

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
Federal State Autonomous Educational Institution of Higher Education
"NORTH CAUCASUS FEDERAL UNIVERSITY"

SUGGESTIONS FOR TEACHING
for the performance of practical work
in the discipline "Probability Theory and Mathematical Statistics"
for bachelors of training
11.03.02 Infocommunication Technologies and Communication Systems
Profile "Intelligent Data Processing in Infocommunication Systems and Networks"

Stavropol 2026

Practical Lesson 1

Elements of set theory

Educational questions of the lesson:

1. Basic operations on sets and events
2. Axiomatics of probability theory

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to the performance of basic operations on sets and events.

Checking the readiness of students for the lesson

1. Write down the basic formulas of combinatorics.
2. To give a classical definition of probability.
3. Basic properties of probability.
4. Sum, product, difference of events.

1. Basic operations on sets and events

1. Graphically depict the set $A \cup B$.
2. Graphically depict the set $A \cap (A \cup B)$.

3. The vessel has one steering gear, four boilers and two turbines. Event A means that the steering gear is in good working order, B_j ($j = 1, 2, 3, 4$) is the serviceability of the j th boiler, and C_k ($k = 1, 2$) is the serviceability of the k th turbine. Event F – steerable vessel – occurs when the steering gear, at least one boiler and at least one turbine are in good working order. Express the events F_i in terms of A, B_j and C_k .

4. Specify the picture on which the $D \cup E$ event is shaded:

5. Let A, B, C be three arbitrarily selected events. Find expressions for events consisting of the following:

- a) only event A has occurred;
- b) A and C have occurred, but B has not happened;
- c) all three events have occurred;
- d) at least one of these events has occurred;
- e) at least two events have occurred;
- f) one and only one of these events occurred;
- g) two and only two events have occurred;
- h) none of these events occurred;
- i) no more than two events have occurred.

2. Axiomatics of probability theory

1. There are n traffic lights in the city. Everyone can be in one of three states: red, yellow, or green. How many ways can all traffic lights be lit?

2. Ten subjects are studied in the class. There are five lessons on Tuesday, and the lessons

are different. How many ways can you make a schedule for Tuesday?

3. The signalman has four different flags at his disposal. A signal consisting of at least two flags is hoisted on the flagpole. How many different signals can be hoisted on the flagpole if the order of the flags in the signal is taken into account?

4. A student must pass three (different) exams within six days. A maximum of one exam per day is allowed. How many ways can exams be organized?

5. The subscriber forgot the last digit of the phone number and is forced to dial it at random. Find the probability that he will have to call no more than three numbers. How will the probability change if the subscriber remembers that the last digit of the number is even?

6. There are 10 white, 15 black, 20 red balls in the urn. From the urn 9 balls are taken at random. To find:

- a) How many different ways can 9 balls be removed?
- b) In how many different ways can 9 balls be drawn, including 6 white and 3 black?
- c) How many different ways can 9 balls be drawn, including 2 white, 3 black and 4 red balls?

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 2

Probabilistic spaces

Educational questions of the lesson:

1. Probabilistic space with geometric probability type
2. Conditional probability

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are offered to perform practical tasks related to the performance of basic operations in probability spaces with a geometric type of probability.

Checking the readiness of students for the lesson

1. To define the algebra of events.
2. To define σ -algebra.
3. To define Borel algebra.
4. To define probability in a measurable space.
5. To give a definition of probabilistic space.
6. Properties of probability.

1. Probabilistic space with geometric probability type

1. The search area for the parachutist is specified, which is a square with a side a . In the search area there is a lake, the surface of which can be approximately considered to coincide with the surface of the circle of radius $R = \frac{a}{2}$. Assuming that the parachutist could have landed at any point of the square in question, find the probability that the parachutist splashed down on the surface of the lake.

2. On the telephone line connecting points A and B, the distance between which is l , a wire break occurred. Determine the probability that the point of break C is located from point A at a distance of at least $\frac{l}{3}$, assuming that the position of the break C on the telephone line is equally possible.

3. At any moments of the time interval T , two independent signals are equally possible to enter the receiver. Signals are distorted if the difference between the moments of their arrival is less than τ . Determine the probability that the signals will be distorted.

4. A rod of length l is broken into three parts, and the breaking points are chosen at random. Find the probability that the resulting parts can be made into a triangle.

5. Two friends have agreed to meet at a certain place between two and three o'clock in the afternoon. The one who comes first waits for the other for 10 minutes, after which he leaves. What is the probability of meeting friends if the arrival of each of them during the specified hour can occur at any time?

2. Conditional probability

1. There are two groups of students in the lesson. In one group there are 20 students, and 5 of them are excellent students; in the second group there are 25 students, and 6 of them are excellent students. To solve the example, one excellent student was called to the blackboard. Find the probability that a student from the first group was called to the blackboard.

2. Let event B roll 4 or 6 points on the dice, event A roll an even number of points, event A2 roll 3, 4 or 5 points, event A3 roll an odd number of points. Find the conditional probabilities of events A, A2 and A3 under the condition of event B.

3. On seven cards are written the letters that form the word "NIGHTINGALE". The cards are shuffled and three cards are taken out of them at random and laid out from left to right. Find the probability that the word "OX" (event A) will be obtained.

4. Each letter of the word "MATHEMATICS" is written on a separate card. The cards are carefully mixed. Four cards are taken out sequentially. Find the probability of event A – get the word "TOPIC"?

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 3

Full probability, Bayes, Bernoulli formulas

Educational questions of the lesson:

1. Full probability formula. Bayes' formula
2. Bernoulli's formula

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to the use of perfect probability formulas, Bayes' and Bernoulli's formula.

Checking the readiness of students for the lesson

1. How is the probability of the sum and product of events calculated?
2. Explain the formula for total probability.
3. Explain Bayes' formula.
4. Explain Bernoulli's formula.

1. Full probability formula. Bayes' formula

1. There are five rifles in the shooting range (No1, No2, No3, No4, No5), the probabilities of hitting from which for this shooter are 0.5, respectively; 0,6; 0,7; 0,8; 0,9.

a) Determine the probability of hitting the target with one shot, if the shooter takes one of the rifles at random.

b) The shooter takes one of the rifles at random, fires a shot and hits the target. Find the probability that the shooter fired from rifle No3.

2. Two out of three (No1, No2, No3) independently operating lamps of the device failed. Find the probability that lamp No1 failed, if the probabilities of failure of lamps No1, No2, No3, respectively, are equal to: $p_1=0.1$; $p_2=0.2$; $p_3=0.3$.

3. After examining the patient, the doctor believes that one of the two diseases is possible, which are encrypted with numbers 1 and 2, and the degree of his confidence in the correctness of the diagnosis is estimated as 0.4 and 0.6, respectively. To clarify the diagnosis, the patient is sent for analysis, the outcome of which gives a positive reaction in disease 1 - in 90% of cases and in disease 2 - in 20% of cases. The analysis gave a positive reaction. How will the doctor's opinion change after the analysis?

4. Electric lamps are manufactured at three factories. The first plant produces 30% of the total number of electric lamps, the second - 25%, and the third - the rest. The products of the first plant contain 1% of defective electric lamps, the second - 1.5%, the third - 2%. The store receives

products from all three factories. The lamp bought in the store turned out to be defective. What is the probability that it was produced by the first plant?

5. Two of the three independently operating elements of the computing device have failed. Find the probability that the first and second elements have failed if the probabilities of failure of the first, second, and third elements, respectively, are $p = 0.2$; $p_2 = 0.4$; $p = 0.3$.

6. Three daughters - Julia, Marina and Lena - are responsible for washing dishes in the family. Since Julia is the eldest, she has to do 40% of all the work. The remaining 60% of the work falls equally on Marina and Lena. The probabilities of breaking something from the dishes (during one washing) for Julia, Marina and Lena are respectively: 0.02; 0.03; 0.04. Parents do not know who was on duty in the evening, but they heard the clink of broken dishes. What is the probability that the dishes were washed by: a) Julia; b) Marina; c) Lena?

2. Bernoulli's formula

1. On average, 20% of the blocks of shares at auctions are sold at the originally declared price. Find the probability that out of 9 blocks of shares as a result of trading at the originally declared price:

1. 5 packages will not be sold.
2. To be sold:
 - a) less than 2 packets;
 - b) no more than 2 packages;
 - c) at least 2 packages;
 - d) the most likely number of packets.

2. 6 independent shots are fired at the target. The probability of hitting each shot is 0.75. To find:

- a) the probability of exactly 5 hits;
- b) probability of at least 5 hits;
- c) the probability of less than 3 hits.

3. There are 200 capacitors in the box, and 2 of them are of the required capacity. Randomly, 1 capacitor is taken out of the box and, after determining its capacity, is returned back to the box. Find out how many times you need to carry out the specified operation so that the probability of meeting at least one capacitor of the required capacity is at least 0.95.

4. The device consists of 3 independently operating main elements. The device fails if at least one element fails. The probability of failure of each element in time t is 0.1. Find the probability of failure of the device in time t if:

- a) only the main elements work;
- b) one backup element is included;
- c) 2 reserve elements are included.

It is assumed that the backup elements operate in the same mode as the main ones, the probability of failure of each backup element is 0.1, and the device fails less than 3 elements.

5. The probability of producing a standard part on an automatic machine is 0.8. Find the probabilities of the possible number of defective parts among 5 selected ones.

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 4

Discrete random variables

Educational questions of the lesson:

1. The law of distribution of a discrete random variable
2. Distribution function of a discrete random variable

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to discrete random variables.

Checking the readiness of students for the lesson

1. To give a definition of a random variable.
2. What random variable is called discrete?
3. What random variable is called continuous?
4. What is the law of distribution of a random variable? Methods of setting.
5. What is meant by the distribution function of a random variable? Methods of setting.

1. The law of distribution of a discrete random variable

1. The discrete random variable X is given by the law of distribution

a)

X	1	3	6	8
p	0,2	0,1	0,4	0,3

b)

X	2	4	5	6
p	0,3	0,1	0,2	0,4

c)

X	10	15	20
p	0,1	0,7	0,2

Construct a polygon of distribution.

2. The device consists of three independently operating elements. The probability of failure of each element in one experiment is 0.1. Draw up a law for the distribution of the number of failed elements in one experiment.

3. The batch contains 10% of non-standard parts. Four parts are selected at random. Write a binomial distribution law of a discrete random variable X – the number of non-standard parts among the four selected ones and build a polygon of distribution.

4. Write a binomial law of distribution of the discrete random variable X – the number of the appearance of the "coat of arms" during two coin tosses.

5. After the student answers the questions on the exam ticket, the examiner asks the student additional questions. The teacher stops asking additional questions as soon as the student discovers knowledge of the question asked. The probability that the student will answer any additional question asked is 0.9. It is necessary to make a law of distribution of the discrete random variable X – the number of additional questions that the teacher will ask the student, to find the most likely number of additional questions asked to the student.

6. Two guns are fired at the target alternately until the first hit by one of the guns. The probability of hitting the target with the first gun is 0.3, the second - 0.7. The first gun starts firing. Draw up a law of distribution of the discrete random variable X and Y - the number of shells spent by the first and second guns, respectively.

7. The textbook was published in a circulation of 100000 copies. The probability that the textbook is bound incorrectly is 0.0001. Find the probability that the print run contains exactly five defective books.

8. The store received 1000 bottles of mineral water. The probability that the bottle will be broken during transportation is 0.003. Find the probability that the store will receive broken bottles: a) exactly two; b) less than two; c) more than two; d) at least one.

2. Distribution function of a discrete random variable

1. For all tasks in step 1, construct a distribution function of a random variable X .

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 5

Numerical Characteristics of Discrete Random Variables

Educational questions of the lesson:

1. Mathematical expectation of a discrete random variable
2. Dispersion of a discrete random variable

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to finding numerical characteristics of discrete random variables.

Checking the readiness of students for the lesson

1. To give a definition of a random variable.
2. What random variable is called discrete?
3. What is called expected value? Properties.
4. Define the dispersion of a discrete random variable? Properties.
5. Give a definition of the standard deviation?

1. Mathematical expectation of a discrete random variable

1. The discrete random variable X is given by the law of distribution

A)

X	1	3	6	8
p	0,2	0,1	0,4	0,3

B)

X	2	4	5	6
R	0,3	0,1	0,2	0,4

C)

X	10	15	20
R	0,1	0,7	0,2

Calculate the expected value.

2. Find the mathematical expectation of a discrete random variable Z if the mathematical expectations X and Y are known: a) $Z=X+2Y$, $M(X)=5$, $M(Y)=3$; b) $Z=3X+4Y$, $M(X)=2$, $M(Y)=6$.

3. The discrete random variable X takes three possible values $x=4$ with a probability of $p=0.5$; $x=6$ with a probability of $p=0.3$; x with a probability of p . Find x and p , knowing that $M(X)=8$.

4. The test consists of three questions. Each question has 4 answers. Compile and graphically depict the law of distribution of the number of correct answers by simple guessing. Find the mathematical expectation of this random variable.

5. There are 3 problems in the ticket. The probability of the correct solution of the first problem is 0.9, the second – 0.8, and the third – 0.7. Compile and graphically depict the distribution law of the number of correct solutions to problems in the ticket and calculate the mathematical expectation of this random variable.

6. Two high-risk deposits were made: 10 thousand rubles to company A and 15 thousand rubles to company B. Company A promises 50% per annum, but may burst with a probability of 0.2. Company B – 40% per annum, but may burst with a probability of 0.15. Draw up a law of distribution of a random variable – the total amount of profit (loss) received from two companies in a year. Find its mathematical expectation.

7. Find the expected value of lottery tickets that will win if 20 tickets are purchased, and the probability of winning one ticket is 0.3.

8. Find the expected value of a discrete random variable X distributed according to Poisson's law:

9.

X	0	1	2	...	k
p	$\exp(-\lambda)$	$\frac{\lambda \exp(-\lambda)}{1!}$	$\frac{\lambda^2 \exp(-\lambda)}{2!}$	...	$\frac{\lambda^k \exp(-\lambda)}{k!}$

Take into account that

2. Dispersion of a discrete random variable

1. For tasks No1, 3, 4, 5 p. 1, calculate the variance of the distribution of the random variable X.

2. The random variable X is given by the law of distribution

X	2	3	10
p	0,1	0,4	0,5

Find the standard deviation (X).

3. Find the variance of the discrete random variable X, the number of occurrences of event A in five independent trials, if the probability of occurrence of events A in each trial is 0.2.

4. N dice rolled. Find the variance of the sum of the number of points that can appear on all the faces rolled.

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 6

Numerical Characteristics of Continuous Random Variables

Educational questions of the lesson:

1. Mathematical expectation of a continuous random variable
2. Variance of a Continuous Random Variable

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to finding numerical characteristics of continuous random variables.

Checking the readiness of students for the lesson

1. What random variable is called continuous?
2. What is called probability density?
3. What is called the mathematical expectation of a continuous random variable?
4. Define the variance of a continuous random variable?
5. To define the initial theoretical moment of order k ?
6. To define the central theoretical moment of order k ?

1. Mathematical expectation of a continuous random variable

1. The random variable X is given by the distribution density $f(x)=x/2$ in the interval $(0, 2)$; outside this interval $f(x)=0$. Find the mathematical expectation of the value of X .

2. The random variable X is given by the density of distribution in the interval $(0; 1)$; outside this interval, $f(x)=0$. Find the mathematical expectation of the function (without first finding the distribution density Y).

3. Find the mathematical expectation of a random variable X given by a distribution function:

4. The random variable X is given by the probability density $f(x)=$ in the interval $(0; 1)$. Outside this interval, $f(x)=0$. Find the parameter C .

5. The random variable X on the interval $(2, 4)$ is given by the distribution density $f(x)=-\frac{3}{4}x+\frac{9}{2}x-6$; outside this interval, $f(x)=0$. Find the mode, the expected value, and the median of X .

6. The function is defined as:

To find:

- a) the value of the constant A , at which the function will be the probability density of some random variable X ;
- b) expression of the distribution function $F(x)$;
- c) calculate the probability that the random variable X will take a value on the segment $[2; 3]$;
- d) find the mathematical expectation and variance of the random variable X .

2. Variance of a Continuous Random Variable

1. The random variable X is given by the distribution density $f(x)=1/()$ in the interval $(-c; c)$; outside this interval $f(x)=0$. Find the variance of the value X .

2. The random variable X on the interval $(0,)$; is given by the distribution density $f(x)=(1/2)\sin x$; outside this interval $f(x)=0$. Find the variance of the value X .

3. The random variable X is given by the distribution density $f(x)=0.5x$ in the interval $(0, 2)$; outside this interval $f(x)=0$. Find the initial and central moments of the first, second, third, and fourth orders.

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 7

Primary static processing of random sampling

Educational questions of the lesson:

1. Calculation of the main characteristics of samples. Variation and statistical series
2. Estimation of general numerical characteristics using a grouped statistical series

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to determining the characteristics of samples.

Checking the readiness of students for the lesson

1. What is the general population, sampling, selection?
2. Formulate a definition of simple random selection.
3. What types of real elections do you know?
4. Which sample is called representative, homogeneous?
5. What is a variational, statistical series?
6. Define the extreme elements of the variation series, range, median, quartiles.
7. What is a frequency polygon?
8. Define a sample distribution function.
9. What is the evaluation of a general numerical characteristic?

1. Calculation of the main characteristics of samples. Variation and statistical series

- 1.** As a result of the experiment, a sample of the volume was obtained:

2; 4; 2; 4; 3; 3; 3; 2; 0; 6; 1; 2; 3; 2; 2;
4; 3; 3; 5; 1; 0; 2; 4; 3; 2; 2; 3; 3; 1; 3;
3; 3; 1; 1; 2; 3; 1; 4; 3; 1; 7; 4; 3; 4; 2;
3; 2; 3; 3; 1; 4; 3; 1; 4; 5; 3; 4; 2; 4; 5;
3; 6; 4; 1; 3; 2; 4; 1; 3; 1; 0; 0; 4; 6; 4;
7; 4; 1; 3.

Construct a variation and statistical series, depict a frequency polygon, determine the extremes, sample range, sample median, lower and upper quartiles, find and construct a sample distribution function. Calculate the main sample estimates: sample mean, 2nd order sample initial and central moments, sample variance, and sample standard deviation.

2. Estimation of general numerical characteristics using a grouped statistical series

1. The maximum capacitance of the 20 trim capacitors has been measured and the measurement results (in picofarads) are shown in the table:

Capacitor Number	1	2	3	4	5	6	7	8	9	10
Capacity, pcs	4,40	4,31	4,40	4,40	4,65	4,56	4,71	4,54	4,36	4,56
Capacitor Number	11	12	13	14	15	16	17	18	19	20
Capacity, pcs	4,31	4,42	4,60	4,35	4,50	4,40	4,43	4,48	4,42	4,45

Construct a grouped statistical series and plot it as a histogram. Evaluate general numerical characteristics using a grouped statistical series.

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 8

Point estimation of numerical characteristics and distribution parameters of the general population

Educational questions of the lesson:

1. Calculation of point estimates of the general average
2. Calculation of point estimates of the general variance of the general population

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to the determination of point estimates of the numerical characteristics of the general population.

Checking the readiness of students for the lesson:

1. What is the general population, sampling, selection?
2. Formulate a definition of simple random selection.
3. What types of real elections do you know?
4. Which sample is called representative, homogeneous?

5. What is a variational, statistical series?
6. Define the extreme elements of the variation series, range, median, quartiles.
7. What is a frequency polygon?
8. Define a sample distribution function.
9. What is the evaluation of a general numerical characteristic?

1. Calculation of point estimates of the general average

1. A sample of the volume $n = 50$ was extracted from the general population:

Elements	x_i	2	5	7	10
Frequency	n_i	16	12	8	14

Find the unbiased estimate of the general average. Is the median an unbiased score? Why?

2. A sample of the volume $n=60$ was extracted from the general population:

Elements	x_i	1	3	6	26
Frequency	n_i	8	40	10	2

Find the unbiased estimate of the general average. Is the half-sum of the interquartile latitude an unbiased estimate?

3. Find the sample mean for a given distribution of the sample volume $n = 10$ using the transition to conditional options:

Elements	x_i	1250	1270	1280
Frequency	n_i	2	5	3

4. Find the sample mean for a given distribution of the sample volume $n = 20$ using the transition to conditional options:

Elements	x_i	2560	2600	2620	2650	2700
Frequency	n_i	2	3	10	4	1

5. There is a random sample ($X, \dots, X_5$) of the volume $n = 5$ from the general population X , distributed according to the exponential law:

The arithmetic mean of the extreme terms of the variation series is taken as a point estimate of the mathematical expectation. Is the estimate biased?

2. Calculation of point estimates of the general variance of the general population

1. For the volume sample $n=41$, an offset estimate equal to 3 of the general variance was found. Find an unbiased estimate of the variance of the general population.

2. As a result of five measurements of the rod length with one device (without systematic errors), the following results were obtained (in mm): 92; 94; 103; 105; 106. Find: a) the sample length of the rod; b) the selective and corrected variance of the device errors.

3. Find the sample variance over a given sample distribution of the volume $n=10$:

Elements	x_i	186	192	194
Frequency	n_i	2	5	3

4. Find the sample variance over a given distribution of the sample size $n=10$ using the transition to conditional options:

Elements	x_i	0,01	0,04	0,08
Frequency	n_i	5	3	2

5. Find the sample variance over a given distribution of the sample size $n=50$ using the transition to conditional options:

Elements	x_i	18,4	18,9	19,3	19,6
Frequency	n_i	5	10	20	15

6. Find the corrected sample variance for a given sample distribution of volume $n=10$ using the transition to conditional options:

Elements	x_i	23,5	26,1	28,2	30,4
Frequency	n_i	2	3	4	1

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 9

Obtaining estimates of the parameters of the general distribution using the methods of moments and maximum likelihood

Educational questions of the lesson:

1. Obtaining estimates of the parameters of general distribution by the method of moments
2. Obtaining estimates of the parameters of the general distribution by the method of maximum likelihood

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to the determination of point estimates of the numerical characteristics of the general population.

Checking the readiness of students for the lesson:

1. What is the general population, sampling, selection?
2. Formulate a definition of simple random selection.
3. What types of real elections do you know?
4. Which sample is called representative, homogeneous?
5. What is a variational, statistical series?
6. Define the extreme elements of the variation series, range, median, quartiles.

7. What is a frequency polygon?
8. Define a sample distribution function.
9. What is the evaluation of a general numerical characteristic?
10. What assessment is called sound, unbiased, robust, effective?

1. Obtaining estimates of the parameters of general distribution by the method of moments

1. A random variable has a Poisson distribution.

where is the unknown parameter. As a result of independent observations, a random sample was obtained. Find a point estimate of the parameter by the method of moments and make sure that this estimate is unbiased and consistent.

2. Suppose a random sample of the volume from the general population is given, which has a uniform law of distribution

with unknown parameters and . Find point estimates of these parameters by the moment method.

2. Obtaining estimates of the parameters of the general distribution by the method of maximum likelihood

1. Let a random sample of the volume be given from the general population distributed according to the binomial law

with an unknown parameter (the probability of an event occurring in one trial). Using the method of maximum likelihood, find a point estimate of the parameter .

2. Using the method of maximum likelihood from a random sample, find the estimate of the parameter in the Pareto distribution:

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

Practical Lesson 10

Interval estimation of numerical characteristics

Educational questions of the lesson:

1. Construction of a confidence interval for a sample mean of the normal population

2. Estimating the confidence interval for expected value

CONTENT OF THE LESSON

Introduction

In today's practical lesson, you are invited to perform practical tasks related to the determination of point estimates of the numerical characteristics of the general population.

Checking the readiness of students for the lesson:

1. Define the confidence interval.
2. What is the accuracy and reliability of the assessment?
3. How is the accuracy and reliability of estimating the probability of an event determined by relative frequency?
4. Construct a confidence interval for the mathematical expectation m of the normal population
5. Construct a confidence interval for the mathematical expectation m of any population with a large sample size.
6. Find the quantile of the Student's distribution (according to Table III) of the order of 0.9 and 0.99 with 10 degrees of freedom.

1. Construction of a confidence interval for a sample mean of the normal population

1. Using a voltmeter, the accuracy of which is characterized by a standard deviation of 0.2 V, 10 measurements of the onboard battery voltage were carried out. Find the confidence interval for the true value of the battery with a coefficient $= 0.95$, if the arithmetic mean of the observation results $= 50.2$ V. The monitored feature has a normal distribution.

2. From a large batch of electric lamps, 400 pieces were randomly selected to determine the average duration of burning. The sample average duration of burning of lamps turned out to be equal to 1220 hours. Find with a confidence factor of $= 0.997$ the confidence interval for the average duration of burning of an electric lamp for the entire batch, if the standard deviation of the duration of burning is 35 hours.

2. Estimating the confidence interval for expected value

1. As a result of the launches of 10 missiles, the values of lateral deviations of the points of impact from the aiming points were obtained (in conventional units) (see Table).

Rocket Number	1	2	3	4	5	6	7	8	9	10
Deviation	1,0	2,0	1,0	-0,1	-0,5	5,0	-1,0	3,0	0,5	1,0

Assuming that the random variable X (the random lateral deviation of the impact points from the aiming points) has a normal distribution. Construct a confidence interval for its mathematical expectation with a confidence coefficient

Lesson Reporting

The results of the work should be reflected in the workbook and defended orally by each student. When preparing for the defense, the main attention should be paid to understanding the physical meaning of each item of the task and substantiating the results obtained.

Conclusion

An interview is conducted with each student based on the results of work. Students who have successfully completed the calculation of all practical tasks are graded.

List of recommended literature

List of basic literature:

- Kolemaev, V.A. Teoriya veroyatnostey i matematicheskaya statistika Elektronnyi resurs: uchebnik [Theory of Probability and Mathematical Statistics] Elektronnyi resurs: uchebnik [Elektronnyi resurs: uchebnik] / V.N. Kalinina / V.A. Kolemaev; ed. by V.A. Kolemaev. - Probability Theory and Mathematical Statistics, 2020-10-10. - Moscow: UNITI-DANA, 2017. - 352 p. - The book is in the basic version of the IPRbooks EBS. - ISBN 5-238-00560-1
- Probability Theory and Mathematical Statistics: A Course of Lectures: A Textbook. E. O. Tarasenko; Author: I. V. Zaitseva; Author: P. K. Korneev; Author: A. V. Gladkov; Ministry of Science and Higher Education of the Russian Federation; Federal State Autonomous Educational Institution of Higher Education "North Caucasus Federal University". - Stavropol: NCFU, 2018. - 229 p.: ill.

List of additional literature:

- Ventsel E. S. Teoriya probrozvotel'nostey: uchebnik dlya vuzov [Probability theory: a textbook for higher education institutions]. - 9th ed., ster. - Moscow: Vysshaya shkola, 2003. - 576 p.: ill. - Grief: Rek. MO. - Appendix. pp. 551-563. - Predm. decree. pp. 564-567. - ISBN 5-7695-0984-8
- Gmurman V. E. Teoriya probrozavnostey i matematicheskaya statistika: uchebnik dlya prikladnogo bakalavrata [Probability Theory and Mathematical Statistics: A Textbook for Applied Bachelor's Degree]. - 12th ed. - Moscow: Yurayt, 2016. - 480 p.: ill. - (Bachelor). - Grief: Rek. UMO i MO. - Predm. Op. cit.: p. 474-479. - ISBN 978-5-9916-6110-2
- Matalytskiy M.A., Khatskevich G.A. Teoriya probrozavnostey i matematicheskaya statistika: uchebnik [Probability Theory and Mathematical Statistics: Textbook]. - Minsk: Higher School, 2017. - 592 p. - <http://biblioclub.ru/>. - Bibliogr. in. - ISBN 978-985-06-2855-8
- Gmurman V. E. Rukovodstvo k resheniya zadach po teorii verobnostey i matematicheskoy statistiki: uchebnoe posobie dlya prikladnogo bakalavratka: dlya studentov vuzov [A guide to solving problems in the theory of probability and mathematical statistics: a textbook for applied bachelor's degree: for university students]. - 11th ed., revised and supplemented - Moscow: Yurayt, 2016. - 405 p.: ill., tabl. - (Bachelor's Degree). - Appendix: pp. 388-404. - ISBN 978-5-9916-6505-6

List of Educational and Methodological Support for Students' Independent Work in a Discipline (Module)

- Probability Theory and Mathematical Statistics: A Course of Lectures: A Textbook. E. O. Tarasenko; Author: I. V. Zaitseva; Author: P. K. Korneev; Author: A. V. Gladkov; Ministry of Science and Higher Education of the Russian Federation; Federal State Autonomous Educational Institution of Higher Education "North Caucasus Federal University". - Stavropol: NCFU, 2018. - 229 p.: ill.

List of resources of the information and telecommunication network "Internet" necessary for mastering the discipline (module)

- [https://ru.wikipedia.org/wiki/ Probability Theory](https://ru.wikipedia.org/wiki/Probability_Theory). Wikipedia Free Encyclopedia.
- <https://intuit.ru/studies/courses/2263/219/info> Probability Theory and Mathematical Statistics: INTUIT National Open University
- <http://exponenta.ru/> Educational mathematical website EXPONENTA.ru

Appendices. Quantile tables

Table 1. Normal distribution quantiles

p	0,90	0,95	0,975	0,99	0,995	0,999	0,9995
u_v	1,282	1,645	1,960	2,326	2,576	3,090	3,291

Table 2. Quantiles of the Student's distribution, k is the number of degrees of freedom of distribution; the order of quantiles.

R	0,750	0,900	0,950	0,975	0,990	0,995	0,999
n							
1	1,000	3,078	6,314	12,706	31,821	63,657	318
2	0,816	1,886	2,920	4,303	6,965	9,925	22,3
3	0,765	1,638	2,353	3,182	4,541	5,841	10,2
4	0,741	1,533	2,132	2,776	3,747	4,604	7,173
5	0,727	1,476	2,015	2,571	3,365	4,032	5,893
6	0,718	1,440	1,943	2,447	3,143	3,707	5,208
7	0,711	1,415	1,895	2,365	2,998	3,499	4,785
8	0,706	1,397	1,860	2,306	2,896	3,355	4,501
9	0,703	1,383	1,833	2,262	2,821	3,250	4,297
10	0,700	1,372	1,812	2,228	2,764	3,169	4,144
11	0,697	1,363	1,796	2,201	2,718	3,106	4,025
12	0,695	1,356	1,782	2,179	2,681	3,055	3,930
13	0,694	1,350	1,771	2,160	2,650	3,012	3,852
14	0,692	1,345	1,761	2,145	2,624	2,977	3,787
15	0,691	1,341	1,753	2,131	2,602	2,947	3,733
16	0,690	1,337	1,746	2,120	2,583	2,921	3,686
17	0,689	1,333	1,740	2,110	2,567	2,898	3,646
18	0,688	1,330	1,734	2,101	2,552	2,878	3,610
19	0,688	1,328	1,729	2,093	2,539	2,861	3,579
20	0,687	1,325	1,725	2,086	2,528	2,845	3,552
21	0,686	1,323	1,721	2,080	2,518	2,831	3,527
22	0,686	1,321	1,717	2,074	2,508	2,819	3,505
23	0,685	1,319	1,714	2,069	2,500	2,807	3,485
24	0,685	1,318	1,711	2,064	2,492	2,797	3,467
25	0,684	1,316	1,708	2,060	2,485	2,787	3,450
26	0,684	1,315	1,706	2,056	2,479	2,779	3,435
27	0,684	1,314	1,703	2,052	2,473	2,771	3,421
28	0,683	1,313	1,701	2,048	2,467	2,763	3,408
29	0,683	1,311	1,699	2,045	2,462	2,756	3,396
30	0,683	1,310	1,697	2,042	2,457	2,750	3,385
40	0,681	1,303	1,684	2,021	2,423	2,704	3,307
60	0,679	1,296	1,671	2,000	2,390	2,660	3,232
120	0,677	1,289	1,658	1,980	2,358	2,617	3,160
∞	0,674	1,282	1,645	1,960	2,326	2,576	3,090