these tab. 2 construct four schedules of dependence of SKO that numbers of weighting coefficients for four groups of files.

2.4.17. Make the conclusion in what cases are more reasonable to apply linear filters and in what rank. Explain behavior of the constructed four schedules. Call types of linear and nonlinear filters except those which are used in this laboratory work.

3. MAIN ALGORITHMS OF PROCESSING OF IMAGES (PART 2)

3.1. Input data:

– the image of the television table – tv.dat.

Files with weighting coefficients of filters:

Table 3

File name	<i>lapl.msk</i>	<i>lff.msk</i>	<i>ddf.msk</i>	<i>gradz.msk</i>
Weight filter coefficients	-1 -1 -1	1/9 1/9 1/9	1 -2 1	1 1 -1
	-1 8 -1	1/9 1/9 1/9	-2 4 -2	1 -2 -1
	-1 -1 -1	1/9 1/9 1/9	1 -2 1	1 1 -1

File name	<i>grads.msk</i>	<i>gor.msk</i>	<i>vert.msk</i>	<i>diag.msk</i>
Weight filter coefficients	1 1 1	-1 -1 -1	-1 2 -1	2 -1 -1
	1 -2 1	2 2 2	-1 2 -1	-1 2 -1
	-1 -1 -1	-1 -1 -1	-1 2 -1	-1 -1 2

3.2. Work performance order

3.2.1. Check the file of the initial image tv.dat using the view program. Gather in the command line:

view tv.dat

3.2.2. For viewing of oscillograms of lines of the image it is necessary to normalize the file the norm program. Gather in the command line:

norm/v tv.dat

At implementation of the norm program the histogram of the image will appear. For implementation of the normalization press <Enter>.

3.2.3. The cut program check oscillograms of characteristic sites of the image of the television table. Gather in the command line:

cut tv.dat tv0.dat

At implementation of the cut program the contour of the white square on the image will appear. Keys <down> and <Shift of the Shooter up> establish to Shift of the Shooter the necessary sizes of the square by keys <the arrow down> and <the arrow up> move the square to the necessary site of the image. For viewing of oscillograms on the allocated site press the F3 key. Sketch oscillograms. After viewing press the <Esc> keys <Enter>.

3.2.4. Process initial the images masks provided in tab. 3. Carry out processing the fir program, similar to item 2.4.1. The view program see the visual differences of images received as a result of processing. Describe them and explain the origin. Normalize the received images as shown in item 3.2.2. The cut program check oscillograms of lines on characteristic sites of the received images similar to item 3.2.3. Sketch oscillograms and compare them to oscillograms of the initial image, and also between yourself. In the analysis of diagrams explain differences which occurred after processing of the image of the television table different masks.

3.2.5. As a result of processing of the initial image 8 masks you have to receive 8 files of images.

Contents of the report

1. Name of laboratory work.
2. Work purpose.
3. Algorithms of performance of part 1 and part 2 works.

4. To give experimental results in the form of table 2.
5. To construct in one drawing four schedules of dependence of SKO on number of weighting coefficients for four groups of files.
6. To sketch oscillograms of characteristic sites of images as regards 2 works, to give qualitative explanations on the nature of transformation of the output image.
7. Conclusions by part 1 and part 2.

List of references

1. Rabiner L., Gould B. Theory and application of digital processing of signals. – M.: World, 1978. – 848 pages.
2. Citizens of Kazan GD, Kuryachy MI, Pustynsky IN Measuring television. – M.: The higher school, 1994. – 287 pages.
3. Lindli K. Practical processing of images in the SI language: The lane with English – M.: World, 1996. – 512 pages.

WORK NO. 4 (SECTION 1)

DIGITAL LINEAR FILTERING OF IMAGES

Work purpose: **Studying pulse, transitional and amplitude-frequency characteristics of linear filters. Assessment of quality of linear filtering of the image, noise the normal and impulse noise, depending on value of weighting coefficients and their number in the mask.**

1. INTRODUCTION

The two-dimensional discrete signal is the function defined on set of couples of integral numbers:

$$x = \{x(n_1, n_2), 0 \leq (n_1, n_2) < \infty\}.$$

Now widely use the two-dimensional filters corresponding to space structure of the image to processing of images. For linear two-dimensional filters, physically implementable and invariant in time, the output signal registers in the form of two-dimensional discrete convolution (1.1)

$$y(n_1, n_2) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} x(i_1, i_2) h(n_1 - i_1, n_2 - i_2), \quad (1.1)$$

where $h(n_1, n_2)$ - pulse response of the filter.

The linear digital filter is the device in which the current output counting of the signal is presented in the form of linear combination of the current entrance counting of the filter and the previous entrance and output counting.

Function of transfer of the two-dimensional filter can be written down in the look:

$$H(z_1, z_2) = \frac{\sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} z_1^{-i_1} z_2^{-i_2}}{1 - \sum_{j_1=0}^{N_1} \sum_{j_2=0}^{N_2} b_{j_1 j_2} z_1^{-j_1} z_2^{-j_2}}, \quad (j_1, j_2) \neq (0, 0). \quad (1.2)$$

where values $a_{i_1 i_2}$ and $b_{j_1 j_2}$ determine filtering parameters.

Let's enter the concept of basic areas of the filter (the set of values of the signals used at calculations) on input and output signals respectively:

$$\{(i_1, i_2) | 0 \leq i_1 < M_1, 0 \leq i_2 < M_2\},$$

$$\{(j_1, j_2) | 0 \leq j_1 < N_1, 0 \leq j_2 < N_2, (j_1, j_2) \neq (0, 0)\}.$$

At $b_{j_1 j_2} = 0$ nonrecursive algorithms of data processing are implemented by the two-dimensional filter, and expression for discrete convolution of the input signal and the pulse response of the filter has the appearance:

$$y(n_1, n_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} \cdot x(n_1 - i_1, n_2 - i_2). \quad (1.3)$$

In such way, counting of the output signal of the two-dimensional nonrecursive filter represents the weighed sum of counting of the input signal in basic area. At sale of such filter for formation of each of entrance counting it is necessary to execute $(M_1+1)(M_2+1)$ of intelligence and $(M_1M_2+M_1+M_2)$ of additions over the delayed counting of the input signal. Reading of video signal in raster systems is made “the element behind the element” and “the line after line”, and in this case when processing of the operators Who is this video? in real time for implementation z_1^{-1} and z_2^{-1} lag lines are used. And operator z_1^{-1} the operator is implemented in the form of the lag line for the element of decomposition (Tae), and z_2^{-1} in the form of the lag line for the line of decomposition (Tc). The block diagram of the linear two-dimensional filter working on the television signal in real time according to difference equation (1.3) is provided on the figure 1.

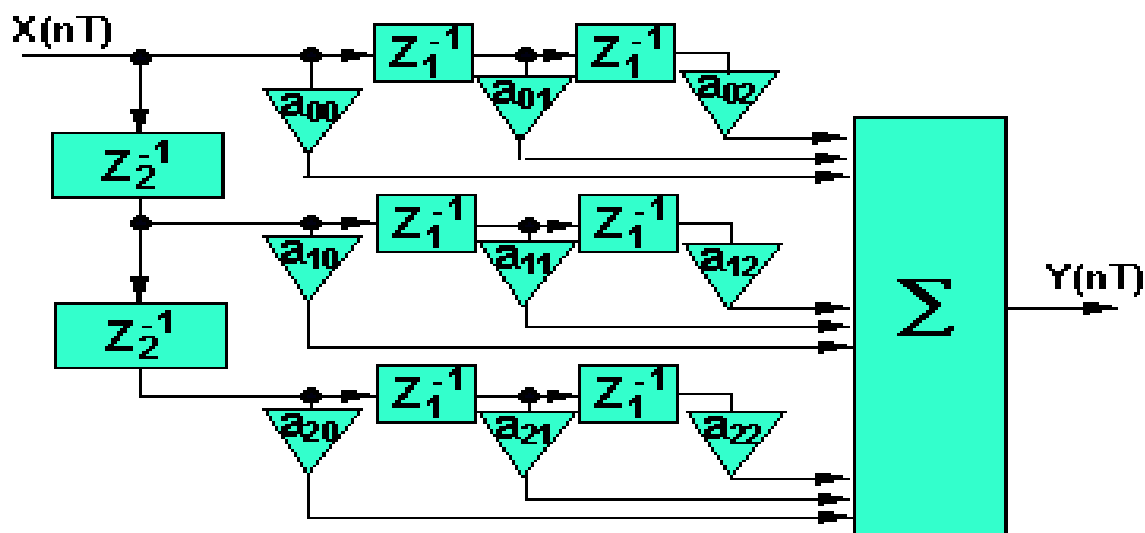


Fig. 1. – Block diagram of the linear two-dimensional filter

3*3 elements in size

The set of lines of delays with the corresponding communications, provides access to the current counting of the image in each instant. Different types of linear filters differ in the weight functions and the normalizing coefficients. Apertures of 3*3 elements in size are usually used; increase in the sizes of the aperture significantly increases the volume of calculations while quality of processing improves slightly. However, with growth of productivity of COMPUTERS processors computing expenses limit the size of the applied apertures less, and recently apertures of 5*5 in size and even 7*7 elements enter the use.

Let's give examples the most often used at information processing in television measuring systems of nonrecursive operators, the decomposition 3*3 elements in size.

In the drawing 1.2 coefficients of two-dimensional filtering $a_{i_1 i_2}$ are provided in the type of the masks describing pulse responses of the corresponding filters [2].

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

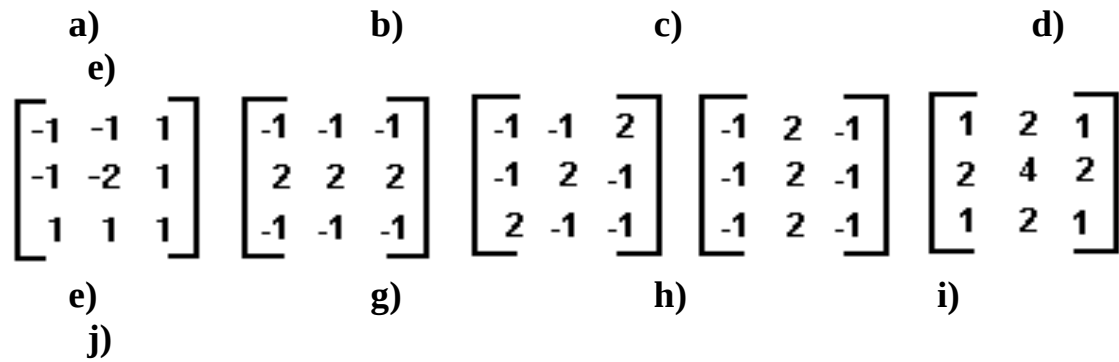


Fig. 2. – Pulse responses of linear two-dimensional filters

"Sliding average" filter (two-dimensional filter
lower frequencies)

The filter which is summing up counting of the input signal with the equal weight realizes the calculation algorithm of “the sliding average”, (the figure 1.2, a). When using such filter for processing of the video information it is possible to increase the relation signal/noise in the output signal [2]. The output signal like “the sliding average” is calculated according to (1.1) at coefficients $a_{i_1 i_2} = 1$ also has the appearance:

$$y(n_1, n_2) = x(n_1, n_2) + x(n_1 - 1, n_2) + x(n_1 - 2, n_2) + x(n_1, n_2 - 1) + x(n_1 - 1, n_2 - 1) + x(n_1 - 2, n_2 - 1) + x(n_1, n_2 - 2) + x(n_1 - 1, n_2 - 2) + x(n_1 - 2, n_2 - 2).$$

Low-frequency two-dimensional filters leave low-frequency components untouched and weakened high-frequency components. Such filters are used for decrease in the visual noise which is contained in the image and also for removal of high-frequency components from the image it was possible to investigate carefully contents low-frequency the component. It means that all values of elements of the image constant or slowly changing. As the low-frequency mask passes through area of the image, new value of convertible elements of the image are calculated according to (1.1). If all values of elements of the image in the field of the adjunction are constant (are identical), their new values will be the same, as well as initial. Thus, when processing low-frequency components remain, any fast changes of intensity are averaged with the remained image elements in the field of the adjunction and by that level high-frequency the component goes down. Visual result of low-frequency filtering is weak unsharpness of the image [3].

Allocation of “edge” (difference of brightness in the image)

Allocation of “edge” is used as the preliminary step in the course of extraction of signs of the image. Though allocation of “edge” is generally used in machine sight, it, of course, has also other applications. For example, information on “edge” is obtained in the course of its allocation, maybe, is used in the initial image for strengthening of its clarity. Allocation of “edge” can be used as the method for

production of original images which can then be retouched in programs of drawing for creation of the highly artistic image [3].

Allocation of “edge” by method of the directed gradient

For allocation of differences of the certain orientation the weight functions called by course gradient masks are used depending on the required direction (the figure 1.2d, e).

The name of the course speaks about the direction of the difference in brightness causing the maximum response of the filter. For highlighting of “edges” of all there are 8 different masks and are called as parts of the world: “East”, “North”, “South”, “southeast”, etc.

Intensity of the output element of the image will depend on the gradient of change of brightness (the more the inclination, the is brighter the element). For example, the gradient the East will strengthen “edge” which contains transition from black to white from left to right.

Allocation of lines

The filter presented in the figure 1.2zh selects horizontal, and at turn for 45 degrees, diagonal lines of details of the image the figure 1.2z, then vertical the figure 1.2i. Difference equation for the filter allocating vertical details of the image has the appearance:

$$y(n_1, n_2) = -x(n_1, n_2) - x(n_1 - 1, n_2) - x(n_1 - 2, n_2) + 2x(n_1, n_2 - 1) + 2x(n_1 - 1, n_2 - 1) + 2x(n_1 - 2, n_2 - 1) - x(n_1, n_2 - 2) - x(n_1 - 1, n_2 - 2) - x(n_1 - 2, n_2 - 2).$$

For underlining of lines of determinate direction, masks similar shown in the figure 1.2k can be used. In this case weight function emphasizes with large scales four coherent elements of the initial image, ie horizontal and verticals.

Allocation of “edge” according to Laplace (two-dimensional filters high frequencies)

High pass filters increase the level of small-sized details in the image. Such filtering is used when it is necessary to investigate high-frequency structure of the object [3].

The method of strengthening of “edge” according to Laplace differs from other methods in the fact that “edges” are highlighted irrespective of the direction. Function $f(x, y)$ Laplace registers in the look:

$$L(f(x, y)) = d^2f / dx^2 + d^2f / dy^2,$$

where d^2f / dx^2 – the second partial differential coefficient on x , and d^2f / dy^2 – the second partial differential coefficient on y .

For discrete functions second differential coefficients can be approximated as follows:

$$d^2f / dx^2 = f(x + 1, y) - 2 \cdot f(x, y) + f(x - 1, y),$$

$$d^2f / dy^2 = f(x, y + 1) - 2 \cdot f(x, y) + f(x, y - 1).$$

Thus, the Laplacian can be written down in the following look:

$$L(f(x, y)) = f(x + 1, y) + f(x - 1, y) + f(x, y - 1) + f(x, y + 1) - 4 \cdot f(x, y).$$

This expression can be considered as the pulse response of the filter which is written down in the look $f(x, y)$. For convenience signs before coefficients change for opposite. The Laplacian for “four neighbors”, it is presented in the figure 1.2, for “eight neighbors” in the figure 1.2, century.

Operators of processing of two-dimensional data (the figure 1.2b, in, d) realize function of allocation of signals of small-sized objects from the background component in video signal.

2. DESCRIPTION OF THE PROGRAM

Laboratory work No. 4 (section 1) “Digital linear filtering of images” is realized in the form of the program in the environment of multimedia.

The program consists of three sections:

- 1. "Introduction".**
- 2. "Examination".**
- 3. "Laboratory work".**

Each section consists of pages. On each page, above, the name of laboratory work, the name of the section, number of the page and their quantity in this section is highlighted. In the lower part of the screen buttons for transition to the following page, previous and for the exit from the program are located. In the second section there is the additional button for the exit from examination and summing up. In laboratory work is the button “Task”, “Reference” and “*Number/half tone*” (transfers the image in windows to numbers or half tone).

The basic concepts of linear filtering, the characteristic of linear filters with results of filtering of images different filters are presented in the section “Introduction”.

In examination five tasks are provided to you for control of knowledge. In the first two you need “to click” the correct answer (ie guide at it the cursor using a mouse and quickly to press and release its left key), as a result of opposite the chosen answer “tick” will appear. In the third and fourth move numbers (having pressed the left key of the mouse) to boxes according to the task. The fifth task represents the block diagram of the two-dimensional linear filter. You should “click” with the mouse the necessary block (the chosen blocks will be led round by the red frame).

After exposed answers, press the Results of Examination button and you reach results of testing.

On each page of the section “Laboratory Work” there is the block diagram for the research of characteristics of the filter and the window in which images are output from different parts of the scheme.

Work on each page of the third section is performed in following the lock:

- 1. Choose the window in which it is necessary to output the image, for this purpose “click” the name (the name of the active window will be led round by the red frame).**
- 2. “Click” the scheme block parameters of which you need to set (on active blocks the cursor changes).**
- 3. “Click” the circle (yellow with the blue frame) at the block exit, as a result, the image will be output from this part of the scheme in the active window.**

3. Work performance order

In the manager of programs, in the group “Appendices”, “double click” on the pictogram of «LAB1».

3.1. Task for the first page

3.1.1. Set the F1 filter = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Remove the pulse response.

3.1.2. Set the F1 filter = $\begin{bmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Remove the pulse response.

3.1.3. Draw the conclusion on the type of the pulse response on the basis of item 1 and item 2.

3.2. Task for the second page

3.2.1. Set the F1 filter = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and F2 = $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Remove pulse

responses for F1, for F2 and for F1-F2. Draw the conclusion on the type of the pulse response for series connection of the chain of filters.

3.2.2. Set coefficients of F1 and F2 filters in the look $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Remove

the pulse response of F1-F2.

3.2.3. Explain the result.

3.3. Task for the third page

3.3.1. Choose three filters (according to instructions of the teacher) and measure their amplitude-frequency characteristic in the direction of the axis of frequencies of X.

3.3.2. For from the chosen filters measure two-dimensional AChH.

3.3.3. Enter observed data in the table. Construct two schedules of dependence of the transfer ratio of filters on frequency.

3.3.4. Explain the result.

3.4. Task for the fourth page

3.4.1. Set coefficients of filters so that to allocate from the presented line digits which are present at it.

3.4.2. Establish thresholds so that the digit at the adder exit in the best way repeated initial.

3.4.3. Enter result of processing of digit in the report (in the numerical look).

3.4.4. Explain the result.

3.5. Task for the fifth page

3.5.1. Remove dependence of SKO, the image noisy the impulse noise, from the number of coefficients in the filter (not equal to zero) for four values of percent of filling $G=7,10,20,30$ at $A=100$ (And - amplitude of impulses, can accept values ot0 do255).

3.5.2. In work use the filters given in table 1.

Table 1 – Rectifier filters

Filter No.	1	2	3	4	5	6
Weight coefficient s of the filter	$\begin{bmatrix} 00000 \\ 00000 \\ 00100 \\ 00000 \\ 00000 \end{bmatrix}$	$\begin{bmatrix} 00000 \\ 00000 \\ 01110 \\ 00000 \\ 00000 \end{bmatrix}$	$\begin{bmatrix} 00000 \\ 00100 \\ 01110 \\ 00100 \\ 00000 \end{bmatrix}$	$\begin{bmatrix} 00000 \\ 01110 \\ 01110 \\ 01110 \\ 00000 \end{bmatrix}$	$\begin{bmatrix} 00100 \\ 01110 \\ 11111 \\ 01110 \\ 00100 \end{bmatrix}$	$\begin{bmatrix} 11111 \\ 11111 \\ 11111 \\ 11111 \\ 11111 \end{bmatrix}$
Number of weight coefficient	1	3	5	9	13	25

3.5.3. Repeat item 3.5.1 for the image noisy normal noise. To take dispersion in limits: from 30 to 70.

3.5.4. Enter observed data in the table similar to table 2.

3.5.5. On the basis of data of the table construct 9 schedules of dependences of SKO on number of nonzero counting in the filter (all 9 schedules to arrange in one drawing).

3.5.6. Upon termination of work, press the Exit from the Program button.

Table 2

Number of weighting coefficients in the filter	1	3	5	9	13	25
SKO, at G=7						
SKO, at G=10						
SKO, at G=20						
SKO, at G=30						
SKO, at ☐ =						
SKO, at ☐ =						
SKO, at ☐ =						
SKO, at ☐ =						
SKO, at ☐ =						

Contents of the report

1. Name of laboratory work.
2. Work purpose.
3. Introduction.
4. The observed data presented in table 2 form.
5. To construct in one drawing of 9 schedules of dependence of SKO on number of weighting coefficients in the filter.
6. Conclusions.

List of references

1. Rabiner L., Gould B. Theory and application of digital processing of signals. - M.: World, 1978.-848 pages.
2. Pratt U. Digital processing of images. In 2 - x books. The lane with English - M.: World, 1982. - Book of 1 - 312 page, Book 2 - 480c.
3. Citizens of Kazan GD, Kuryachy MI, Pustynsky IN Measuring television: Studies. the grant for higher education institutions / - M.: Vyssh. school, 1994. - 288 pages.

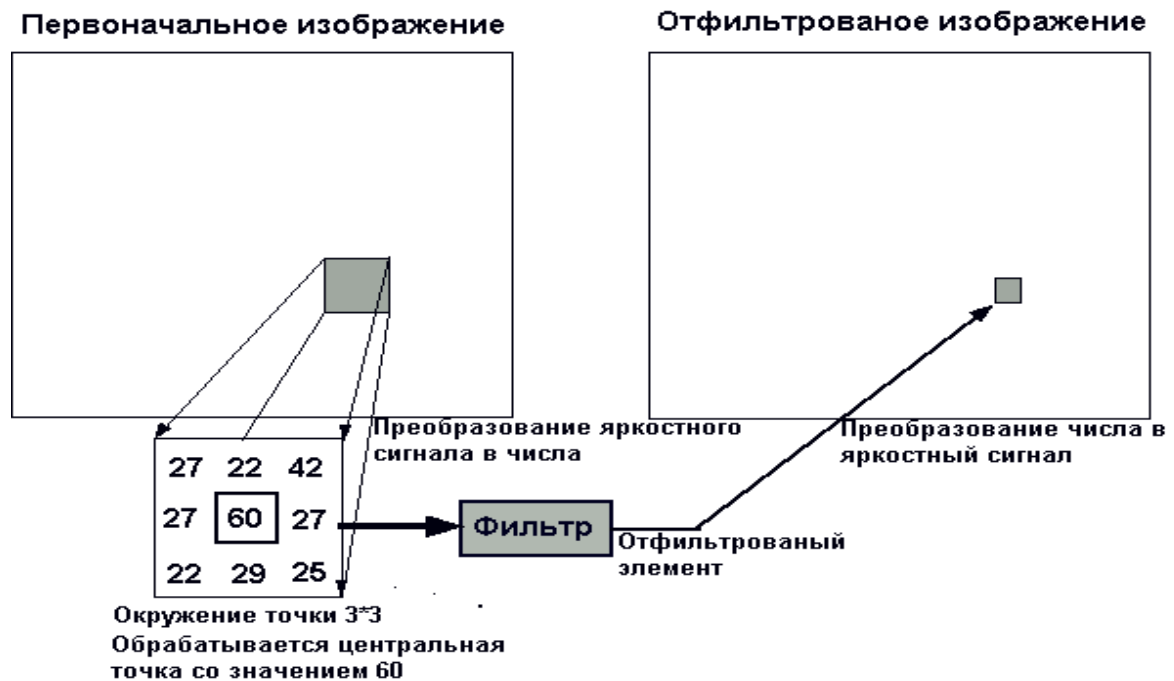
WORK NO. 4 (SECTION 2)

DIGITAL NONLINEAR PROCESSING OF IMAGES

Work purpose: Studying of characteristics of the nonlinear filters giving the idea of nonlinear processing of images. Assessment of quality of nonlinear filtering of the image, noise the normal, impulse or ambient noise, depending on parameters of nonlinear filters.

1. INTRODUCTION

Now widely use the two-dimensional filters corresponding to space structure of the image to processing of images. Let's consider the principle of data processing in the two-dimensional filter on the example:



Nonlinear filters

The basic concepts of the theory of local filtering applied to linear two-dimensional processing of images are fair also for local nonlinear filtering. Concepts of process of filtering, the aperture, ways and algorithms of movement of the aperture according to the image mean; weight function is applied not always. The main difference from linear filtering consists that the output signal of the nonlinear filter forms in a nonlinear way according to the initial image.

Nonlinear filtering includes two classes of filters - it is the rank filters received as a result of ranging of elements of the processed image and operations over them both the homomorphic filters including the block of predistortions and the compensating block allowing to improve the quality of the perceived image, there are also filters which cannot be carried to one of above-mentioned classes.

In laboratory work three filters are considered: rank, extreme and filter of allocation of the small-sized object.

Rank filter

Rank data processing call transformation of set of reports of the signal to the variational series in which reports of the signal are placed on the importance (increase of values of amplitudes). The first (minimum) rank in the varying series of numbers occupies counting of the signal with the minimum amplitude. The last

(maximum) rank in the varying series of numbers occupies counting of the signal with the minimum amplitude.

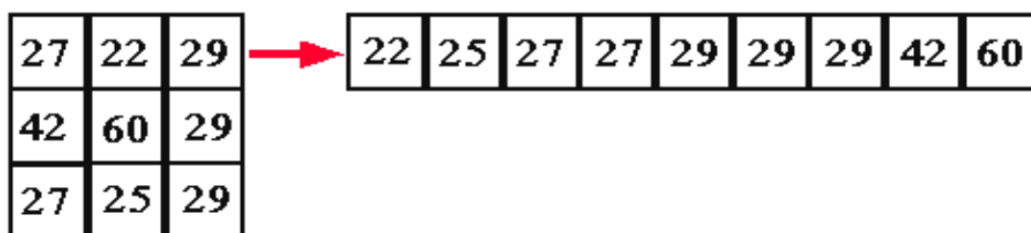


Рис. 1. – Работа рангового фильтра

Median Filter

The median filter belongs to rank filtering. The median represents the central element in the variable series received from the data which are in aperture limits.

Median filters are used to smoothing of images and to noise suppression, as well as linear low-frequency filters. Median filters on the properties differ from them. First, median filters keep sharp differences whereas linear low-frequency filters grease them. Secondly, median filters are very effective when smoothing the impulse noise, but can lead to total disappearance of fine details of the image at the inadequate choice of parameters of the filter.

Median filters are also used for detection of borders and allocation of objects.

Kind of the median filter is deliberately - the median filter. In such filter weight function is used, but it is interpreted differently than in linear filters. Here weighting coefficients show how many time it is necessary to consider the image pixels which got to the aperture.

If to appropriate to the exit of the filter not value of the median of the data which are in the aperture, but value any r_y ($r=1,2,\dots n$ where n is the total number of elements of the aperture) serial statistics, then it is possible to receive n of the filters called protsentilny.

Function the describing communication of output and entrance signals of the filter:



Рис. 2. – Работа медианного фильтра

Allocation of the local maximum (top) two-dimensional object

According to the image the window of given size moves. The pixels which got to the window are compared in size to the pixel corresponding to the center of

the window. If the central pixel is more or is equal to the environment, then it without change is transferred to the output image. If at least one pixel in the window exceeds central, to the output field zero registers.



Рис. 3. – Работа экстремального фильтра

The filter allocates flat top of the object. The filter of allocation of the local maximum of the two-dimensional object belongs to extreme filters.

Function the describing communication of output and entrance signals of the filter:

$$S_{ij} = \begin{cases} S_{ij}, \text{ при } S_{ij} \geq \{S_{i+1,j+1}; S_{i+1,j-1}; S_{i+1,j}; S_{i-1,j}; S_{i-1,j-1}; S_{i-1,j+1}; S_{i,j+1}; S_{i,j-1}\} \\ 0, \text{ во всех остальных случаях.} \end{cases}$$

Allocation of the small-sized object against the background of

At the time of coincidence of the center of the small-sized object and the central element of the aperture of the filter of amplitude of peripheral reports are close to zero, and differences between amplitudes central and peripheral elements are close to amplitude of the central counting. Such sign is not inherent to signals from background educations as they have the correlated reports of signals of large extent. This filter carries out, depending on characteristics, allocation of the small-sized object and suppression of the background.

Let's consider the principle of data processing in filters of this kind on the specific example. Let's assume that the SMA sizes are equal 7*7 elements.

The type of the aperture is conditionally represented in fig. 4, and the counting used at calculations is marked with their coordinates.

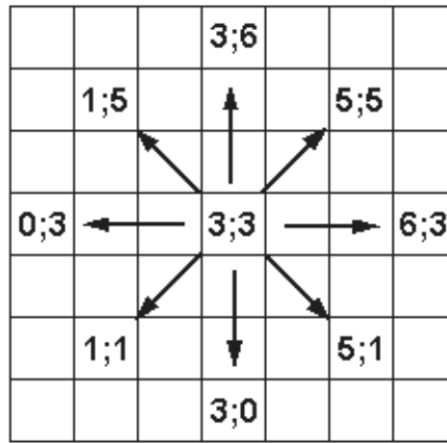


Рис. 4. – Вид апертуры экстремальных двумерных фильтров

Before data processing in the register of the mask through information entrance the binary code (fig. 5) registers. This code defines the area form from the vicinity of the central element of the aperture on which data processing is conducted. From the exit of the SMA block 7*7 signals are given at the same time on information entrances of the multichannel switch of signals which is controlled from the mask code.

On exits of the switch of signals there pass only those reports of the aperture which are resolved by the mask code from the register. The mask code for the aperture given on fig. 4 represents the 49-digit sequence of zero and units.



Рис. 5. – Структурная схема фильтра

Function the describing communication of output and entrance signals of the filter shown in fig. 5:

$$\begin{aligned}
 y(n_1, n_2) = f[& \min\{ [x(n_1-3, n_2-3) - x(n_1-1, n_2-1)]; [x(n_1-3, n_2-3) - \\
 & - x(n_1, n_2-3)]; [x(n_1-3, n_2-3) - x(n_1-1, n_2-5)]; [x(n_1-3, n_2-3) - \\
 & - x(n_1-3, n_2-6)]; [x(n_1-3, n_2-3) - x(n_1-5, n_2-5)]; [x(n_1-3, n_2-3) - \\
 & - x(n_1-6, n_2-3)]; [x(n_1-3, n_2-3) - x(n_1-5, n_2-1)]; [x(n_1-3, n_2-3) - \\
 & - x(n_1-3, n_2)] \}],
 \end{aligned}$$

2. DESCRIPTION OF THE PROGRAM

Laboratory work No. 4 (section 2) “Digital nonlinear processing of images” is realized in the form of the program in the environment of multimedia.

The program consists of three sections:

1. Prolog
2. Examination
3. Experimental part

Each section consists of pages. On each page, above, the name of laboratory work, the name of the section, number of the page and their quantity in this section is highlighted. In the lower part of the screen buttons for transition to the following page, previous and for the exit from the program are located. In the second section there is the additional button for the exit from examination and summing up. In laboratory work is the Task button and “*Number/half tone*” (transfers the image in windows to numbers or half tone).

The basic concepts about nonlinear processing of images, characteristics of nonlinear filters with results of filtering of images different filters and bright examples of operation of filters are presented in the section “Introduction”.

In examination six tasks are provided to you for control of knowledge. All tasks can be divided into two categories: in the first you need “to click” the correct answer (ie guide at it the cursor using a mouse and quickly to press and release its left key), as a result of opposite the chosen answer “tick” will appear, in the second to move numbers (having pressed the left key of the mouse) to boxes according to the task.

After exposed answers, press the Results of Examination button and you reach results of testing.

On each page of the section “Laboratory Work” there is the block diagram for the research of characteristics of the filter and the window in which images are output from different parts of the scheme.

Work on each page of the third section is performed in the following order:

1. Choose the window in which it is necessary to output the image, for this purpose “click” the name (the name of the active window will be led round by the red frame).
2. “Click” the scheme block parameters of which you need to set (on active blocks the cursor changes).
3. “Click” the letter at the block exit, as a result, the image will be output from this part of the scheme in the active window.

3. WORK PERFORMANCE ORDER

In the manager of programs, in the group “Appendices”, “double click” on the pictogram of “LAB2”.

3.1. Task for the first page

3.1.1. Set the filter [1 1 1]. Changing the rank of the filter process it images: Single, Lines horizontal - 3, the vertical - 3. Explain the received results.

3.1.2. Set the filter $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Perform for it the task of item 3.1.1. Explain The received results.

3.1.3. Set the filter $\begin{bmatrix} 1 & & \\ 1 & 1 & 1 \\ & 1 & \end{bmatrix}$. Perform for it the task of item 3.1.1. Explain The received results.

3.1.4. For the filter 3*3 changing the rank of the filter process images: Single, Rectangle 3*3. Too most do for the filter 5*5. Explain the results received.

3.1.5. Draw conclusions on the done work.

3.2. Task for the second page

3.2.1. Set the filter 3*3, process it the image “Lena”, changing the rank of the filter from 1 to 9 for different coefficients of the impulse noise.

3.2.2. Fill in table 1, construct schedules of dependence of SKO on serial statistics of the filter.

Table 1

Percent of G at A=70	10%	30%	50%	70%
SKO, at r=1				
SKO, at r=3				
SKO, at r=4				
SKO, at r=5				
SKO, at r=6				
SKO, at r=7				
SKO, at r=9				

Issue result in the form of schedules.

3.2.3. Draw conclusions on the done work.

3.3. Task for the third page

3.3.1. Set the filter 2*2, process it the image “Lena” and remove SKO value, too most do for the filter 3*3, explain the received results.

3.3.2. Fill in tables 2 and 3, construct schedules of dependence of SKO on noise parameters.

Table 2

Number of elements of the aperture in the filter	2*2	3*3	4*4	5*5
SKO, at G=0 σ=50				
SKO, at G=10				
SKO, at G=20				

SKO, at G=40				
SKO, at G=70				
SKO, at G=100				
SKO, at $\sigma=10$ G=50				
SKO, at $\sigma = 20$				
SKO, at $\sigma = 30$				
SKO, at $\sigma = 50$				
SKO, at $\sigma = 100$				

Table 3

Number of elements of the aperture in the filter	2*2	3*3	4*4	5*5
SKO, at G=10 A=50				
SKO, at G=30				
SKO, at G=50				
SKO, at G=70				
SKO, at G=100				
SKO, at A=10 G=50				
SKO, at A=20				
SKO, at A=50				
SKO, at A=70				
SKO, at A=100				

Issue result in the form of schedules.

3.3.3. Draw conclusions on the done work.

3.4. Task for the fourth page

3.4.1. Set the filter 3*3, process it images of unit, gaussoidy and the rectangle, explain the results received.

3.4.2. For process the gaussoida filters 5*5 and 7*7, explain the received results.

3.4.3 . 2*2 and 3*3 process rectangles filters 3*3 - 5 and 3*3 - 9.

3.4.4. Choose the filter aperture so that it allocated the head of the person.

3.4.5. Setting different coefficients for the ambient noise and choosing different filters select images of unit, the rectangle 3*3.

3.4.6. Draw conclusions on the done work.

3.5. Task for the fifth page

3.5.1. Process the image of the gaussoida the filter 3*3.

3.5.2. Draw conclusions on the done work.

3.6. Task for the sixth page

3.6.1. Process the image of the rectangle 3*3.

3.6.2. Changing noise dispersion, process the filter of the image of the rectangle 2×2 with different values of amplitude.

3.6.3. Fill 4 amplitudes of rectangles which are unambiguously determined depending on dispersion of noise and the sizes of the filter in the table:

Table 4

The aperture size in the filter	3*3	5*5	7*7	9*9
Amplitude, at $\sigma = 1$				
Amplitude, at $\sigma = 3$				
Amplitude, at $\sigma = 5$				
Amplitude, at $\sigma = 10$				

Issue result in the form of the schedule.

3.6.4. Draw conclusions on the done work.

Contents of the report

1. Name of laboratory work.
2. Work purpose.
3. Introduction.
4. Results of experiments.
5. Conclusions.

List of references

1. Rabiner L., Gould B. Theory and application of digital processing of signals. - M.: World, 1978.-848 pages.
2. Bates R., Mac-Donnell of M. Vostanovleniye and reconstruction of images: The lane with English - M.: World, 1989.-336 pages, silent.
3. Citizens of Kazan GD, Kuryachy MI, Pustynsky IN Measuring television: Studies. the grant for higher education institutions. - M.: Vyssh. school, 1994. - 288 pages.
4. Yashin VV, Of Kalinin. A. Processing of images in the SI language for IBM PC: Algorithms and programs. - M.: World, 1994. - 238 pages .

WORK No. 4 (section 3)

DIGITAL METHODS OF CORRECTION OF IMAGES

Work purpose: **To improve quality of the image, namely: to increase contrast and sharpness of the image, to suppress the VCh-additive noise, to filter the pulse hindrance.**

1. INTRODUCTION

The two-dimensional discrete signal is the function defined on set of couples of integral numbers:

$$x = \{x(n_1, n_2), 0 \leq (n_1, n_2) < \infty\}.$$

Now widely use the two-dimensional filters corresponding to space structure of the image to processing of images. For linear two-dimensional filters, physically implementable and invariant in time, the output signal registers in the form of two-dimensional discrete convolution:

$$y(n_1, n_2) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} x(i_1, i_2) \cdot h(n_1 - i_1, n_2 - i_2),$$

where $h(n_1, n_2)$ – pulse response of the filter.

Function of transfer of the two-dimensional filter can be written down in the look:

$$H(z_1, z_2) = \frac{\sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} \cdot z_1^{-i_1} z_2^{-i_2}}{1 - \sum_{j_1=0}^{N_1} \sum_{j_2=0}^{N_2} b_{j_1 j_2} \cdot z_1^{-j_1} z_2^{-j_2}}, (j_1, j_2) \neq (0, 0),$$

where values $a_{i_1 i_2}$ and $b_{j_1 j_2}$ determine filtering parameters.

At $b_{j_1 j_2} = 0$ nonrecursive algorithms of data processing are implemented by the two-dimensional filter, and expression for discrete convolution of the input signal and the pulse response of the filter has the appearance:

$$y(n_1, n_2) = \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_2} a_{i_1 i_2} \cdot x(n_1 - i_1, n_2 - i_2). \quad (1)$$

Reading of video signal in raster systems is made “the element behind the element” and “the line after line”, and in this case when processing of the operators Who is this video? in real time for implementation z_1^{-1} and z_2^{-1} lag lines are used. And operator z_1^{-1} the operator is implemented in the form of the lag line for the element of decomposition (Tae), and z_2^{-1} in the form of the lag line for the line of decomposition (Tc). The block diagram of the linear

two-dimensional filter working on the television signal in real time according to difference equation (1) is provided on the figure 1.

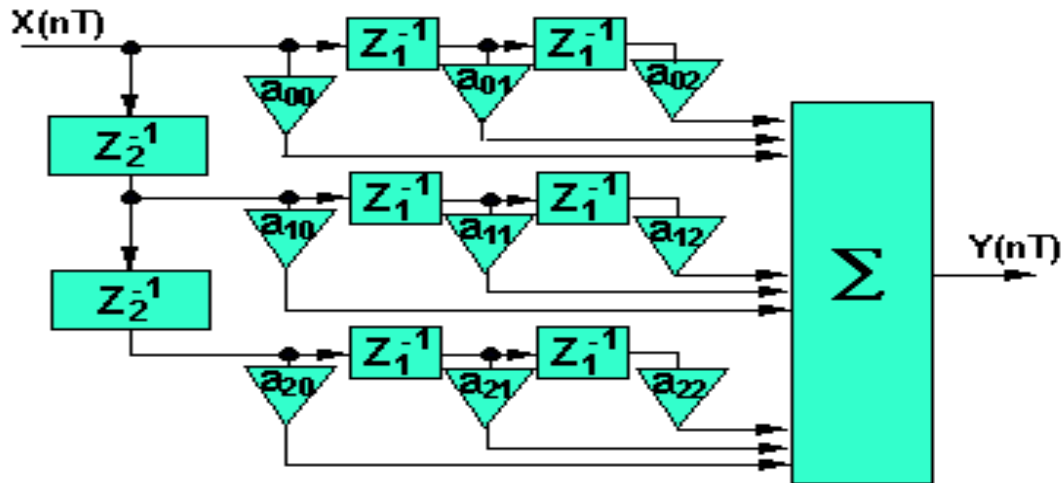


Fig. 1. – Block diagram of the linear two-dimensional filter 3*3 elements in size

The set of lines of delays with the corresponding communications, provides access to the current counting of the image in each instant. Different types of linear filters differ in the weight functions and the normalizing coefficients.

Initial premises at the solution of problems of correction of the image is the assumption that there is some function, will tell $a(n1, n2)$ describing the image at the exit of the ideal representing system, and $n1=0..., m-1$; $n2=0..., n-1$ – integer coordinates of elements of the image, ie number of the element in line and the line number, and m, n – number of elements in line and lines in the shot, and action of the real representing system it is possible to describe some transformation F of the ideal image to really observable:

$$b(n1, n2) = F[a(n1, n2)].$$

Then the problem of correction comes down to that, knowing these or those parameters of transformation F to find such F observed images adjusting transformation that its result

$$\hat{a}(n1, n2) = \hat{O}[a(n1, n2)]$$

was in sense of some set criterion of fidelity of reproduction or the metrics, perhaps, is closer to the ideal image. In such statement this task is called still recovery or restoration of images.

From positions of the theory of static decisions the corrected image will be assessment of the initial image on observable. It is necessary to aim to receive the best assessment, ie to carry out optimum correction.

There is the question of optimality criterion, of what sense to put in the concept “best”. It is reasonable to take proximity of the corrected image to initial as the measure of quality of correction, but the concept “proximity of images ” or “distance between images” can make a different sense too. In

broadcasting television the proximity of images would be natural to be connected with the degree of their visual similarity.

In systems with the automatic analysis of images the proximity of images can be measured by probability of the error of detection or the error of measurement of coordinates of the set objects in the image, etc. The theory of static decisions shows that assessment, optimum from the point of view of, any criterion, is close to optimum and for other reasonable criterion.

It is convenient to begin with the quadratic measure of proximity – normal euclidovy distance though as it was already noted that the distance between images not always corresponds to the degree of their visual similarity. Such choice is justified by the fact that the quadratic measure in case of linear correction leads to very simple final formulas. The square of Euclidean distance between images $and(n1, n2)$ and $\hat{a}(n1, n2)$ is the square of norm of their difference:

$$\|\Delta\|^2 = \sum_{n_1=0}^{m-1} \sum_{n_2=0}^{n-1} \Delta^2(n_1, n_2), \quad (2)$$

where differential image

$$\Delta(n1, n2) = and(n1, n2) - \hat{a}(n1, n2), \quad n1 = 0..., m-1, \quad n2 = 0..., n-1.$$

To receive the measure of quality of the adjusting transformation for some set of initial images in general which would not depend on implementation of the hindrance it is convenient to take the ensemble average of size (2), having averaged it on the set of initial images, and on different implementations of the hindrance. The size received thus Δ

$$\varepsilon = \|\Delta\| = \sqrt{\frac{\sum_{n_1=0}^{m-1} \sum_{n_2=0}^{n-1} \varepsilon^2(n_1, n_2)}{m * n}},$$

where $\Delta^2(n1, n2) = \langle \hat{a}(n1, n2) - and(n1, n2) \rangle^2$, is called the mean square error (SKO) of correction. We use SKO as optimality criterion of correction. Let's consider the best corrected image it for which SKO has the minimum value.

However in applied and scientific tasks often it appears insufficient to present to the observer the object by means of the ideal representing system. At the solution of the complex challenges demanding the careful analysis of the image (the search, identification of objects, any definition of quantitative characteristics generalizing descriptions, etc.) it is desirable to arm sight, ie to give to the observer of means, the easier interpretations of images, extraction of information from them.

The processing acting as supportive application of visual interpretation of images is called preparation of images. For example, use of preparation of the image in medicine.

1.1. Correction of amplitude characteristics

One of the simplest types of distortion of video signal in the representing systems is its nonlinear transformation as a result of which the observed brightness of image at the exit of system differs from that which would be at the exit of ideal system, and the difference in each point depends only on the size of video of the signal in this point. Such distortions are described by amplitude characteristics of system – the functions showing dependence of size of the output signal from size entrance:

$$b = [a(n_1, n_2)].$$

The ideal amplitude characteristic of the ideal system $[a(n_1, n_2)]$ is considered set. Most often it is symmetric function. Thus, the problem of correction of the amplitude characteristic consists in search of the adjusting $\square$ k transformation $[b(n_1, n_2)]$ which turns the characteristic of system with correction into set.

At digital correction the main source of noise – the quantization noise. In view of the fact that the number of levels of quantization is limited, as a result of nonlinear transformation of the quantized signal at its correction the values incoincident with the quantization scale can turn out that leads to increase in the quantization noise. This phenomenon is illustrated by the figure 2 where scales of values of the adjusted signal of b and result of correction $\hat{a}$ are quantized evenly on 32 levels.

Because the steepness of the $\square$ k line as necessary is more than unit, correction leads to increase in the error of quantization.

Addition of dummy levels of quantization destroys false contours of quantization, visually quality of images is evaluated above. Increase in contrast of images can be the example of amplitude equalization. Resort to increase in contrast when digital video signal occupies for one reason or another only the part of the range of values allocated for it. It can be treated as a result of action of system with the amplitude characteristic similar to that which is shown in the figure 3 by the $\square$ line for the case when video signal accepts values only between levels 12 and 26 of possible range from 0 to 32. The adjusting characteristic for this case is shown in the drawing by the $\square$ k line. Brightness of the signal is distributed as follows: to zero value of the signal there corresponds level black, maximum – level white.

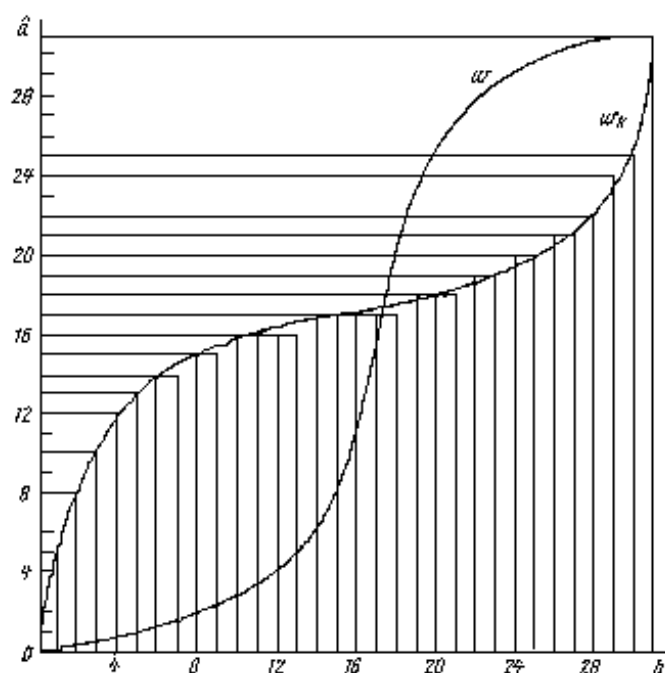


Fig. 2. – The schedule of the distorting and adjusting functions

1.2. Filtering of pulse interferences

Pulse interferences lead to the fact that with some probability (probability of the mistake) value of video signal is replaced with the random quantity. On the image such hindrances look the isolated points contrast.

Statistical properties of pulse interference differ from statistical image properties markedly. Smooth, little changes of value of video signal from the element to the element are characteristic of images, and fast, intermittent changes are rare and form extended contours. Pulse hindrances represent the single isolated emissions, significant in size. The fact that it is visually very easy to distinguish emissions of the hindrance from the image though such hindrances and have the strong disturbing effect is explained by it.

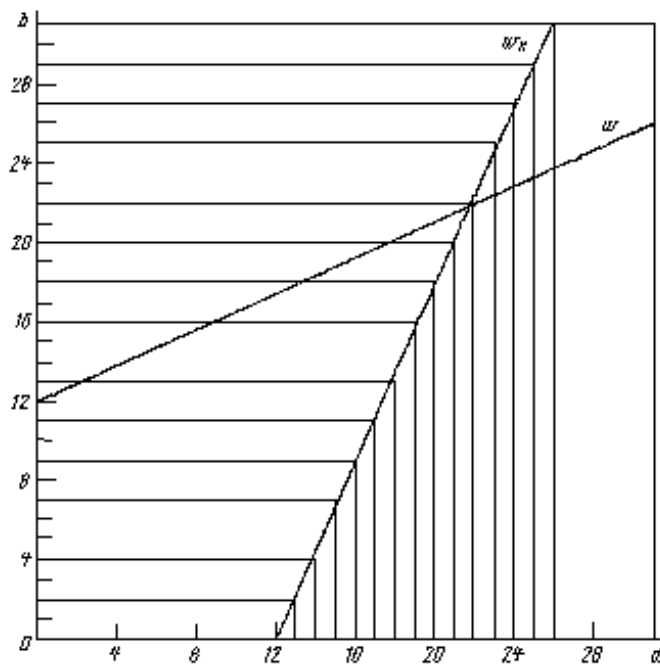


Fig. 3. – The schedule of the distorting and adjusting functions with dummy levels

Thanks to such sharp difference of hindrances from images the algorithm of filtering of pulse interferences is very simple. It consists of two operations: detection of the distorted elements of the image and correction of value of video signal in these elements. As value of video signal in each element of the image most likely is close to values of video signal for the elements, next to it, it is possible to find noise “emissions”, having compared in each element value of video signal to its value predicted on the next elements. If distinction exceeds some threshold of $\Delta 1$ on the module, then the decision on noise existence is made. The rule of correction of the damaged values can be also based on smoothness of images. Using it, it is possible to replace the distorted value of video signal predicted with addition of some constant $\Delta 2$ which is selected for the defocusing exception as a result of averaging of video signal. The matter is that as a result of correction contrast (difference) of this element in relation to people around decreases to the size $\Delta \Delta$. Thus, the constant $\Delta 2$ has to be less threshold of visual detection for single details more visual threshold of detection of extended differences. In this case “deleting” of the impulse noise will not lead to visual impression of loss of the extended differences which are available on the image.

As the values of video signal used for the prediction in turn can be distorted, the algorithm of detection has to be the iterative, beginning with great values of the threshold of detection Δ_1 . To avoid it, processing can be made the recursive filter, using not all elements of the vicinity which but only are already corrected, preceding this for the prediction.

Digital implementation of the described procedure is simple. According to it, value a_{n_1, n_2} video signal in the element (n_1, n_2) the digital image it will be transformed as follows:

$$b_{n_1, n_2} = \begin{cases} a_{n_1, n_2}, & \text{если } |a_{n_1, n_2} - \bar{a}_{n_1, n_2}| < \delta_1 \\ a_{n_1, n_2} + \delta_2 * (a_{n_1, n_2} - \bar{a}_{n_1, n_2}), & \text{если } |a_{n_1, n_2} - \bar{a}_{n_1, n_2}| > \delta_1 \end{cases},$$

where $\bar{a}_{n_1, n_2}$ – the value of video signal in the analyzed image element predicted on the next, already corrected elements.

For calculation of this predicted value it is possible to use the simple linear prediction when this value is as the sum of the values of the next elements taken with some scales. Good results are yielded by the following formula of the prediction on four the next at the left and from above on the raster to image elements (means that processing is made along lines from left to right consistently, since the zero line from top to down):

$$\bar{a}_{n_1, n_2} = 0,3 * (b_{n_1, n_2-1} + b_{n_1-1, n_2}) + 0,2 * (b_{n_1-1, n_2-1} + b_{n_1+1, n_2-1}).$$

Despite the simplicity, this algorithm is very effective remedy of improvement of quality of the images distorted by pulse interferences.

1.3. Correction of linear distortions (increase in clarity)

The majority of the representing systems can be considered as linear. One of the main characteristics of such systems is pulse reaction or its Fourier transform – the frequency characteristic showing how spatial harmonics of the image are transferred in this system. Ideal it is possible to call such representing system which does not change the space range of the signal, ie system with the uniform frequency characteristic within area of the spatial frequencies occupied with the range of images.

Characteristics of the real representing systems (optical, photographic, television) differ from ideal. As a result of the image at the exit of such systems undergo distortions. They are called linear distortions. Most often these distortions are that the representing system weakens upper spatial frequencies of the image. Visually it leads to deterioration in sharpness of the image.

As Fourier's range of the signal at the exit of the linear filter is equal to the work of the range of the input signal on the frequency characteristic of the filter, the problem of correction of linear distortions comes down to search of the linear filter with the frequency characteristic, the return frequency characteristic of the system in the field of frequencies occupied with the image. As the frequency characteristic of the representing system is in

advance unknown, in advance it is impossible to define the characteristic of the adjusting linear filter precisely.

Three typical masks for performance of high frequency filtering are given below:

mask 1	mask 2	mask 3
$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

Distinctive feature of these masks is that the sum of their elements is equal to unit.

1.4. Filtering of periodic hindrances

Action of periodic hindrances is shown in the figure 4a. In the figure 4b the schedule of intensivnost horizontal spectral the component for this image is shown in logarithmic scale.

Apparently from the drawing, to peaks there corresponds the synchronous hindrance. For filtering of this sort of hindrances, it is necessary to remove these peaks. It can be made by means of the rejection filter.

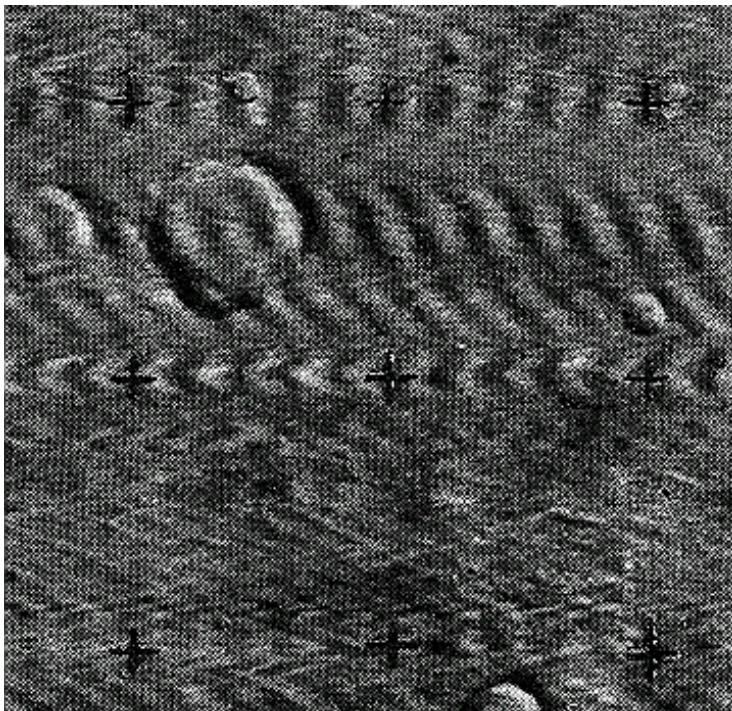


Fig. 4a. – Example of action of the synchronous hindrance

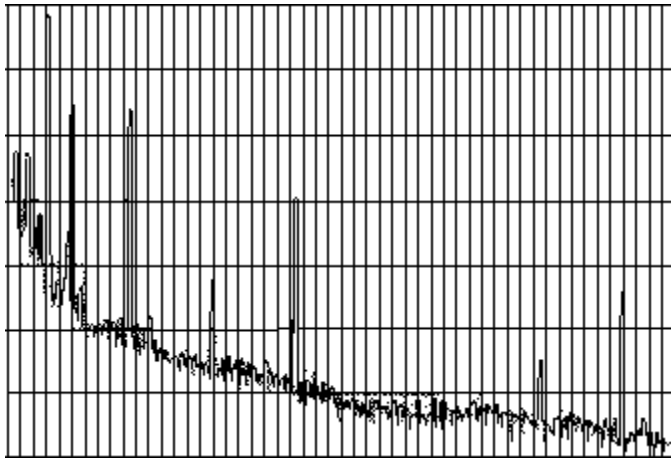


Fig. 4b. – Intensity spectral component

1.5. Filtering of the LF additive noise

Action of the LF additive noise is shown in the figure 5.

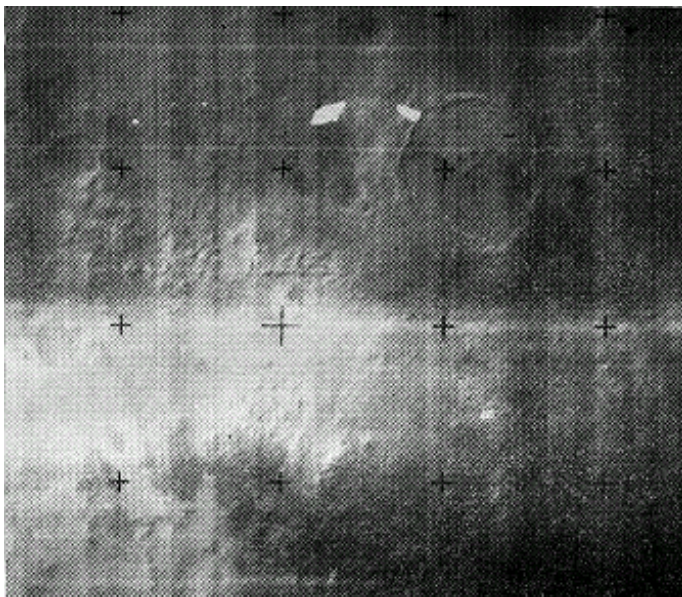


Fig. 5 – Example of action of the LF additive noise

Discrete range of this sort of Fourier of hindrances is concentrated in the field of very low spatial frequencies. For correction of such distortions it is necessary to limit the image range at low frequencies.

2. DESCRIPTION OF THE PROGRAM

Laboratory work no. 4 (section 3) “Digital methods of correction of images” is realized in the form of the multimedia program.

The program consists of three sections:

- 1. "Introduction".**
- 2. "Examination".**
- 3. "Laboratory work".**

Each section consists of pages. On each page, above, the name of laboratory work, the name of the section, number of the current page and their quantity in this section is highlighted. In the lower part of the screen buttons for transition to the following page, return on previous and the exit button from the program are located. In the second section there is the additional button for the exit from examination and summing up. In the third section of laboratory work there are Task and Reference buttons.

In the section "Introduction" the main methods of correction of images with examples are stated.

In examination five tasks are provided to you for control of knowledge. In the first two you need "to click" the correct answer (ie guide at it the cursor using a mouse and quickly to press and release its left key), as a result of opposite the chosen answer "tick" will appear. In the third and fourth move numbers (having pressed the left key of the mouse) to boxes according to the task. The fifth task represents the block diagram of the two-dimensional linear filter. You should "click" with the mouse the necessary block (the chosen blocks will be allocated with the red frame).

After exposed answers, press the Results of Examination button and results of testing will be shown to you.

On each page of the third section there is the block diagram for the research and the window in which images are output from different parts of the scheme.

Work on each page of the third section is performed in the following order:

- 1. Choose the window in which it is necessary to output the image, having clicked it (the name of the active window will be led round by the red frame).**
- 2. "Click" the scheme block parameters of which you need to set (on active blocks the cursor changes appearance).**
- 3. "Click" the circle (yellow, with blue the frame) at the block exit, as a result the image will be output from this part of the scheme in the active window.**

3. WORK PERFORMANCE ORDER

In the manager of programs, in the group "Appendices", "double click" on the pictogram of «LAB3».

3.1. Task for the first page

3.1.1. Select the initial image.

3.1.2. Having chosen the linear distorting and adjusting functions, the proinvertiruyta the initial image, changing the K1 and K2 parameters, equating them 0, 255, at first at distorting, and then and at the adjusting function. Remove SKO values.

3.1.3. For all available functions remove dependence of SKO on K1 value, changing K1 from 0 to 254, at invariable K2 = 255.

3.1.4. Explain the result.

3.2. Task for the second page

3.2.1. Select the initial image.

3.2.2. Make comparison of the filtering properties of linear and nonlinear methods of filtering, without forgetting to expose the correct coefficient of the normalization for the linear method. For the choice of the current method of filtering press the linear button for the linear method, or the nonlinear button for the nonlinear method.

3.2.3. Remove dependence of SKO for the image, noise at first the normal, and then impulse noise for the linear method of filtering. For the impulse noise to take dispersion ranging from 30 to 70. To present observed data in the form of table 1 and table 2.

3.2.4. To do point 3.2.3. for the nonlinear method of filtering. To present observed data in the form of table 1 and table 2.

3.2.5. Explain the result.

3.3. Task for the third page

3.3.1. Select the initial image.

3.3.2. Choose the low-frequency mask which imitates the rasfokusirovaniye of the initial image.

3.3.3. Choose the high-frequency mask.

3.3.4. Remove SKO values in the presence of normal noise (the impulse noise is absent), changing low-frequency and high-frequency masks. To pay attention that the normalization is made automatically Observed data to present in table 3 form.

3.3.5. To do point 3.3.4. in the presence of the impulse noise (normal noise is absent). To take dispersion ranging from 30 to 70. To present observed data in table 4 form.

3.3.6. Explain the result.

Table 1 – Dependence of SKO on LF masks in the presence of normal noise

	$\square = 0$	$\square = 10$	$\square = 20$	$\square = 30$	$\square = 40$	$\square = 50$	$\square = 60$
LF mask 1							
LF mask 2							
LF mask 3							

Table 2 – Dependence of SKO on LF masks in the presence of the impulse noise

	$\square = 0$	$\square = 10$	$\square = 20$	$\square = 30$	$\square = 40$	$\square = 50$	$\square = 60$
LF mask 1							
LF mask 2							
LF mask 3							

Table 3 – Dependence of SKO when focusing images

from LF - and VCh-masks in the presence of normal noise

	$\square = 0$	$\square = 10$	$\square = 20$	$\square = 30$	$\square = 40$	$\square = 50$	$\square = 60$
LF mask 1 VCh-mask 1							
LF mask 1 VCh-mask 2							
LF mask 1 VCh-mask 3							
LF mask 2 VCh-mask 1							
LF mask 2 VCh-mask 2							
LF mask 2 VCh-mask 3							
LF mask 3 VCh-mask 1							
LF mask 3 VCh-mask 2							

LF mask 3 VCh-mask 3							
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Table 4 – Dependence of SKO when focusing images

from LF - and VCh-masks in the presence of the impulse noise

	$\square = 0$	$\square = 10$	$\square = 20$	$\square = 30$	$\square = 40$	$\square = 50$	$\square = 60$
LF mask 1 VCh-mask 1							
LF mask 1 VCh-mask 2							
LF mask 1 VCh-mask 3							
LF mask 2 VCh-mask 1							
LF mask 2 VCh-mask 2							
LF mask 2 VCh-mask 3							
LF mask 3 VCh-mask 1							
LF mask 3 VCh-mask 2							
LF mask 3 VCh-mask 3							

- 1. Name of laboratory work.**
- 2. Work purpose.**
- 3. Introduction.**
- 4. The observed data presented in the form of tables.**
- 5. According to ready tables to construct their graphic interpretation. To submit data on each of tables on the separate schedule.**
- 6. Conclusions.**

List of references

- 1. Rabiner L., Gould B. Theory and application of digital processing of signals. - M.: World, 1978.-848 pages.**
- 2. Pratt U. Digital processing of images. In 2 - x books. The lane with English - M.: World, 1982. - Book of 1 - 312 page, Book 2 - 480c.**
- 3. Citizens of Kazan GD, Kuryachy MI, Pustynsky IN Measuring television: Studies. the grant for higher education institutions. - M.: Vyssh. school, 1994. - 288 pages.**

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN
FEDERATION FEDERAL STATE AUTONOMOUS EDUCATIONAL
INSTITUTION OF HIGHER EDUCATION
NORTH CAUCASIAN FEDERAL UNIVERSITY

Methodological guidelines

for students on organizing and conducting independent work
in the discipline "Digital Signal Processing / Цифровая обработка
сигналов"

for students majoring in 02.04.01 Mathematics and Computer Science
profile "Training Program for Electronic Information"

Stavropol
2026

CONTENT

1. General Provisions
2. The purpose and objectives of independent work
3. Technological map of independent student work
4. Control points and types of reporting on them
5. Procedure for completing independent work by a student
6. List of literature for independent work

1. General Provisions

Independent work is the planned educational, educational and research, or scientific research work of students, carried out outside of class (in class) time, as assigned and under the methodological guidance of the teacher, but without his or her direct participation (with partial direct participation of the teacher, leaving the leading role to the students' work). To Vladislav Vyacheslavovich

Student independent work (SIW) at a university is an important form of student learning and research activity. Student independent work plays a significant role in the rating system of learning.

The main types of independent work for students include:

- formation and assimilation of the content of lecture notes based on the educational literature recommended by the lecturer, including information educational resources (electronic textbooks, electronic libraries, etc.);
- writing reports;
- preparation for seminars, practical and laboratory work, their design;
- compilation of an annotated list of articles from relevant journals in the fields of knowledge (pedagogical, psychological, methodological, etc.);
- implementation of educational and research work, project activities;
- preparation of practical developments and recommendations for solving problematic situations;
- completing homework assignments in the form of solving individual problems, performing typical calculations, computer calculations and individual work on individual sections of the content of disciplines, etc.;
- computer-based current self-monitoring and progress monitoring based on electronic training and assessment tests;
- completion of coursework (projects) within the framework of disciplines;
- completion of final qualifying work, etc.

The methodology for organizing students' independent work depends on the structure, nature, and characteristics of the discipline being studied, the number of hours for its study, the type of assignments for students' independent work, the individual qualities of students, and the conditions of educational activities.

The process of organizing students' independent work includes the following stages:

- preparatory (defining goals, drawing up a program, preparing methodological support, preparing equipment);
- basic (implementation of the program, use of techniques for searching for information, assimilation, processing, application, transfer of knowledge, recording of results, self-organization of the work process);
- final (assessment of significance and analysis of results, their systematization, assessment of the effectiveness of the program and work methods, conclusions on areas of labor optimization).

Independent work on the subject "FOREIGN LANGUAGE" is aimed at developing the following **competencies** :

Code	Formulation
UK-2	Capable of managing a project at all stages of its life cycle
OPK-2	Capable of conducting methodological and expert work in the field of mathematics and computer science

2. The purpose and objectives of independent work

The main goal of organizing and implementing independent work coincides with the goal of student education – the formation of a set of general scientific, professional and universal competencies of the future bachelor.

When organizing independent work, developing the ability to work independently to acquire knowledge, skills, and the ability to organize educational and research activities is an important and essential condition. The goal of students' independent work is to master fundamental knowledge, professional skills, and relevant professional activities, as well as experience in creative and research activities. Students' independent work promotes the development of independence, responsibility, self-discipline, and a creative approach to solving academic and professional problems.

The objectives of the SRS are:

- systematization and consolidation of the acquired theoretical knowledge and practical skills of students;
- deepening and expanding theoretical knowledge;
- development of skills in using regulatory, legal, reference documentation and specialized literature;
- development of students' cognitive abilities and activity: creative initiative, independence, responsibility and organization;
- development of independent thinking, abilities for self-development, self-improvement and self-realization;
- development of research skills;
- use of material collected and obtained during independent work and laboratory classes.

3. Technological map of independent student work

Codes of implemented competencies	Type of student activity	The final product of independent work	Assessment tools and technologies	Volume of hours, including (astr.)		
				SRS	Contact work with the teacher	Total
2nd semester						
UK-2 OPK-2	Self-testing, preparation for testing	Testing	Test tasks	28.0	8.0	28.0
UK-2 OPK-2	Preparation for the practical lesson	Cospit	Interview	30.0	10.0	30.0
Total for 1 semester				58.0	18.0	58.0
Total				58.0	18.0	58.0

4. Control points and types of reporting on them

Quality control and deadlines for independent work are assessed in accordance with the academic schedule and are formatted according to the assignment. The following knowledge rating is provided:

Current control

5. Procedure for completing independent work by a student

1.1 List of discussion topics for the reports:

Basic level

My Computer

Achievements in Mathematics
The Most Famous Mathematicians Internet in our Life
Mathematicians – Nobel Prize Winners
I Dream to be a Mathematician/Programmer

Advanced level

Computer in my Life
The Latest Achievements in Mathematics
The Most Famous Mathematicians of the Modern World Internet and Study
Mathematicians – Nobel Prize Winners
I Dream to be a Mathematician/Programmer

1.2 List of discussion questions for the interview:

Base level

Speak on this topic:

Computer Literacy
What do you Know about History of Computers? Career and Life
My Future Profession is a Mathematician/Programmer

Advanced level

Write on this topic:

Computer Literacy
What do you Know about History of Computers? Career and Life
My Future Profession is a Mathematician/Programmer

2. Recommendations for types of independent work:

MESSAGE

A message is a prose work of small volume and free composition , expressing individual impressions and thoughts on a specific occasion or issue and clearly not claiming to be a definitive or exhaustive interpretation of the subject.

The purpose of the essay is to develop skills such as independent creative thinking and written expression of one's own thoughts.

Structure and plan of the message

The structure is determined by the requirements:

- I. the author's thoughts on the problem are presented in the form of brief theses (T);
- II. The idea must be supported by evidence, therefore the thesis is followed by arguments (A).

Arguments are facts, social phenomena, events, life situations and experiences, scientific evidence, references to the opinions of scientists, etc. It is best to provide two arguments in support of each thesis: one argument seems unconvincing, three arguments can "overload" a presentation written in a genre focused on brevity and imagery.

Thus, the essay acquires a ring structure (the number of theses and arguments depends on the topic, the chosen plan, and the logic of thought development):

- introduction; - thesis, arguments; - thesis, arguments; - thesis, arguments; - conclusion.

Rules for writing a message

- Of the formal rules, only one can be named - the presence of a title.
- The internal structure can be arbitrary. Since this is a short written work, a mandatory repetition of conclusions at the end is not required; they can be included in the main text or in the title.

- Argumentation may precede the problem statement. The problem statement may coincide with the final conclusion.
- Unlike an essay, which is addressed to any reader and therefore begins with “I want to talk about” and ends with “I have come to the following conclusions...”, an essay
 - This is a remark addressed to a prepared reader (listener), that is, someone who already has a general idea of what will be discussed. This allows the author to focus on revealing new information and not clutter the presentation with unnecessary details.

INTERVIEW

An interview is a form of educational activity, a special conversation between a teacher and a student on topics related to the discipline being studied, designed to determine the extent of the student's knowledge on a specific section, topic, problem, etc., allowing for an assessment of their ability to argue their own point of view, presupposing a comprehensive discussion of any issue, problem, or comparison of information, ideas, opinions, and proposals.

REPORT

A report is a study on a specific problem. Preparing a report involves not only familiarizing yourself with the literature on the chosen topic but also independently researching specific issues.

A scientific report can be prepared for presentation at a seminar, at a conference, or as part of a round table.

Preparation of a report includes several stages of work:

1. Selecting a report topic
2. Selection of materials
3. Making a plan. Working on the text
4. Design of materials
5. Preparing for the performance

The report consists of the following parts:

- front page
- table of contents
- introduction
- main part
- conclusion
- list of references.

Introduction

The introduction must first explain the relevance of this study.

Main part

This is, in essence, your report. Typically, the main body consists of several sections (chapters, paragraphs), each devoted to a distinct aspect of the topic being studied. Each chapter, unlike the introduction and conclusion, should have its own title.

Conclusion

In the conclusion, it's important to draw general conclusions on the topic covered. These should be extremely clear and specific.

Working on the text

Develop a plan. As you gain more knowledge of the material being studied, the initial plan can be expanded, refined, and refined.

Writing the text of your future speech is one of the most difficult and crucial stages of preparing a report. It is at this stage that you must analyze and evaluate the collected material and formulate a final plan.

When starting to work on the text of the report, you should take into account the structure of its construction.

REQUIREMENTS FOR THE PRESENTATION AND REGISTRATION OF THE RESULTS OF THE SWS

After completing the independent work, students present:

- notes on grammar sections;
- the results of studying topics within the course program that are not covered in practical classes;
- completed control exercises;
- thematic reports and messages on problematic topics;
- creative translation works;
- individual dictionaries on the topics studied;
- online articles on the topic being studied;
- language portfolio.

During classroom work, testing is carried out to carry out intermediate control of the independent work system.

Structure and content of independent work

Title page - 1 page:

- 1) Name of the educational institution (centered, capital letters)
- 2) Independent work on the topic of the work. (centered)
- 3) Completed by: Full name of the author of the work, faculty, course, group
- 4) Supervisor: position and rank of the teacher, full name (left aligned)
- 5) Stavropol, 20_(center aligned)

Requirements for the design of the SRS

Independent work is completed by the student in English or Russian (according to the objectives of the assignments) on standard white A4 sheets of paper. The text of the work must be typed on a computer (Times New Roman font, font size - 12-14 pt). Left margin - 2.5 cm; top, right, bottom margins - 2 each.

See Alignment - by width, with all paragraphs highlighted.

All pages must be numbered at the top center. The title page is unnumbered, and the following page is numbered starting with number 2.

Line spacing is single. The total length of the work is 12-14 sheets (not including the title page).

The work is filed page by page in a plastic binder with a transparent cover. The tutor's review is filed first, followed by the title page, and so on.

Evaluation criteria:

A student receives an "Excellent" grade if they have thoroughly and thoroughly mastered the program material, present it comprehensively, consistently, clearly, and logically, are able to closely link theory with practice, handle questions and other applications of knowledge fluently, and are comfortable answering modified assignments, utilize material from monographs in their answers, correctly justify their decisions, possess versatile skills and techniques for answering practice-oriented questions, make sound management decisions, possess practical problem-solving skills and techniques, and complete test assignments 100 percent. Learning outcomes in this discipline, as part of mastering UK-4 competencies, are achieved at a high level.

A "good" grade is awarded to a student if they have a solid knowledge of the material, present it clearly and to the point, avoid significant inaccuracies in answering questions, correctly apply theoretical principles to solve practical problems, possess the necessary skills and techniques for answering them, correctly apply theoretical principles to solve practical problems, possess the necessary skills and techniques for completing them, and complete 70 percent of the test assignments. Learning outcomes in this discipline, as part of mastering UK-4 competencies, are achieved at a good level.

A "satisfactory" grade is awarded to a student who has mastered only the core material but has not grasped its details, makes inaccuracies, uses inaccurate wording, lacks logical consistency in the presentation of the program material, has difficulty answering questions and completing practical assignments and case studies, and completes test assignments only 50 percent. The learning outcomes for this course, as part of mastering UK-4 competencies, are achieved at a basic level.

A student receives a grade of "unsatisfactory" if they lack a significant understanding of the curriculum material, make significant errors, answer questions with uncertainty and difficulty, make significant errors when solving practical-level assignments, and score 49 percent or less on test assignments. The learning outcomes for the course, as part of mastering UK-4 competencies, have not been achieved.

task	Level of completion of the control	Rating score (as a percentage of the maximum score for the test task)
	Great	100
	Good	80
	Satisfactory	60
	Unsatisfactory	0

6. List of literature for completing independent work

Main literature:

1. Umnyashkin S. V. Fundamentals of the Theory of Digital Signal Processing / Цифровая обработка сигналов: a tutorial. Moscow: Tekhnosfera, 2016. URL : https://biblioclub.ru/index.php?page=book_red&id=444859&sr=1

2. Rozhentsov A. A., Baev A. A., Chernyshev D. S. Development of signal processing devices on programmable logic integrated circuits: laboratory practical training. Yoshkar-Ola: Perm State Technical University, 2016. URL : https://biblioclub.ru/index.php?page=book_red&id=461634&sr=1

Further reading:

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