

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
FEDERAL STATE AUTONOMOUS EDUCATIONAL
INSTITUTION OF HIGHER EDUCATION
"NORTH CAUCASUS FEDERAL UNIVERSITY"

SUGGESTIONS FOR TEACHING

for the performance of practical work
in the discipline "Linear Algebra"
for students of the field of study

11.03.02 Infocommunication Technologies and Communication Systems
Profile

"Intelligent Data Processing in Infocommunication
Systems and Networks"

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INTRODUCTION

The purpose of practical classes in the discipline "Algebra" is the formation of the general professional competence of OPK-1 of the future bachelor in the direction of training 11.03.02 - Infocommunication technologies and communication systems.

Among the tasks of mastering the discipline It can be attributed to:

- Both providing fundamental training in one of the most important areas of modern mathematics;
- Familiarization with the basics of classical algebra;
- teaching basic algebraic methods of problem solving.

The discipline "Algebra" belongs to the disciplines of the compulsory part and is studied in the 1st semester.

In the process of studying the discipline, students form the following competence (in accordance with the educational standard):

OPK-1 - Able to use the provisions, laws and methods of natural sciences and mathematics to solve problems of engineering activity.

The methodological instructions for the implementation of practical work in the discipline "Algebra" describe the sequence and main content of practical classes in the discipline. For each practical lesson, theoretical tasks, practical tasks to be solved in the classroom, and practical tasks for independent work at home are given.

Topic 1. Basic algebraic structures.

Practical lesson No1.

Binary algebraic operations and their basic properties.

The purpose of the lesson: to study binary algebraic operations and their basic properties.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Definition of algebraic operation.
- Examples of algebraic operations.
 - The concept of algebra as a set with operations.
- Examples of algebras.
- Definition of a binary algebraic operation.
- Basic properties of binary algebraic operations.

1. Practical tasks solved in the classroom:

Problem 1. Are the following operations R^+ defined on sets $N, Z, Q, 2Z, 2Z+1, R$:

a) $(a,b) \rightarrow ab$,

b) $(a,b) \rightarrow \frac{a}{b}$,

c) $(a,b) \rightarrow \frac{a+b}{2}$,

d) $(a,b) \rightarrow \min\{a,b\}$?

Task 2. Which of the operations from exercise 1 have the property of commutativity?

Task 3. Which of the operations from exercise 1 have the property of associativity?

Task 4. For the operations in Exercise 1 defined on the set R , find out if a neutral element exists. If it does, specify all the elements to be symmetrized.

3. Homework:

Problem 1. Are the following operations defined on sets $N, Z, Q, 2Z, 2Z+1, R$: R^+

a) $(a,b) \rightarrow a + b$,

b) $(a,b) \rightarrow a - b$,

c) $(a,b) \rightarrow \sqrt{ab}$,

d) $(a, b) \rightarrow ab - ba$,

e) $(a, b) \rightarrow \max\{a, b\}$?

Task 2. Which of the operations from exercise 1 have the property of commutativity?

Task 3. Which of the operations from exercise 1 have the property of associativity?

Problem 4. For the operations in Exercise 1 defined on the set R , find out if a neutral element exists. If it does, specify all the elements to be symmetrized.

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

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Practical lesson No2.

Groups.

Purpose of the lesson: Definition of the group, the main properties of groups.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Group definition.
- Examples of groups.
- Additive groups.
- Multiplicative groups.
- Basic properties of groups.
- Definition of a subgroup.

2. Practical tasks solved in the classroom:

Problem 1. Which of the following sets of mappings of the set $M = \{1, 2, \dots, n\}$ into itself forms a group of relative multiplication:

- a) the set of all mappings;
- b) the set of all injective mappings;
- c) the set of all surjective mappings;
- d) the set of all bijective mappings;
- e) the set of all even permutations;
- f) the set of all odd permutations.

Problem 2. Prove that the set of functions of the form $y = \frac{ax+b}{cx+d}$, where $a, b, c, d \in \mathbb{R}$ and $ad - bc \neq 0$, is a group with respect to the operation of the composition of functions.

3. Homework:

Problem 1. Which of the following sets of mappings of the set $M = \{1, 2, \dots, n\}$ into itself forms a group of relative multiplication:

- a) the set of all bijective mappings;
- b) the set of all even permutations;
- c) the set of all odd permutations.

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Practical lesson No3.

Rings and fields.

Purpose of the lesson: definition of the ring, the main properties of the rings. The concept of the field, the main properties of the fields.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1, knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Ring definition.
- Examples of rings.
- Basic properties of rings.
- Zero divisors.
- Multiplicative group of a ring with one.
- The concept of the field.
- Examples of fields.
- Basic properties of fields.

2. Practical tasks solved in the classroom:

Problem 1. Which of the following numerical sets form a ring relative to the usual operations of addition and multiplication:

- a) the set of real numbers of the form $x + y\sqrt{2}$, where $x, y \in \mathbb{Q}$;
- b) the set of real numbers of the form $x + y\sqrt[3]{2}$, where $x, y \in \mathbb{Q}$;
- c) the set of real numbers of the form $x + y\sqrt[3]{2} + z\sqrt[3]{4}$, where $x, y, z \in \mathbb{Q}$.

Task 2. Which of the rings in problem 1 are fields?

3. Homework:

Problem 1. Which of the following numerical sets form a ring relative to the usual operations of addition and multiplication:

- a) set $\mathbb{Z}$;
- b) a set $n\mathbb{Z} (n > 1)$;
- c) a set of non-negative integers;
- d) set $\mathbb{Q}$.

Task 2. Which of the rings in problem 1 are fields?

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

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Practical lesson No4.

Algebraic form of a complex number and operations on complex numbers in algebraic form. Geometric representation and trigonometric form of notation of complex numbers of a number.

Purpose of the lesson: representation of complex numbers in algebraic form, operations on complex numbers in algebraic form; Geometric representation and trigonometric form of writing complex numbers.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Representation of complex numbers in algebraic form.
- Addition and multiplication of complex numbers in algebraic form.
- Conjugate complex numbers.
- Division of complex numbers in algebraic form.
- Extraction of the square root of a complex number in algebraic form.
- Geometric representation of complex numbers.
- Geometric representation of actions on complex numbers.
- Determination of the modulus of the complex number.
- Define a complex number argument.
- Trigonometric form of a complex number.

2. Practical tasks solved in the classroom:

Task 1. Calculate the expressions:

a) $(2 + i)(3 - i) + (2 + 3i)(3 + 4i)$;

b) $\frac{(1 + 3i)(8 - i)}{(2 + i)^2}$;

c) $\frac{(1 + i)^5}{(1 - i)^3}$.

Problem 2. Calculate $i^{77}, i^{98}, i^{-57}, i^n$, where n is an integer.

Task 3. Solve equations:

a) $z^2 = 5 - 12i$; b) $z^2 - 5z + 4 + 10i = 0$.

Task 4. Draw on a plane the points corresponding to the numbers 5, -2, -3i, $\pm 1 \pm i\sqrt{3}$.

Problem 5. Indicate the geometric meaning of the expression $|z_1 - z_2|$, where z_1 and z_2 are the given complex numbers.

Problem 6. Draw on a plane a set of points corresponding to complex numbers z that meet the following conditions:

a) $\arg z = \frac{\pi}{3}$; b) $|z| \leq 2$; c) $|z - 1 - i| < 1$; d) $1 \leq |z - 2i| < 2$.

Problem 7. Solve equation $|z| + z = 8 + 4i$.

Task 8. Find the trigonometric form of the number:

a) -2 ; b) $-3i$; c) $1 - i$; d) $1 + i\sqrt{3}$; e) $-1 + i\sqrt{3}$; f) $-1 - i\sqrt{3}$;
g) $1 + \cos \varphi + i \sin \varphi$.

3. Homework:

Task 1. Calculate the expressions:

a) $(2 + i)(3 + 7i) - (1 + 2i)(5 + 3i)$; b) $(3 + i)^3 - (3 - i)^3$;
c) $\frac{(2 + i)(4 + i)}{1 + i}$.

Task 2. Solve equations:

a) $z^2 = 3 - 4i$; b) $z^2 - (1 + i)z + 6 + 3i = 0$; c) $z^2 + (2i - 7)z + 13 - i = 0$.

Problem 3. Find all complex numbers conjugate to their square. **Problem 4.** Find complex numbers corresponding to the vertices of the square centered at the origin, with sides of length 1 parallel to the coordinate axes.

Problem 5. Draw on a plane a set of points corresponding to complex numbers z that meet the following conditions:

a) $|z| = 1$; b) $|z + 3 + 4i| \leq 5$; c) $2 < |z| < 3$.

Problem 6. Solve equation $|z| - z = 8 + 12i$.

Task 7. Find the trigonometric form of the number:

a) 5; b) i ; c) $1 + i$; d) $1 - i\sqrt{3}$; e) $-\sqrt{3} + i$; f) $-\sqrt{3} - i$; g) $\cos \alpha - i \sin \alpha$.

Recommended reading

1. Reference citations:

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Practical lesson No5.

Multiplication and division of complex numbers in trigonometric form. Extraction of the nth root from a complex number.

Purpose of the lesson: actions on complex numbers in trigonometric form.

The relevance of the lesson lies in the fact that during its conduct, students form competencies; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson :

- Multiplication of complex numbers in trigonometric form.
- Division of complex numbers in trigonometric form
- Moivre's formula.
- Extraction of the nth degree root from a complex number.
- Geometric representation of roots of nth degree from a complex number.
- Extraction of the nth root from one.
- Primitive roots of the nth degree from one.
- Multiplicative group of nth degree roots from one.

2. Practical tasks solved in the classroom:

Task 1. Calculate the expressions:

$$\text{a) } (1+i)^{1000}; \text{ b) } \left(\frac{1-i\sqrt{3}}{1+i} \right)^{12}; \text{ c) } \frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}} + \frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}.$$

Task 2. When $n \in \mathbb{Z}$ calculating the expressions:

$$\text{a) } \left(\frac{1-itg\alpha}{1+itg\alpha} \right)^n; \text{ b) } (1+\cos\varphi+i\sin\varphi)^n.$$

Task 3. Represent in the form of polynomials of the $\sin x$ and $\cos x$ function:

$$\text{a) } \sin 5x; \text{ b) } \cos 5x.$$

$n \in Z$.

Task 4. Calculate:

a) $\sqrt[6]{i}$; b) $\sqrt[8]{2\sqrt{2}(1-i)}$; c) $\sqrt[4]{-72(1-i\sqrt{3})}$;
d) $\sqrt[4]{\frac{7-2i}{1+i\sqrt{2}} + \frac{4+14i}{\sqrt{2}+2i}} - (8-2i)$.

Problem 5. Solve the equation: $(z+1)^n + (z-1)^n = 0$.

Task 6. Calculate:

a) $\sqrt[3]{1}$; b) $\sqrt[4]{1}$; c) $\sqrt[6]{1}$.

Task 7. Find all the original roots of the n th degree from one, if a) $n=2$, b) $n=3$, c) $n=4$, d) $n=6$.

3. Homework:

Task 1. Calculate the expressions:

a) $(1+i\sqrt{3})^{130}$; b) $(\sqrt{3}+i)^{30}$; c) $\left(\frac{\sqrt{3}+i}{1-i}\right)^{30}$.

Task 2. When $n \in Z$ calculating the expressions:

a) $(1+i)^n$; b) $\left(\frac{1-i\sqrt{3}}{2}\right)^2$.

Task 3. Represent in the form of polynomials of the $\sin x$ and $\cos x$ function:

a) $\sin 4x$; b) $\cos 4x$.

Task 4. Prove equality

a) $\cos x + \cos 2x + \dots + \cos nx = \frac{\sin \frac{nx}{2} \cos \frac{(n+1)x}{2}}{\sin \frac{x}{2}}, (x \neq 2k\pi, k \in Z);$

b) $\sin x + \sin 2x + \dots + \sin nx = \frac{\sin \frac{nx}{2} \cos \frac{(n+1)x}{2}}{\sin \frac{x}{2}}, (x \neq 2k\pi, k \in Z).$

Task 5. Calculate:

a) $\sqrt[3]{i}$; b) $\sqrt[4]{-4}$; c) $\sqrt[6]{64}$; d) $\sqrt[4]{8\sqrt{3}i-8}$;
e) $\sqrt[3]{\frac{1-5i}{1+i} - 5\frac{1+2i}{2-i}} + 2$.

Problem 6. Solve the equation: $(z+i)^n + (z-i)^n = 0$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

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Topic 2. Ring of matrices above the ring. Matrices above the field.

Practical lesson No6.

Matrices on the ring and operations on them.

The purpose of the lesson: operations on matrices and their main properties.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- The concept of a matrix.
- Rectangular and square matrices.
- Addition of matrices of the same type.
- Multiplication of matrices by a number.
- Multiplication of matrices.
- Basic properties of operations on matrices.

2. Practical tasks solved in the classroom:

Task 1. Multiply the matrices:

$$\text{a) } \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 29 \\ 2 & 18 \\ 0 & -3 \end{pmatrix}; \quad \text{b) } \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix}; \quad \text{c) } \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$$

Task 2. Perform actions:

$$\begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & -1 & 2 \\ -2 & -1 & 1 & 2 \\ 2 & 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -4 & 6 & 1 \\ 2 & 2 & -5 & -2 \\ 2 & -2 & 6 & 4 \\ 1 & 3 & 0 & 1 \end{pmatrix}.$$

Task 3. Calculate:

$$\text{a) } \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}^2, \quad \text{b) } \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2.$$

Problem 4. Calculate the value of the polynomial $f(x)$ from matrix A :

$$f(x) = x^3 - 3x + 2, \quad A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

Task 5. Solve a system of matrix equations:

$$2X - Y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad -4X + 2Y = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}.$$

3. Homework:

Task 1. Multiply the matrices:

$$\text{a) } \begin{pmatrix} 1 & 5 & 3 \\ 2 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{pmatrix}; \quad \text{b) } \begin{pmatrix} 1 & -1 & 3 \\ -1 & 1 & -3 \\ 2 & - & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 & 2 \\ 0 & 3 & -1 \\ 2 & 1 & -1 \end{pmatrix};$$

$$\text{h) } \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -3 & 1 \end{pmatrix}.$$

Task 2. Perform actions:

$$\begin{pmatrix} 3 & 0 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 2 & 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & 1 \\ -1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & -3 \\ 0 & 6 & -3 \\ 5 & -2 & 8 \end{pmatrix}.$$

Task 3. Calculate:

$$\text{a) } \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 \quad \text{b) } \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}^2.$$

Problem 4. Calculate the value of the polynomial $f(x)$ from matrix A :

$$f(x) = x^3 - 2x + 2, \quad A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Task 5. Solve a system of matrix equations:

$$X + Y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad 2X + 3Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

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4. Chegolín A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No 7.

Definition and properties of the determinant of a square matrix of the n th order.

Purpose of the lesson: application of the definition and properties of the determinant of a square matrix of the n th order for its calculation.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Permutations of a finite set.
- Parity of the permutation.
- Substitutions.
- Parity of substitution.
- Determination of the determinant of a square matrix of the n th order.
- Calculation of determinants of the 2nd order.
- Calculation of determinants of the 3rd order.
- Transformations of the determinant of a square matrix of the n th order that do not change it .
- Equality of rows and columns of the determinant.
- Transformations of the determinant of a square matrix of the n th order, turning it into 0.
- The most general case of equality of the determinant to zero.
- Transformations of the determinant of the n th order square matrix, exposing it to easily traceable changes.

2. Practical tasks solved in the classroom:

№ 1. Determine the number of inversions in permutations (if there are no special instructions, the position 1,2,3 is taken as the initial location of the permutation,... in ascending order):

- a) 2,3,5,4,1.
- b) 1,9,6,3,2,5,4,7,8
- c) 1,3,5,7,...,2n-1, 2,4,6,8,...,2n.

№2. Find out which of the following works are included in the determinants of the corresponding orders and with which signs:

- a) $a_{43}a_{21}a_{35}a_{12}a_{54}$; b) $a_{61}a_{23}a_{45}a_{36}a_{12}a_{54}$; c) $a_{27}a_{36}a_{51}a_{74}a_{25}a_{43}a_{62}$.

No 3. Choose the values of i and k so that the product $a_{47}a_{63}a_{11}a_{55}a_{7k}a_{24}a_{31}$ is included in the 7th order determinant with a plus sign.

No 4. Find the 4th order terms of the determinant that contain an element a_{32} and are included in the determinant with a plus sign.

№ 6. Calculate the determinants of the 2nd order:

- a) $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$, b) $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$.

№ 7. Calculate the determinants of the 3rd order:

$$\text{a) } \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}, \text{ b) } \begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}.$$

Task 8. Calculate determinants using properties and definition:

$$\text{a) } \begin{vmatrix} 1 & 2 & 3 & 4 \\ -3 & 2 & -5 & 13 \\ 1 & -2 & 10 & 4 \\ -2 & 9 & -8 & 25 \end{vmatrix}, \text{ b) } \begin{vmatrix} 7 & 6 & 9 & 4 & -4 \\ 1 & 0 & -2 & 6 & 6 \\ 7 & 8 & 9 & -1 & -6 \\ 1 & -1 & -2 & 4 & 5 \\ -7 & 0 & -9 & 2 & -2 \end{vmatrix}, \text{ c) } \begin{vmatrix} 1 & 5 & 3 & 5 & -4 \\ 3 & 1 & 2 & 9 & 8 \\ -1 & 7 & -3 & 8 & -9 \\ 3 & 4 & 2 & 4 & 7 \\ 1 & 8 & 3 & 3 & 5 \end{vmatrix}.$$

3. Homework:

№ 1. Determine the number of inversions in permutations (if there are no special instructions, the position 1,2,3 is taken as the initial location of the permutation,... in ascending order):

a) 6,3,1,2,5,4;

b) 7,5,6,4,1,3,2;

c) 2,4,6,...,2n,1,3,5,...,2n-1.

№ 2. Find out which of the following works are included in the detailed expression of the determinants of the corresponding orders and with what signs:

a) $a_{13}a_{22}a_{31}a_{46}a_{55}a_{64}$; b) $a_{31}a_{13}a_{52}a_{45}a_{24}$; c) $a_{34}a_{21}a_{46}a_{17}a_{73}a_{54}a_{62}$

No 3. Choose the values of i, j, k so that the product $a_{51}a_{i6}a_{1j}a_{35}a_{44}a_{6k}$ is included in the expanded expression of the sixth order with a minus sign.

№ 4. Using the partition, calculate the following determinants:

$$\text{a) } \begin{vmatrix} a & 3 & 0 & 5 \\ 0 & b & 0 & 2 \\ 1 & 2 & c & 3 \\ 0 & 0 & 0 & d \end{vmatrix}, \text{ b) } \begin{vmatrix} 1 & 0 & 2 & a \\ 2 & 0 & b & 0 \\ 3 & c & 4 & 5 \\ d & 0 & 0 & 0 \end{vmatrix}.$$

№ 5. Calculate the determinants:

$$\text{a) } \begin{vmatrix} 3 & 5 \\ 5 & 3 \end{vmatrix}, \text{ b) } \begin{vmatrix} ab & ac \\ bd & cd \end{vmatrix}.$$

№ 6. Calculate the determinants:

$$\text{a) } \begin{vmatrix} 1 & 2 & 3 \\ 5 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}, \text{ b) } \begin{vmatrix} -1 & 5 & 4 \\ 3 & -2 & 0 \\ -1 & 3 & 6 \end{vmatrix}.$$

Task 7. Calculate determinants using properties and definition:

$$\begin{array}{lcl}
 \text{a)} \begin{vmatrix} 1 & -1 & 1 & -2 \\ 1 & 3 & -1 & 3 \\ -1 & -1 & 4 & 3 \\ -3 & 0 & -8 & -13 \end{vmatrix} & \text{b)} \begin{vmatrix} 4 & 4 & -1 & 0 & -1 & 8 \\ 2 & 3 & 7 & 5 & 2 & 3 \\ 3 & 2 & 7 & 5 & 2 & 3 \\ 1 & 2 & 2 & 1 & 1 & 2 \\ 1 & 7 & 6 & 6 & 5 & 7 \\ 2 & 1 & 1 & 2 & 2 & 1 \end{vmatrix} & \text{c)} \begin{vmatrix} -5 & -7 & -2 & 2 & -2 & 16 \\ 0 & 0 & 4 & 0 & -5 & 0 \\ 2 & 0 & -2 & 0 & 2 & 0 \\ 6 & 4 & 6 & -1 & 15 & -5 \\ 5 & -4 & 10 & 1 & 14 & 6 \\ 3 & 0 & -2 & 0 & 3 & 0 \end{vmatrix} .
 \end{array}$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

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Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

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Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No8.

Minors and algebraic complements. Calculation of the nth order determinant.

Purpose of the lesson: calculation of the determinant of the nth order using decomposition by elements of a row or column.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Minors.
- Algebraic complement of the determinant element.

- Theorem of the Product $a_{ij}A_{ij}$.
- Decomposition of the determinant by elements of a row or column.
- Calculation of the nth order determinant.

2. Practical tasks solved in the classroom:

Task 1. Decomposing by the third line, calculate the determinant

$$\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$$

Task 2. Calculate the determinants:

$$\text{a) } \begin{vmatrix} 1 & 2 & 3 & 4 \\ -3 & 2 & -5 & 13 \\ 1 & -2 & 10 & 4 \\ -2 & 9 & -8 & 25 \end{vmatrix}, \text{ b) } \begin{vmatrix} -5 & -7 & -2 & 2 & -2 & 16 \\ 0 & 0 & 4 & 0 & -5 & 0 \\ 2 & 0 & -2 & 0 & 2 & 0 \\ 6 & 4 & 6 & -1 & 15 & -5 \\ 5 & -4 & 10 & 1 & 14 & 6 \\ 3 & 0 & -2 & 0 & 3 & 0 \end{vmatrix}, \text{ c) } \begin{vmatrix} 4 & -2 & 0 & 5 \\ 3 & 2 & -2 & 1 \\ -2 & 1 & 3 & -1 \\ 2 & 3 & -6 & -3 \end{vmatrix}$$

$$\text{d) } \begin{vmatrix} 3 & 2 & 4 & 5 \\ 4 & -3 & 2 & -4 \\ 5 & -2 & -3 & -7 \\ -3 & 4 & 2 & 9 \end{vmatrix}, \text{ e) } \begin{vmatrix} 6 & 3 & 4 & 5 \\ 4 & -3 & 2 & -4 \\ 5 & -2 & -3 & -7 \\ -3 & 4 & 2 & 9 \end{vmatrix}, \text{ f) } \begin{vmatrix} 27 & 44 & 40 & 55 \\ 20 & 64 & 21 & 40 \\ 13 & -20 & -13 & 24 \\ 46 & 45 & -55 & 84 \end{vmatrix}.$$

3. Homework:

Task 1. Decomposing by the second column, calculate the determinant

$$\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}.$$

Task 2. Calculate the determinants:

$$\text{a) } \begin{vmatrix} 1 & -1 & 1 & -2 \\ 1 & 3 & -1 & 3 \\ -1 & 1 & 4 & 3 \\ -3 & 0 & -8 & -13 \end{vmatrix}, \text{ b) } \begin{vmatrix} 7 & 6 & 9 & 4 & -4 \\ 1 & 0 & -2 & 6 & 6 \\ 7 & 8 & 9 & -1 & -6 \\ 1 & -1 & -2 & 4 & 5 \\ -7 & 0 & -9 & 2 & -2 \end{vmatrix}, \text{ c) } \begin{vmatrix} 1001 & 1002 & 1003 & 1004 \\ 1002 & 1003 & 1001 & 1002 \\ 1001 & 1001 & 1001 & 999 \\ 1001 & 1000 & 998 & 999 \end{vmatrix},$$

$$\text{d) } \begin{vmatrix} 4 & 3 & 3 & 5 \\ 3 & 4 & 3 & 2 \\ 3 & 2 & 5 & 4 \\ 2 & 4 & 2 & 3 \end{vmatrix}, \text{ e) } \begin{vmatrix} 14 & 13 & 3 & -13 \\ -7 & -4 & 2 & 10 \\ 21 & 23 & 0 & -23 \\ 7 & 12 & -2 & -6 \end{vmatrix}, \text{ f) } \begin{vmatrix} 2 & 4 & 6 & -5 \\ 1 & 6 & 5 & 4 \\ -3 & 2 & 4 & 6 \\ 4 & 5 & 2 & 3 \end{vmatrix}.$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

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Practical lesson No9.

Inverse matrix and its calculation.

The purpose of the lesson: calculating the inverse matrix using the formula for calculating the inverse matrix.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Inverse matrix.
- Uniqueness of the inverse matrix.
- Calculation of the inverse matrix using elementary transformations.
- Formula for calculating the inverse matrix.
- Conditions for the existence of an inverse matrix.
- Non-degenerate matrices.

2. Practical tasks solved in the classroom:

Task 1. Find the inverse of the matrix using the attached matrix:

$$\text{a) } \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \text{b) } \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}, \quad \text{c) } \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}, \quad \text{d) } \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}, \quad \text{e) } \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

Task 2. Solve matrix equations:

$$\text{a) } \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \text{b) } \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix},$$

$$\text{c) } \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} X \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

3. Homework:

Task 1. Find the inverse of the matrix using the attached matrix:

$$\text{a) } \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \text{b) } \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \text{c) } \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}, \quad \text{d) } \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix},$$

Task 2. Solve matrix equations:

$$\text{a) } X \begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 3 & 4 \end{pmatrix}, \quad \text{b) } \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix},$$

$$\text{c) } X \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}, \quad \text{d) } \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix} X = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{pmatrix}.$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

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Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

Practical lesson No10.

The rank of the matrix and its calculation.

The purpose of the lesson: reducing matrices to a stepped form using elementary transformations; calculating the rank of the matrix using elementary transformations.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Matrix rank.
- Step Matrix Rank
- Elementary matrix transformations.
- Equivalent matrices.
- Bringing the matrix to a stepped view. .
- Calculating the rank of the matrix using elementary transformations.

2. Practical tasks solved in the classroom:

1. Bring the matrices to a stepped form with the help of elementary transformations:

$$\text{a) } \begin{pmatrix} 8 & 2 & 2 & -1 & 1 \\ 1 & 7 & 4 & -2 & 5 \\ -2 & 4 & 2 & -1 & 3 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} 4 & 1 & 7 & -5 & 1 \\ 0 & -7 & 1 & -3 & -5 \\ 3 & 4 & 5 & -3 & 2 \\ 2 & 5 & 3 & -1 & 3 \end{pmatrix}$$

$$\text{c) } \begin{pmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{pmatrix}$$

$$\text{d) } \begin{pmatrix} 3 & 1 & 1 & 2 & -1 \\ 0 & 2 & -1 & 1 & 2 \\ 4 & 3 & 2 & -1 & 1 \\ 12 & 9 & 8 & -7 & 3 \\ -12 & -5 & -8 & 5 & 1 \end{pmatrix}$$

2. Find the rank of the following matrices using elementary transformations

$$\text{a) } \begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}, \text{ b) } \begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}, \text{ c) } \begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}.$$

3. Find the values λ at which the matrix $\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$ has the lowest rank. What is the rank for the found values λ and what is it equal for other values λ ?

3. Homework:

1. Bring the matrices to a stepped form with the help of elementary transformations:

$$\text{a) } \begin{pmatrix} 1 & 7 & 7 & 9 \\ 7 & 5 & 1 & -1 \\ 4 & 2 & -1 & -3 \\ -1 & 1 & 3 & 5 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} 8 & -4 & 5 & 5 & 9 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \\ 3 & -1 & 3 & 2 & 5 \end{pmatrix}$$

$$\text{c) } \begin{pmatrix} 77 & 32 & 6 & 5 & 3 \\ 32 & 14 & 3 & 2 & 1 \\ 6 & 3 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{d) } \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 \\ 1 & 4 & 1 & 1 \\ 5 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$$

2. Find the rank of the following matrices using elementary transformations:

$$\text{a) } \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}, \text{ b) } \begin{pmatrix} 47 & -67 & 32 & 201 & 155 \\ 26 & 98 & 23 & -294 & 86 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix},$$

$$\text{c) } \begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}.$$

3. What is the rank of the matrix at $\begin{pmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix}$ different values λ ?

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra.- 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

Available at: <http://www.iprbookshop.ru/68568.html>

5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No11.

Arithmetic n-dimensional vectors.

Purpose of the lesson: operations on arithmetic n-dimensional vectors,
Linear dependence of the system of arithmetic n-dimensional vectors, finding the basis and rank of the system of arithmetic n-dimensional vectors.

The relevance of the lesson lies in the fact that during its conduct, students form the competencies of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson

- Arithmetic n-dimensional vectors.
- Operations on arithmetic n-dimensional vectors.
- Linear dependence of the system of arithmetic n-dimensional vectors.
- Basis and rank of a system of arithmetic n-dimensional vectors.

2. Practical tasks solved in the classroom:

1. Find vector x from the equation:

$$4a_1 + 2a_2 + 3a_3 + 4x = 0, \text{ where } a_1 = (5, -8, -1, 2), a_2 = (2, -1, 4, -3), a_3 = (-3, 2, -5, 4)$$

2. Suppose a linearly independent system of vectors is given $a_1, \dots, a_k$. Find out whether a system of vectors is linearly dependent

$$b_1 = 3a_1 + 2a_2 + a_3 + a_4, b_2 = 2a_1 + 5a_2 + 3a_3 + 2a_4, b_3 = 3a_1 + 4a_2 + 2a_3 + 3a_4.$$

3. Find all the bases of the vector system:

$$a_1 = (1, 2, 0, 0), a_2 = (1, 2, 3, 4), a_3 = (3, 6, 0, 0).$$

4. Find some basis for a system of vectors

$$a_1 = (5, 2, -3, 1), a_2 = (4, 1, -2, 3), a_3 = (1, 1, -1, -2), a_4 = (3, 4, -1, 2), a_5 = (7, -6, -7, 0) \text{ and express the rest of the vectors of the system through this basis.}$$

3. Homework:

1). 1. Find vector x from the equation:

$$3(a_1 - x) + 2(a_2 + x) = 5(a_3 + x), \text{ where } a_1 = (2, 5, 1, 3), a_2 = (10, 1, 5, 10), a_3 = (4, 1, -1, 1)$$

2. Suppose a linearly independent system of vectors is given $a_1, \dots, a_k$. Find out whether a system of vectors is linearly dependent

$$b_1 = 3a_1 + 4a_2 - 5a_3 - 2a_4 + 4a_5, b_2 = 8a_1 + 7a_2 - 2a_3 + 5a_4 - 10a_5, b_3 = 2a_1 - a_2 + 8a_3 - a_4 + 2a_5.$$

3. Find all the bases of the vector system:

$$a_1 = (4, -1, 3, -2), a_2 = (8, -2, 6, -4), a_3 = (3, -1, 4, -2), a_4 = (6, -2, 8, -4).$$

4. Find some basis of a system of vectors and express the rest of the vectors of the system through this basis:

$$a) a_1 = (2, -1, 3, 5), a_2 = (4, -3, 1, 3), a_3 = (3, -2, 3, 4); a_4 = (4, -1, -15, 17)$$

$$b) a_1 = (2, 3, -4, -1), a_2 = (1, -2, 1, 3), a_3 = (5, -3, -1, 8); a_4 = (3, 8, -9, -5)$$

$$c) a_1 = (2, 2, 7, -1), a_2 = (3, -1, 2, 4), a_3 = (1, 1, 3, 1).$$

2). Prepare for an interview on the topics "Basic Algebraic Structures" and "Ring of Matrices over a Ring. Matrices Over a Field"

on the following issues :

Topic 1. Basic algebraic structures

1. Algebraic operation (definition, examples).
2. Basic properties of binary algebraic operations.
3. The concept of algebra as a set with algebraic operations. Examples of algebras.
4. The concept of a group (definition, examples). Subgroup. Basic properties of groups.
5. The concept of a ring (definition, examples). The main properties of rings.
6. Reversible elements of a ring with one. Multiplicative group of a ring.
7. The concept of a field. A field of rational and a field of real numbers.
8. Field of complex numbers (construction).
9. Algebraic form of a complex number and operations on complex numbers in algebraic form.
10. Geometric representation of complex numbers and operations with them.
11. Trigonometric form of writing complex numbers.
12. Operations on complex numbers in trigonometric form.
13. Extraction of n -degree roots from a complex number.
14. A group of n -degree roots from one.

Topic 2. Ring of matrices above the ring. Matrices above the field.

1. Matrices of the same type, equal, square. Zero matrix. Addition of matrices and its properties.
2. Additive group of real matrices of the same type.
3. Multiplication of a matrix by a number. Product of matrices. Properties of these operations.
4. Ring of square real matrices of n th order.
5. Permutations. Parity of permutation.
6. Substitutions. Parity of substitution.
7. Determinant of a square matrix of the n th order. Calculation of determinants of the 2nd and 3rd order.
8. Properties of determinants of a square matrix of the n th order.
9. Minor and the algebraic complement of the determinant element. Theorem on the product $a_{ij} A_{ij}$.
10. Decomposition of the determinant by elements of a row or column. Calculation of the determinant of the n th order.
11. Inverse matrix. Uniqueness of the inverse matrix.
12. Formula for calculating the inverse matrix. Conditions for the existence of the inverse matrix.
13. A group of square non-degenerate matrices of the n th order.
14. The rank of the matrix. The rank of the step matrix
15. Elementary transformations of matrices. Equivalent matrices.
16. Bringing the matrix to a stepped form.

17. Calculating the rank of the matrix using elementary transformations.
18. Arithmetic n -dimensional vectors.
19. Operations on arithmetic n -dimensional vectors.
20. Linear dependence of the system of arithmetic n -dimensional vectors.
21. Basis and rank of the system of arithmetic n -dimensional vectors.

3). Prepare for the interview on the completed homework on the topics "Basic algebraic structures" and "Ring of matrices over the ring. Matrices over the field"

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

Available at: <http://www.iprbookshop.ru/68568.html>

5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No 12.

Final lesson on the topics "Basic algebraic structures" and "Ring of matrices over a ring. Matrices over a field".

The purpose of the lesson: to test knowledge, skills and abilities on the topics "Basic algebraic structures" and "Ring of matrices over a ring. Matrices over a field"

The relevance of the lesson lies in the fact that during its conduct, students are

checked for the level of competence formation of OPK-1; knowledge, skills and abilities on the topics

"Basic Algebraic Structures" and "Ring of Matrices over a Ring. Matrices over a Field", which will become the basis of their professional activity.

In the lesson, students undergo:

1. Interview on the topics "Basic algebraic structures" and "Ring of matrices over the ring. Matrices over the field" on theoretical issues.
2. Interview on the completed homework on the topics "Basic algebraic structures" and "Ring of matrices over the ring. Matrices over the field"

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra.- 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

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Topic 3. Systems of linear equations over the field. Systems of linear inequalities.

Practical lesson No 13.

Basic concepts of the theory of solving systems of linear equations. Solving systems of linear equations in matrix form.

The purpose of the lesson: solving systems of linear equations in matrix form.

The relevance of the lesson lies in the fact that during its conduct, students form the competence of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Basic concepts of the theory of solving systems of linear equations
- Kronecker-Capelli theorem.
- A criterion for the certainty of a system of linear equations.
- Writing systems of linear equations in matrix form.
- Solution of systems of linear equations in matrix form.
- Conditions under which it is possible to solve a system of linear equations in matrix form.

2. Practical tasks solved in the classroom:

Solve systems of linear equations in matrix form:

$$\begin{array}{lll} \text{a)} \left\{ \begin{array}{l} 2x - y + 3z = 9, \\ 3x - 5y + z = -4, \\ 4x - 7y + z = 5. \end{array} \right. & \text{b)} \left\{ \begin{array}{l} 2x - y - 6z + 3t + 1 = 0, \\ x - 2y - 4z + 9t - 5 = 0, \\ 7x - 4y + 2z - 15t + 32 = 0, \\ x - y + 2z - 6t + 8 = 0. \end{array} \right. & \text{c)} \left\{ \begin{array}{l} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6, \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6. \end{array} \right. \\ \\ \text{d)} \left\{ \begin{array}{l} 2x_1 + 5x_2 + 4x_3 + x_4 = 20, \\ x_1 + 3x_2 + 2x_3 + x_4 = 11, \\ 2x_1 + x_2 + 3x_3 + 2x_4 = 6, \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37. \end{array} \right. & \text{e)} \left\{ \begin{array}{l} 7x_1 + 9x_2 + 4x_3 + 2x_4 = 0, \\ 2x_1 - 2x_2 + x_3 + x_4 - 6 = 0, \\ 5x_1 + 6x_2 + 3x_3 + 2x_4 - 3 = 0, \\ 2x_1 + 3x_2 + x_3 + x_4 = 0. \end{array} \right. & \end{array}$$

3. Homework:

Solve systems of linear equations in matrix form:

$$\begin{array}{ll} \text{a)} \left\{ \begin{array}{l} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6, \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6. \end{array} \right. & \text{b)} \left\{ \begin{array}{l} 3x_1 + 4x_2 + x_3 + 2x_4 + 3 = 0, \\ 3x_1 + 5x_2 + 3x_3 + 5x_4 + 6 = 0, \\ 6x_1 + 8x_2 + x_3 + 5x_4 + 8 = 0, \\ 3x_1 + 5x_2 + 3x_3 + 7x_4 + 8 = 0. \end{array} \right. \\ \\ \text{c)} \left\{ \begin{array}{l} 6x + 5y - 2z + 4t + 4 = 0, \\ 9x - u + 4z - t - 13 = 0, \\ 3x + 4y + 2z - 2t - 1 = 0, \\ 3x - 9y + 2t - 11 = 0. \end{array} \right. & \text{d)} \left\{ \begin{array}{l} 2x + y + 4z + 8t = -1, \\ x + 3y - 6z + 2t = 3, \\ 3x - 2y + 2z - 2t = 8, \\ 2x - y + 2z = 4. \end{array} \right. \\ & \text{e)} \left\{ \begin{array}{l} 2x - 5y + 3z + t = 5, \\ 3x - 7y + 3z - t = -1, \\ 5x - 9y + 6z + 4t = 7, \\ 4x - 6y + 3z + t = 8. \end{array} \right. \end{array}$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

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2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

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4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No 14.

Kramer's theorem on a system of n linear equations with n unknowns.

The purpose of the lesson: the application of Kramer's theorem to solve certain systems of n linear equations with n unknowns.

The relevance of the lesson lies in the fact that during its conduct, students form the competence of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

-Cramer's theorem on systems n of linear equations with n unknowns.

-Conditions under which Cramer's theorem applies.

2. Practical tasks solved in the classroom:

Solve a system of equations according to Kramer's rule:

$$\text{a) } \begin{cases} 2x - y + 3z = 9, \\ 3x - 5y + z = -4, \\ 4x - 7y + z = 5. \end{cases}$$

$$\text{b) } \begin{cases} 2x - y - 6z + 3t + 1 = 0, \\ x - 2y - 4z + 9t - 5 = 0, \\ 7x - 4y + 2z - 15t + 32 = 0, \\ x - y + 2z - 6t + 8 = 0. \end{cases}$$

$$\text{c) } \begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6, \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6. \end{cases}$$

$$d) \begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20, \\ x_1 + 3x_2 + 2x_3 + x_4 = 11, \\ 2x_1 + x_2 + 3x_3 + 2x_4 = 6, \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37. \end{cases}$$

$$e) \begin{cases} x_1 + 9x_2 + 4x_3 + 2x_4 - 2 = 0, \\ 2x_1 - 2x_2 + x_3 + x_4 - 6 = 0, \\ 5x_1 + 6x_2 + 3x_3 + 2x_4 - 3 = 0, \\ 2x_1 + 3x_2 + x_3 + x_4 = 0. \end{cases}$$

3. Homework:

Solve a system of equations according to Kramer's rule:

$$a) \begin{cases} 2x_1 + 2x_2 + 11x_3 + 5x_4 = 2, \\ x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ 2x_1 + x_2 + 3x_3 + 2x_4 = -3, \\ x_1 + x_2 + 3x_3 + 4x_4 = -3. \end{cases}$$

$$b) \begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 + 3 = 0, \\ 3x_1 + 5x_2 + 3x_3 + 5x_4 + 6 = 0, \\ 6x_1 + 8x_2 + x_3 + 5x_4 + 8 = 0, \\ 3x_1 + 5x_2 + 3x_3 + 7x_4 + 8 = 0. \end{cases}$$

$$c) \begin{cases} 6x + 5y - 2z + 4t + 4 = 0, \\ 9x - u + 4z - t - 13 = 0, \\ 3x + 4y + 2z - 2t - 1 = 0, \\ 3x - 9y + 2t - 11 = 0. \end{cases}$$

$$d) \begin{cases} 2x + y + 4z + 8t = -1, \\ x + 3y - 6z + 2t = 3, \\ 3x - 2y + 2z - 2t = 8, \\ 2x - y + 2z = 4. \end{cases}$$

$$e) \begin{cases} 2x - 5y + 3z + t = 5, \\ 3x - 7y + 3z - t = -1, \\ 5x - 9y + 6z + 4t = 7, \\ 4x - 6y + 3z + t = 8. \end{cases}$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

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2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

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3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

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4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No 15.

Solution of systems of linear equations by the Gaussian method.

The purpose of the lesson: solving systems of linear equations using the Gaussian method.

The relevance of the lesson lies in the fact that during its conduct, students form the competence of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson:

- Elementary transformations of systems of linear equations.
- Equivalent systems of linear equations.
- Gaussian method.

2. Practical tasks solved in the classroom:

Find a general solution and one partial solution of a system of linear equations using the Gaussian method:

$$\text{a) } \begin{cases} 5x_1 + 3x_2 + 5x_3 + 12x_4 = 10, \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 = 4, \\ x_1 + 7x_2 + 9x_3 + 4x_4 = 2; \end{cases}$$

$$\text{c) } \begin{cases} -6x_1 + 9x_2 + 3x_3 + 2x_4 = 4, \\ -2x_1 + 3x_2 + 5x_3 + 4x_4 = 2, \\ -4x_1 + 6x_2 + -4x_3 + 3x_4 = 3; \end{cases}$$

$$\text{b) } \begin{cases} -9x_1 + 10x_2 + 3x_3 + 7x_4 = 7, \\ -4x_1 + 7x_2 + x_3 + 3x_4 = 5, \\ 7x_1 + 5x_2 + -4x_3 - 6x_4 = 3; \end{cases}$$

$$\text{d) } \begin{cases} 2x_1 + 5x_2 - 8x_3 = 8, \\ 4x_1 + 3x_2 + -9x_3 = 9, \\ 2x_1 + 3x_2 - 5x_3 = 7, \\ x_1 + 8x_2 - 7x_3 = 12. \end{cases}$$

3. Homework:

Find a general solution and one partial solution of a system of linear equations using the Gaussian method:

$$\text{a) } \begin{cases} -9x_1 + 6x_2 + 7x_3 + 10x_4 = 3, \\ -6x_1 + 4x_2 + 2x_3 + 3x_4 = 2, \\ -3x_1 + 2x_2 - 11x_3 - 15x_4 = 1; \end{cases}$$

$$\text{b) } \begin{cases} 12x_1 + 9x_2 + 3x_3 + 10x_4 = 13, \\ 4x_1 + 3x_2 + x_3 + 2x_4 = 3, \\ 8x_1 + 6x_2 + 2x_3 + 3x_4 = 3; \end{cases}$$

$$\text{c) } \begin{cases} 8x_1 + 6x_2 + 5x_3 + 2x_4 = 21, \\ 3x_1 + 3x_2 + 2x_3 + x_4 = 10, \\ 4x_1 + 2x_2 + 3x_3 + x_4 = 8, \\ 3x_1 + 3x_2 + x_3 + x_4 = 15, \\ 7x_1 + 4x_2 + 5x_3 + 2x_4 = 18; \end{cases}$$

$$\text{d) } \begin{cases} 6x_1 + 4x_2 + 5x_3 + 2x_4 + 3x_5 = 1, \\ 3x_1 + 2x_2 - 2x_3 + x_4 = -7, \\ 9x_1 + 6x_2 + x_3 + 3x_4 + 2x_5 = 2, \\ 3x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 = 3. \end{cases}$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

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Practical lesson No 16.

Homogeneous systems of linear equations. A fundamental system of solutions to a homogeneous system of linear equations.

The purpose of the lesson: finding a fundamental system of solutions to a homogeneous system of linear equations.

The relevance of the lesson lies in the fact that during its conduct, students form the competence of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson

- Non-zero solutions of a homogeneous system of linear equations.
- Properties of solutions of a homogeneous system of linear equations.
- The basis of the set of solutions of a homogeneous system of linear equations.
- Finding a fundamental system of solutions to a homogeneous system of linear equations.
- Expression of the general solution of a system of homogeneous linear equations through a fundamental system of solutions.

2. Practical tasks solved in the classroom:

Find the fundamental system of solutions of a homogeneous system of linear equations and express the general solution of a system of homogeneous linear equations through the fundamental system of solutions:

$$a) \begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0, \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0, \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0, \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0. \end{cases}$$

$$b) \begin{cases} 3x_1 + 2x_2 + x_3 + 3x_4 + 5x_5 = 0, \\ 6x_1 + 4x_2 + 3x_3 + 5x_4 + 7x_5 = 0 \\ 9x_1 + 6x_2 + 5x_3 + 7x_4 + 9x_5 = 0 \\ 3x_1 + 2x_2 + 4x_4 + 8x_5 = 0. \end{cases}$$

$$c) \begin{cases} 6x_1 - 2x_2 + 2x_3 + 5x_4 + 7x_5 = 0, \\ 9x_1 - 3x_2 + 4x_3 + 8x_4 + 9x_5 = 0, \\ 6x_1 - 2x_2 + 6x_3 + 7x_4 + x_5 = 0, \\ 3x_1 - x_2 + 4x_3 + 4x_4 - x_5 = 0. \end{cases}$$

3. Homework:

Find a general solution and a fundamental system of solutions to a system of equations:

$$a) \begin{cases} x_1 + x_2 - 2x_3 + 2x_4 = 0, \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0, \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0, \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0. \end{cases}$$

$$b) \begin{cases} x_1 - x_3 = 0, \\ x_2 - x_4 = 0, \\ -x_1 + x_3 - x_5 = 0, \\ -x_2 + x_4 - x_6 = 0, \\ -x_3 + x_5 = 0, \\ -x_4 - x_6 = 0; \end{cases}$$

$$c) \begin{cases} x_1 - x_3 + x_5 = 0, \\ x_2 - x_4 + x_6 = 0, \\ x_1 - x_2 + x_5 - x_6 = 0, \\ x_1 - x_3 + x_6 = 0, \\ x_1 - x_4 + x_5 = 0; \end{cases}$$

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

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2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

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4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.
5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

Available at: <http://biblioclub.ru/index.php?page=book&id=68974>

Practical lesson No17.

Inhomogeneous systems of linear equations.

The purpose of the lesson: finding a general solution to a heterogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system.

The relevance of the lesson lies in the fact that during its conduct, students form the competence of OPK-1; knowledge, skills and abilities that will become the basis of their professional activity.

1. Theoretical questions for the practical lesson

- Associated systems of linear equations.
- Properties of solutions of an inhomogeneous system of linear equations.
- Finding a general solution to an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system.

2. Practical tasks solved in the classroom:

Find a general solution of an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system :

$$\begin{array}{ll}
 \text{a) } \begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6, \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4, \\ 9x_1 + x_2 + x_3 + 7x_4 = 2. \end{cases} & \text{b) } \begin{cases} 2x_1 - 3x_2 + 5x_3 + 7x_4 = 1, \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 = 2, \\ 2x_1 - 3x_2 - 11x_3 - 15x_4 = 1. \end{cases} \\
 \text{c) } \begin{cases} 2x_1 - 3x_2 + 5x_3 + 7x_4 = 1, \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 = 2, \\ 2x_1 - 3x_2 - 11x_3 - 15x_4 = 1. \end{cases} & \text{d) } \begin{cases} 2x_1 - x_2 + 3x_3 - 7x_4 = 5, \\ 6x_1 - 3x_2 + x_3 - 4x_4 = 7, \\ 4x_1 - 2x_2 + 14x_3 - 31x_4 = 18. \end{cases} \\
 \text{e) } \begin{cases} 9x_1 - 3x_2 + 5x_3 + 6x_4 = 4, \\ 6x_1 - 2x_2 + 3x_3 + x_4 = 5, \\ 3x_1 - x_2 + 3x_3 + 14x_4 = -8. \end{cases} &
 \end{array}$$

3. Homework:

- 1). Find a general solution to an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system:

$$\text{a) } \begin{cases} 5x_1 + 3x_2 + 5x_3 + 12x_4 = 10, \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 = 4, \\ x_1 + 7x_2 + 9x_3 + 4x_4 = 2; \end{cases}$$

$$\text{c) } \begin{cases} -6x_1 + 9x_2 + 3x_3 + 2x_4 = 4, \\ -2x_1 + 3x_2 + 5x_3 + 4x_4 = 2, \\ -4x_1 + 6x_2 + -4x_3 + 3x_4 = 3; \end{cases}$$

$$\text{b) } \begin{cases} 12x_1 + 9x_2 + 3x_3 + 10x_4 = 13, \\ 4x_1 + 3x_2 + x_3 + 2x_4 = 3, \\ 8x_1 + 6x_2 + 2x_3 + 3x_4 = 3; \end{cases}$$

$$\text{d) } \begin{cases} 6x_1 + 4x_2 + 5x_3 + 2x_4 + 3x_5 = 1, \\ 3x_1 + 2x_2 - 2x_3 + x_4 = -7, \\ 9x_1 + 6x_2 + x_3 + 3x_4 + 2x_5 = 2, \\ 3x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 = 3. \end{cases}$$

2). Prepare for the interview on the topic " Systems of linear equations over the field. Systems of linear inequalities ".

on the following issues:

Topic 3. Systems of linear equations over the field. Systems of linear inequalities.

1. Basic concepts of the theory of solving systems of linear equations
2. Kronecker-Capelli theorem.
3. The criterion of definiteness of a system of linear equations.
4. Writing and solving systems of linear equations in matrix form.
5. Conditions under which it is possible to solve a system of linear equations in matrix form.
6. Cramer's theorem on systems n of linear equations with n unknowns.
7. Conditions under which Kramer's theorem applies.
8. Elementary transformations of systems of linear equations.
9. Theorem on equistrong systems of linear equations.
10. Gaussian method.
11. Properties of solutions of a homogeneous system of linear equations.
12. The basis of the set of solutions of a homogeneous system of linear equations.
13. Finding a fundamental system of solutions to a homogeneous system of linear equations.
14. Expression of the general solution of a system of homogeneous linear equations through a fundamental system of solutions.
15. Associated systems of linear equations.
16. Properties of solutions of an inhomogeneous system of linear equations.
17. Finding a general solution of an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system.
18. Basic concepts of the theory of solving systems of linear inequalities.
19. Non-negative solutions of a joint system of linear equations over the field of real numbers.
20. A consequence of the system of linear inequalities.
21. Minkowski's theorem
22. Criterion of the coherence of the system of linear inequalities.

3) Prepare for the interview on the completed homework on the topic " Systems of linear equations over the field. Systems of linear inequalities ".

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

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2. Further reading :

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Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

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5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.

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Practical lesson No18.

Final lesson on the topic "Systems of linear equations over the field. Systems of linear inequalities".

Purpose of the lesson: Testing theoretical knowledge and practical skills on the topic "Systems of linear equations over the field. Systems of linear inequalities".

The relevance of the lesson lies in the fact that during its conduct, students are checked the level of competence formation of OPK-1; knowledge, skills and abilities on the topic " Systems of linear equations over the field. Systems of linear inequalities", which will become the basis of their professional activity.

In the lesson, students undergo:

1. Interview on the topic " Systems of linear equations over the field. Systems of linear inequalities " on theoretical issues.

2. Interview on completed homework on the topic

" Systems of linear equations over a field. Systems of linear inequalities ".

Recommended reading

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

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2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra.- 368 p. - ISBN 978-5-94057-454-5.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63144>

3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

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FEDERATIONS
FEDERAL STATE AUTONOMOUS EDUCATIONAL
INSTITUTION OF HIGHER EDUCATION
"NORTH CAUCASUS FEDERAL UNIVERSITY"

SUGGESTIONS FOR TEACHING
on the organization and conduct of independent work
in the discipline "Linear Algebra"
for students of the field of study
11.03.02 Infocommunication Technologies and Communication Systems
Profile
"Intelligent Data Processing in Infocommunication
Systems and Networks"

Stavropol 2023

Table of contents.

Introduction

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2. Methodical recommendations for the study of theoretical material, preparation for practical classes.
3. Interview questions.
4. Methodical recommendations for preparing for the exam.
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6. Tasks for preparing for the exam.
7. Recommended literature.

Introduction.

Independent work is the planned educational, research, research work of students, carried out in extracurricular (classroom) time on the assignment and under the methodological guidance of the teacher, but without his direct participation (with the partial direct participation of the teacher, leaving the leading role behind the student's work).

Independent work of a student is an important type of educational activity of a student. The Federal State Educational Standard provides for a significant amount of time from the total labor intensity of the discipline for independent work. In this regard, training includes two parts that are almost the same in volume and mutual influence - the learning process and the process of self-learning. Therefore, independent work should become an effective and purposeful work of the student.

The concept of modernization of Russian education defines the main tasks of vocational education - the training of a qualified worker of the appropriate level and profile, competitive in the labor market, competent, responsible, fluent in his profession and oriented in related fields of activity, capable of effective work in the specialty at the level of world standards, ready for constant professional growth, social and professional mobility.

The solution of these problems is impossible without increasing the role of independent work of students on educational material.

Independent work of a student presupposes, first of all, his practical activity, since it is this type of educational work that first of all prepares for the independent performance of professional tasks. The ultimate goal of independent work is the organization of independent cognitive activity, the formation of the ability to independently analyze information, to distinguish the main and secondary.

The main forms of independent work of students in the discipline "Algebra" are:

- preparation for lectures;
- preparation for practical classes;
- preparation for the exam.

In the process of studying the discipline, students form the following competence (in accordance with the educational standard):

OPK-1 - Able to use the provisions, laws and methods of natural sciences and mathematics to solve problems of engineering activity.

1. Schedule of independent work.

Type of student activity	Final product of independent work	Assessment Tools and Technologies	Delivery time
Preparation for lectures on the topic "Basic algebraic structures"	Synopsis	Interview.	Practical lesson No 12
Preparation for practical classes on the topic "Basic algebraic structures"	Performing written tasks for independent work at home.	Interview.	Practical lesson No 12
Preparation for lectures on the topic "Matrices over the ring. Matrices over the field"	Synopsis	Interview.	Practical lesson No 12.
Preparation for practical classes on the topic "Matrices above the ring. Matrices over the field"	Performing written tasks for independent work at home.	Interview.	Practical lesson No 12
Preparation for lectures on the topic "Systems of linear equations over the field. Systems of linear inequalities".	Synopsis	Interview.	Practical lesson No 18
Preparation for practical classes on the topic "Systems of linear equations over a field. Systems of linear inequalities".	Performing written tasks for independent work at home.	Interview.	Practical lesson No 18
Preparation for the exam.	Answer on the exam.	Exam Score	Exam

2. Methodical recommendations for the study of theoretical material, preparation for practical classes.

To prepare for the current practical lesson

At home

1. Read the lecture notes corresponding to the topic of the lesson. Memorize the basic definitions and theorems.
2. Study the recommended literature on the issues to be discussed.
3. Find answers to theoretical tasks. Prepare to answer these questions in class.
4. Take notes of the answers to theoretical tasks that are not contained in your lecture notes on this problem.
5. Study the analyzed examples of solving typical problems and take notes on their solution in a workbook.

In the classroom

6. Give answers to questions from theoretical tasks and solve practical tasks from the tasks solved in the classroom in the workbook.

At home

7. Consolidate the acquired practical skills and abilities by solving practical tasks from tasks for independent work at home.

To prepare for the final practical lesson on the topic

Houses:

1. Prepare for an interview on the topic on the specified theoretical questions of the basic and advanced level.
2. Prepare for the interview on the completed homework on the topic.

3. Interview questions.

Topic 1. Basic algebraic structures

1. Algebraic operation (definition, examples).
2. Basic properties of binary algebraic operations.
3. The concept of algebra as a set with algebraic operations. Examples of algebras.
4. The concept of a group (definition, examples). Subgroup. Basic properties of groups.
5. The concept of a ring (definition, examples). The main properties of rings.
6. Reversible elements of a ring with one. Multiplicative group of a ring.
7. The concept of a field. A field of rational and a field of real numbers.
8. Field of complex numbers (construction).
9. Algebraic form of a complex number and operations on complex numbers in algebraic form.
10. Geometric representation of complex numbers and operations with them.
11. Trigonometric form of writing complex numbers.
12. Operations on complex numbers in trigonometric form.
13. Extraction of n -degree roots from a complex number.
14. A group of n -degree roots from one.

Topic 2. Ring of matrices above the ring. Matrices above the field.

1. Matrices of the same type, equal, square. Zero matrix. Addition of matrices and its properties.
2. Additive group of real matrices of the same type.
3. Multiplication of a matrix by a number. Product of matrices. Properties of these operations.
4. Ring of square real matrices of n th order.
5. Permutations. Parity of permutation.
6. Substitutions. Parity of substitution.
7. Determinant of a square matrix of the n th order. Calculation of determinants of the 2nd and 3rd order.
8. Properties of determinants of a square matrix of the n th order.
9. Minor and the algebraic complement of the determinant element. Theorem on the product $a_{ij} A_{ij}$.
10. Decomposition of the determinant by elements of a row or column. Calculation of the determinant of the n th order.
11. Inverse matrix. Uniqueness of the inverse matrix.
12. Formula for calculating the inverse matrix. Conditions for the existence of the inverse matrix.
13. A group of square non-degenerate matrices of the n th order.

14. The rank of the matrix. The rank of the step matrix
15. Elementary transformations of matrices. Equivalent matrices.
16. Bringing the matrix to a stepped form. .
17. Calculating the rank of the matrix using elementary transformations.
18. Arithmetic n-dimensional vectors.
19. Operations on arithmetic n-dimensional vectors.
20. Linear dependence of the system of arithmetic n-dimensional vectors.
21. Basis and rank of the system of arithmetic n-dimensional vectors.

Topic 3. Systems of linear equations over the field. Systems of linear inequalities.

1. Basic concepts of the theory of solving systems of linear equations
2. Kronecker-Capelli theorem.
3. The criterion of definiteness of a system of linear equations.
4. Writing and solving systems of linear equations in matrix form.
5. Conditions under which it is possible to solve a system of linear equations in matrix form.
6. Cramer's theorem on systems n of linear equations with n unknowns.
7. Conditions under which Kramer's theorem applies.
8. Elementary transformations of systems of linear equations.
9. Theorem on equistrong systems of linear equations.
10. Gaussian method.
11. Properties of solutions of a homogeneous system of linear equations.
12. The basis of the set of solutions of a homogeneous system of linear equations.
13. Finding a fundamental system of solutions to a homogeneous system of linear equations.
14. Expression of the general solution of a system of homogeneous linear equations through a fundamental system of solutions.
15. Associated systems of linear equations.
16. Properties of solutions of an inhomogeneous system of linear equations.
17. Finding a general solution of an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system.
18. Basic concepts of the theory of solving systems of linear inequalities.
19. Non-negative solutions of a joint system of linear equations over the field of real numbers.
20. A consequence of the system of linear inequalities.
21. Minkowski's theorem
22. Criterion of the coherence of the system of linear inequalities.

4. Methodical recommendations for preparing for the exam.

Work through the theoretical material on the questions for the basic and advanced exams.

At the same time, pay special attention to the

- accuracy of the formulation of definitions and theorems;
- the scope of application of methods and rules;
- the procedure for executing algorithms;
- mathematical symbols used in the topics submitted to the exam.

Be sure to set aside time to repeat the methods of solving typical tasks on the material submitted to the exam. At the same time, use the tasks proposed for preparing for the exam.

5. Questions for the exam.

1. Algebraic operation (definition, examples). Basic properties of binary algebraic operations.
2. The concept of algebra as a set with algebraic operations. Examples of algebras.

3. The concept of a group (definition, examples). Semi-group. Basic properties of groups.
4. The concept of a ring (definition, examples). Basic properties of rings. Reversible elements of a ring with one. Multiplicative group of a ring.
5. The concept of a field. A field of rational and a field of real numbers.
6. Field of complex numbers (construction).
7. The algebraic form of a complex number and operations on complex numbers in algebraic form.
8. Geometric representation of complex numbers and operations with them.
9. Trigonometric form of writing complex numbers. Operations on complex numbers in trigonometric form.
10. Extraction of n -power roots from a complex number. A group of n -degrees roots n from unity.
11. Matrices of the same type, equal, square. Zero matrix. Addition of matrices and its properties. Additive group of real matrices of the same type.
12. Multiplication of a matrix by a number. Product of matrices. Properties of these operations. Ring of quadratic real matrices n of n -th order.
13. Determinant of the quadratic matrix n of the n -th order. Calculation of determinants of the 2nd and 3rd order.
14. Properties of determinants of a quadratic matrix n of n -th order.
15. Minor and algebraic complement of determinant element. Product theorem. $a_{ij}A_{ij}$. Decomposition of determinant by elements of a row or column. Calculation of determinant n of n -th order.
16. Inverse matrix. Uniqueness of an inverse matrix. Formula for calculating an inverse matrix. Conditions for the existence of an inverse matrix. A group of square non-degenerate matrices n of n -th order.
17. Elementary transformations of matrices. Equivalent matrices. Reduction of the matrix to a stepped form.
18. Matrix rank. Step matrix rank. Calculate the matrix rank using elementary transformations.
19. Arithmetic-dimensional n vectors. Operations on arithmetic-dimensional n vectors.
20. Linear dependence of the system of arithmetic n -dimensional vectors. Basis and rank of the system of arithmetic n -dimensional vectors.
21. Basic concepts of the theory of solving systems of linear equations. Kronecker-Capelli theorem. Criterion of definiteness of a system of linear equations.
22. Writing and solving systems of linear equations in matrix form. Conditions under which it is possible to solve a system of linear equations in matrix form.
23. Cramer's theorem on systems n of linear equations with n unknowns. Conditions under which Cramer's theorem applies.
24. Elementary transformations of systems of linear equations. Theorem on equivalent systems of linear equations. Gaussian method.
25. Properties of solutions of a homogeneous system of linear equations. Basis of the set of solutions of a homogeneous system of linear equations.
26. Finding the fundamental system of solutions of a homogeneous system of linear equations. Expressing the general solution of a system of homogeneous linear equations through a fundamental system of solutions.
27. Associated systems of linear equations. Properties of solutions of an inhomogeneous system of linear equations.

28. Finding a general solution to an inhomogeneous system of linear equations using the fundamental system of solutions of the corresponding homogeneous system.
29. Nonnegative solutions of a joint system of linear equations over a field of real numbers.
30. Basic concepts of the theory of solving systems of linear inequalities. A consequence of the system of linear inequalities.
31. Minkowski's theorem. Criterion for the coherence of a system of linear inequalities.

7. Tasks for preparing for the exam.

1. Is multiplying a vector by a scalar a binary algebraic operation?
2. Is it a binary algebraic operation to find the arithmetic mean of two integers?
3. Is it a binary algebraic operation to find the arithmetic mean of two rational numbers?
4. Does the multiplication group form a set of all powers of the number 2 with integers?
5. Does a multiplication group form a set of numbers of the form $b + c\sqrt{3}$, where b and c are integers?
6. Determine whether a set of numbers of the form is a ring in relation to the usual operations of addition and multiplication $1, \pm 3, \pm 3^2, \dots, \pm 3^n, \dots$
7. Determine whether a set of numbers of the form is a ring in relation to the usual operations of addition and multiplication $0, 1^2, 2^2, \dots, n^2, \dots$
8. Is the set of numbers of the form where $b + c\sqrt{3}$, and b and c are integers a field relative to addition and multiplication?
9. Is a field with respect to addition and multiplication a set of numbers of the form $b + c\sqrt{3}$, where b and c are rational numbers?
10. Calculate $\frac{(3-i)^2}{1+i} + 3 + 5i$.
11. Solve equation $|z| + z = 8 + 4i$.
12. Calculate $(\sqrt{3} + i)^{30}$.
13. Calculate $\frac{(-3-3i)^{20}}{(4-4i\sqrt{3})^{12}}$.
14. Calculate $\sqrt[6]{-27}$.
15. Calculate $\sqrt[4]{-\frac{18}{1+i\sqrt{3}}}$.
16. Compute the products AB of and BA , if $A = \begin{pmatrix} 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 4 & 5 & 6 & 1 \\ 2 & 1 & 1 & 1 \end{pmatrix}$.
17. Are the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$?
18. Are the matrices $A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -10 \\ -15 & 22 \end{pmatrix}$?
19. Calculate $\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{pmatrix}^2$.

20. Calculate $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & -2 & 3 \end{pmatrix}^3$.

21. Calculate the value of a polynomial $f(x) = x^3 - 2x + 1$ from a matrix $A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$.

22. Choose i and k so that the work $a_{1i}a_{32}a_{4k}a_{25}a_{53}$ is included in the 5th order determinant with a plus sign.

23. Choose i and k so that the product $a_{1i}a_{32}a_{4k}a_{25}a_{53}$ is included in the 5th order determinant with a minus sign.

24. Calculate determinant $\begin{vmatrix} 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{vmatrix}$.

25. Calculate determinant $\begin{vmatrix} 3 & -3 & -2 & -5 \\ 2 & 5 & -4 & 6 \\ 5 & 5 & 8 & 7 \\ 4 & 4 & 5 & 8 \end{vmatrix}$.

26. Calculate determinant $\begin{vmatrix} 4 & 0 & 3 & 5 \\ -2 & 2 & -2 & 0 \\ 1 & -1 & 0 & 3 \\ 3 & 1 & 4 & -2 \end{vmatrix}$.

27. Calculate determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$.

28. Calculate determinant $\begin{vmatrix} 3 & 1 & 4 & -2 \\ -2 & 2 & -2 & 0 \\ 1 & -1 & 0 & 3 \\ 4 & 0 & 3 & 5 \end{vmatrix}$.

29. Using the attached matrix, find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$.

30. Find the inverse matrix for $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$.

31. Find the inverse matrix for $A = \begin{pmatrix} 2 & 2 & 3 \\ -1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$.

32. Solve the matrix equation $X \begin{pmatrix} -1 & 1 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 3 & 4 \end{pmatrix}$.

33. Solve the matrix equation $\begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} X = \begin{pmatrix} 3 & 6 \\ 5 & 10 \end{pmatrix}$.

34. Solve the matrix equation $\begin{pmatrix} -1 & 1 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} -2 & -1 \\ 3 & 4 \end{pmatrix}$.

35. Solve the matrix equation $X \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 5 & 10 \end{pmatrix}$.

36. Find the rank of the matrix $\begin{pmatrix} -1 & 1 & 3 & 5 \\ 7 & 5 & 1 & -1 \\ 4 & 2 & -1 & -3 \\ 1 & 7 & 7 & 9 \end{pmatrix}$.

37. To clarify the question of the linear dependence of the system of vectors $a_1 = [3, -6, 10]$, $a_2 = [2, -5, 8]$, $a_3 = [1, -3, 5]$

38. To clarify the question of the linear dependence of the system of vectors $a_1 = [1, 3, 6, 10]$, $a_2 = [1, 1, 1, 1]$, $a_3 = [1, -3, -8, -18]$, $a_4 = [3, 5, 8, 12]$

39. Find the basis and rank of the system of vectors $a_1 = [3, 1, 1, 1]$, $a_2 = [1, 1, 3, 1]$, $a_3 = [1, 3, -2, 3]$, $a_4 = [1, 3, 1, 1]$

40. Find the basis and rank of the system of vectors $a = [3, 1, 1, 1]$, $b = [1, 1, 3, 1]$, $c = [1, 3, 1, 1]$, $d = [-1, 1, 5, 1]$ and express the rest of the vectors of the system through the basis.

41. Find the basis and rank of the system of vectors $a_1 = [1, 3, 6, 10]$, $a_2 = [1, 1, 1, 1]$, $a_3 = [1, -3, -8, -18]$, $a_4 = [3, 5, 8, 12]$ and express the rest of the vectors of the system through the basis.

42. How many solutions does a system of equations have $\begin{cases} 3x_1 + 4x_2 + 5x_3 + 7x_4 = 1, \\ 2x_1 + 6x_2 - 3x_3 + 4x_4 = 2, \\ 4x_1 + 2x_2 + 13x_3 + 10x_4 = 0, \\ 5x_1 + 21x_3 + 13x_4 = 3. \end{cases} ?$

43. Check whether the system of equations is compatible? $\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1, \\ 3x_1 - 2x_2 + 2x_3 - 3x_4 = 2, \\ 5x_1 + x_2 - x_3 + 2x_4 = -1, \\ 2x_1 - x_2 + x_3 - 3x_4 = 4. \end{cases}$

44. Solve the system in a matrix way $\begin{cases} x_1 - 3x_2 - 4x_3 = 4, \\ 2x_1 + x_2 - 3x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = 11. \end{cases}$

45. Solve the system in a matrix way $\begin{cases} x_1 + 5x_2 - x_3 = 3, \\ 2x_1 + 4x_2 - 3x_3 = 2, \\ 3x_1 - x_2 - 3x_3 = -7. \end{cases}$

46. Solve the system in a matrix way $\begin{cases} x_1 - 3x_2 - 4x_3 = 4, \\ 2x_1 + x_2 - 3x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = 11. \end{cases}$

47. Using Cramer's theorem, find out whether the system of equations has

$$\begin{cases} 3x_1 + 4x_2 + 5x_3 + 7x_4 = 1, \\ 2x_1 + 6x_2 - 3x_3 + 4x_4 = 2, \\ 4x_1 + 2x_2 + 13x_3 + 10x_4 = 0, \\ 5x_1 + 21x_3 + 13x_4 = 3. \end{cases}$$
 a single solution.
48. Using Cramer's theorem, find out whether a system of linear equations has a single solution

$$\begin{cases} 3x_1 + 3x_2 + 4x_3 - 5x_4 = 9, \\ 5x_1 - 7x_2 + 8x_3 + 2x_4 = 18, \\ 4x_1 + 5x_2 - 7x_3 - 3x_4 = -5, \\ 7x_1 + 8x_2 + 3x_3 + 4x_4 = -2. \end{cases}$$
49. Solve the system using Cramer's formulas
$$\begin{cases} 2x_1 - x_2 + 2x_3 = -4, \\ 2x_1 + x_2 + 3x_3 = -2, \\ x_1 + x_2 + 2x_3 = -1. \end{cases}$$
50. Solve the system using Cramer's formulas
$$\begin{cases} x_1 - 3x_2 - 4x_3 = 4, \\ 2x_1 + x_2 - 3x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = 11. \end{cases}$$
51. Solve the system using Cramer's formulas
$$\begin{cases} 2x_1 - x_2 + 2x_3 = -4, \\ 2x_1 + x_2 + 3x_3 = -2, \\ x_1 + x_2 + 2x_3 = -1. \end{cases}$$
52. Solve a system of linear equations
$$\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 1, \\ x_2 - x_3 + x_4 = -3, \\ x_1 - 3x_2 - 3x_4 = 1, \\ -7x_1 + 3x_3 + x_4 = -3. \end{cases}$$
53. Solve a system of linear equations
$$\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1, \\ 3x_1 - 2x_2 + 2x_3 - 3x_4 = 2, \\ 5x_1 + x_2 + x_3 + 2x_4 = -1, \\ 2x_1 - x_2 + x_3 - 3x_4 = 4. \end{cases}$$
54. Find a general solution to a system of equations
$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 6, \\ x_1 - x_2 + x_3 - x_4 + 2x_5 = 8. \end{cases}$$
55. Find a fundamental system of system solutions
$$\begin{cases} 2x_1 - x_2 + 5x_3 + 7x_4 = 0, \\ 4x_1 - 2x_2 + 7x_3 + 5x_4 = 0, \\ 2x_1 - x_2 - x_3 - 5x_4 = 0. \end{cases}$$
56. Find a fundamental system of solutions and a general solution to a system of equations

$$\begin{cases} x_1 - 2x_2 + x_3 + x_4 = 0, \\ 5x_1 - x_2 - 13x_3 - 4x_4 = 0, \\ 2x_1 + 5x_2 - 16x_3 - 7x_4 = 0. \end{cases}$$

57. To find a general solution of a system of linear equations using a fundamental system of solutions of a corresponding homogeneous system of linear equations
$$\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 1, \\ x_2 - x_3 + x_4 = -3, \\ x_1 + 3x_2 - 3x_4 = 1. \end{cases}$$

8. Recommended literature

1. Reference citations:

1. Vinberg E.B. Course of Algebra: Textbook / E.B. Vinberg. - Moscow: MCNMO, 2021. - 591 p. - ISBN 978-5-94057-685-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63299>

2. Remizov A.O., Shafarevich I.R. Linear Algebra and Geometry: Textbook. - Moscow: Fizmatlit, 2019. - 512 p. - ISBN 978-5-9221-1139-3.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=68387>

2. Further reading :

1. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 1. Fundamentals of Algebra. - 273 p. - ISBN 978-5-94057-453-8.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=63140>

2. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 2. Linear Algebra. - 368 p. - ISBN 978-5-94057-454-5.

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3. Kostrikin A.I. Introduction to Algebra: Textbook / A.I. Kostrikin. - Moscow: MCNMO, 2020. - Part 3. Basic Structures of Algebra. - 272 p. - ISBN 978-5-94057-455-2.

Mode of access: <http://biblioclub.ru/index.php?page=book&id=62951>

4. Chegolin A.P. Linear algebra and analytical geometry [Elektronnyi resurs]: uchebnoe posobie [Linear algebra and analytical geometry]: a textbook. - Electronic text data. - Rostov-on-Don: Southern Federal University, 2018. - 149 p. - ISBN 978-5-9275-1728-2.

Available at: <http://www.iprbookshop.ru/68568.html>

5. Ilyin V.A., Poznyak E.G. Linear Algebra: Textbook / V.A. Ilyin, E.G. Poznyak. - 6th ed., stereotype. - Moscow: Fizmatlit, 2021. - 278 p. - (Course of Higher Mathematics and Mathematical Physics. Issue 4). - ISBN 978-5-9221-0481-4.